EFFECTIVE DYNAMIC ISO-SPHERE INOPIN HOLOGRAPHIC RINGS: INQUIRY AND HYPOTHESIS

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Abstract

In this preliminary work, we focus on a particular iso-geometrical, iso-topological facet of iso-mathematics by suggesting a developing, generalized approach for encoding the states and transitions of spherically-symmetric structures that vary in size. In particular, we introduce the notion of "effective iso-radius" to facilitate a heightened characterization of dynamic iso-sphere Inopin holographic rings (IHR) as they undergo "iso-transitions" between "iso-states". In essence, we propose the existence of "effective dynamic iso-sphere IHRs". In turn, this emergence drives the construction of a new "effective iso-state" platform to encode the generalized dynamics of such iso-complex, non-linear systems in a relatively straightforward approach of spherical-based iso-topic liftings. The initial results of this analysis are significant because they lead to alternative modes of research and application, and thereby pose the question: do these effective dynamic iso-sphere IHRs have application in physics and chemistry? Our hypothesis is: yes. To answer this inquiry and assess this conjecture, this developing work should be subjected to further scrutiny, collaboration, improvement, and hard work via the scientific method in order to advance it as such.

Keywords: Geometry and topology; Santilli iso-number; Inopin holographic ring; Iso-radius; Iso-sphere; Dynamic iso-sphere; Effective iso-state.

To the memory and honor of Dr. Andrej "Andy" Inopin.

1 Introduction

The new discipline of *Santilli iso-mathematics* [1, 2, 3, 4, 5] has sparked a revolution in the realm of *universal number classification*. More recently, Santilli also generalized his iso-mathematics to create geno-mathematics and hyper-mathematics [1, 2, 3, 4, 5], which are the heart of hadronic mechanics [6, 7] and state-of-the-art, clean, sustainable, industrial-strength energy sources such as MagneGas Fuel [8, 9, 10, 11, 12, 13, 14, 15] and Intermediate Controlled Nuclear Fusion (Synthesis) [16, 17, 18, 19]. This frontier continues to be explored and expanded, where additional efforts have deployed such iso-topic liftings to initiate, for example, new developments toward a *4D topological iso-string theory* [20], *iso-fractals* such as *Mandelbrot iso-sets* [21, 22], and the *iso-dual tesseract* [23]. Furthermore, similar such rigorous examinations of possible iso-mathematics applications have expedited the *dynamic* iso-topic lifting of iso-spaces to install *dynamic iso-spaces* [24], which form the foundation of dynamic iso-sphere IHRs with the "built-in" exterior and interior inverse iso-duality [25].

In this assignment, we focus on advancing the representation of dynamic *iso-1-sphere* IHRs [25] by forging the *effective iso-radius* ("effective iso-modulus" or "effective iso-amplitude-radius") platform to launch the encoding of their characteristic "iso-transitions" between "iso-states" as they vary in size. The effective iso-radius concept introduced in this paper was originally inspired by the "effective radius" concept from Corda's new framework of black hole effective states [26, 27, 28, 29, 30]. However, this paper is devoted to iso-mathematics rather than physics; thus, the effective representation proposed here targets spherically-symmetric iso-mathematical structures (like IHRs) rather than spherically-symmetric physical structures (like black holes or hadrons). Hence, for now, we limit our investigation to the domain of iso-mathematics [1, 2, 3, 4, 5] but recognize the hypothesis that such dynamic iso-sphere IHRs may possibly be applied to certain aspects of physics in the future. Thus, we launch our investigation with the step-by-step procedural analysis of Section 2 by presenting a systematic construction of the effective iso-radius for a dynamic iso-1-sphere IHR [25] to initiate the characterization of effective iso-states and iso-transitions for dynamic iso-topic liftings [24]—the new *effective dynamic iso-sphere IHR* is submitted. Finally, we conclude with Section 3, where we recapitulate the results of Section 2 with a brief discussion and suggest future modes of research.

2 Procedure

In this section, we define and assemble the effective iso-radius for a dynamic iso-sphere IHR [25] and thereby spark the notion of effective iso-state.

2.1 Initializing the 1-sphere IHR topology

Here, we consider the “iso-1-sphere base case” by instantiating the 1-sphere IHR topology [22, 31, 32, 33] via the following procedure:

1. First, from eq. (7) of [22], let $X = \mathbb{C}$ be the set of all complex numbers, the Euclidean complex space, and the *dual 2D Cartesian-polar coordinate-vector state space*, where the complex number $\vec{x} \in X$ is a *dual 2D Cartesian-polar coordinate-vector state* [22, 31, 32, 33] that is defined by eq. (6) of [22] as

$$x = \vec{x} = \vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}} = (\vec{x}) = (|\vec{x}|, \langle\vec{x}\rangle)_P = (\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C, \quad \forall \vec{x} \in X. \tag{1}$$

   In eq. (1), $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C$ is a *2D Cartesian coordinate-vector state* in the *2D Cartesian coordinate-vector state space* X_C so $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C \in X_C$, while $(|\vec{x}|, \langle\vec{x}\rangle)_P$ is a *2D polar coordinate-vector state* in the *2D polar coordinate-vector state space* X_P so $(|\vec{x}|, \langle\vec{x}\rangle)_P \in X_P$, where X_C and X_P are iso-morphic, dual, synchronized, and interlocking in X [22, 31, 32, 33]. Thus, eq. (1) complies with the constraints imposed by eqs. (8–13) of [22]—see Figure 1.

2. Second, from eq. (16) of [22] we have

$$T^1 = \{\vec{x} \in X : |\vec{x}| = r\}, \tag{2}$$

   where $T^1 \subset X$ is the 1-sphere IHR of amplitude-radius $r > 0$ (with corresponding curvature $\kappa = \frac{1}{r}$) that is centered on the origin $O \in X$ [22, 31, 32, 33]; T^1 is the multiplicative group of all non-zero complex

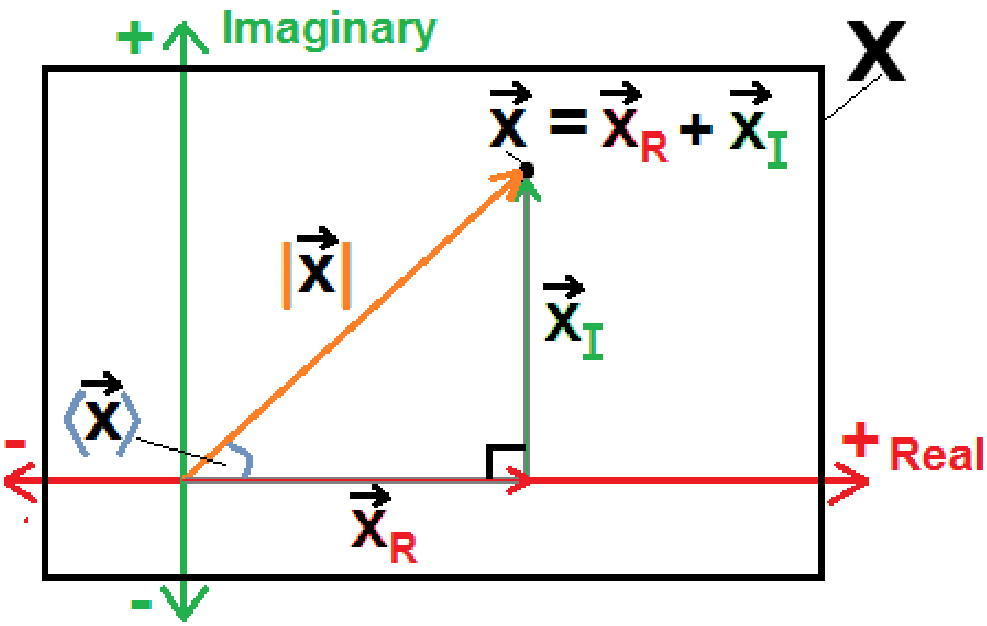

Fig. 1: Complex components for the dual 2D Cartesian-polar coordinate-vector state $\vec{x}$ in the dual 2D Cartesian-polar coordinate-vector state space (and Euclidean complex space) X, such that $\vec{x} \in X$, where $\vec{x}$ is simultaneously treated as a complex number, 2D polar coordinate-vector, and 2D Cartesian coordinate-vector [22, 31, 32, 33]. Specifically, $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C$ is a 2D Cartesian coordinate-vector state in the 2D Cartesian coordinate-vector state space X_C so $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C \in X_C$, while $(|\vec{x}|, \langle\vec{x}\rangle)_P$ is a 2D polar coordinate-vector state in the 2D polar coordinate-vector state space X_P so $(|\vec{x}|, \langle\vec{x}\rangle)_P \in X_P$, where X_C and X_P are iso-morphic, dual, synchronized, and interlocking in X [22, 31, 32, 33]. Note that $\vec{x}_{\mathbb{R}}$ and $\vec{x}_{\mathbb{I}}$ are treated as vectors (with axis-dependent magnitude and direction) so the vector sum is $\vec{x} = \vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}}$ with amplitude $|\vec{x}|$ and direction $\langle\vec{x}\rangle$ [22, 31, 32, 33].

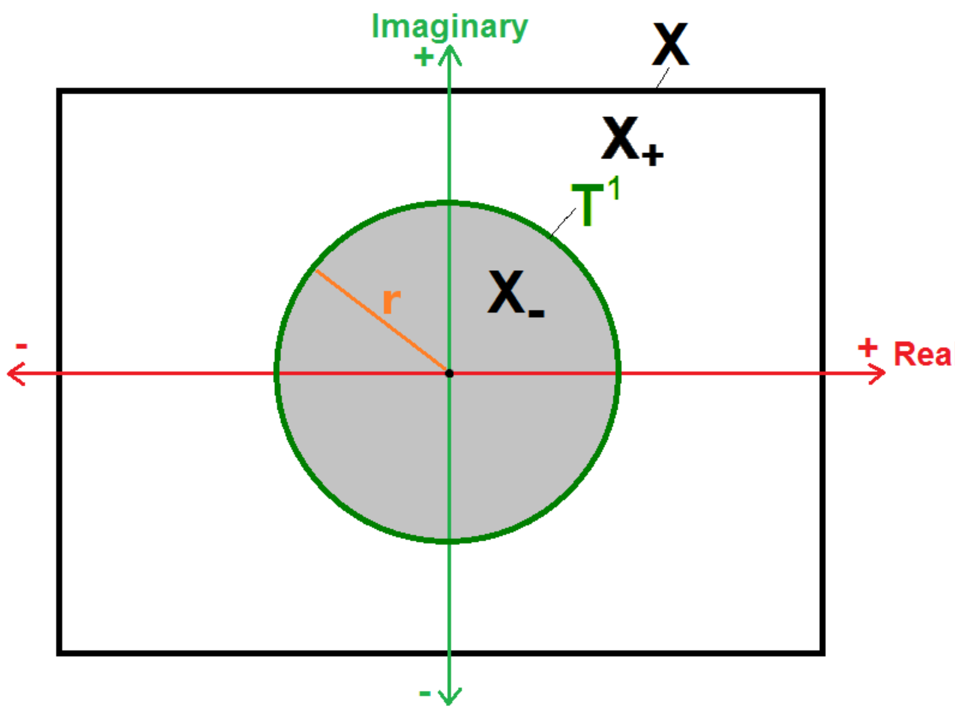

Fig. 2: The 1-sphere IHR topology for the dual 2D Cartesian-polar coordinate-vector state space (and Euclidean complex space) X, where the topological 1-sphere IHR $T^1 \subset X$ is simultaneously dual to two spatial 2-branes [22, 31, 32, 33]: the "2D micro sub-space zone" $X_- \subset X$ and the "2D macro sub-space zone" $X_+ \subset X$ for interior and exterior dynamical systems, respectively [22, 31, 32, 33].

numbers with amplitude-radius r, which is iso-metrically embedded in X and is simultaneously dual to the two complex sub-spaces X_- and X_+ [22, 31, 32, 33]—see Figure 2.

At this point, we've initialized the 1-sphere IHR topology of $T^1 \subset X$ [22, 31, 32, 33]. Therefore, we are ready to explore the proposed the effective iso-radius encoding platform of Section 2.2.

2.2 Constructing the effective iso-radius for the effective iso-state

Here, now that we've initialized the 1-sphere IHR topology of $T^1 \subset X$ [22, 31, 32, 33] in Section 2.1, we are ready to introduce and assemble the effective iso-radius encoding platform for representing iso-sphere IHR [25] iso-states and iso-transitions via the following procedure:

1. First, in conventional mathematics, the number 1 for the multiplicative identity satisfies the original number field axioms [34]. Thus, the number 1 is the fundamental unit that plays important and diverse roles throughout mathematics in general such as, for example, normalization. Therefore, we start by setting the amplitude-radius $r = 1$ so T^1 is the IHR unit-circle with the equivalent curvature $\kappa = \frac{1}{r} = 1$.

2. Second, in iso-mathematics [1, 2, 3, 4, 5, 22], Santilli successfully demonstrated that the multiplicative unit is not limited to the number 1 and can therefore be replaced with the positive-definite iso-multiplicative iso-unit $\hat{r} > 0$ with corresponding inverse $\hat{\kappa} = \frac{1}{\hat{r}} > 0$ for iso-numbers. Hence, for some selected $\hat{r}$, we employ Santilli's iso-methodology [1, 2, 3, 4, 5] to iso-topically lift T^1 via
$$\vec{x}_{\hat{r}} \equiv \vec{x} \times \hat{r},\ \forall \vec{x} \in T^1 \to \forall \vec{x}_{\hat{r}} \in T^1_{\hat{r}}, \tag{3}$$
for the transition and its inverse
$$\begin{aligned} f(\hat{r}, T^1): \quad & T^1 \;\to\; T^1_{\hat{r}} \\ f^{-1}(\hat{r}, T^1_{\hat{r}}): \quad & T^1_{\hat{r}} \;\to\; T^1 \end{aligned} \tag{4}$$
to identify the iso-1-sphere IHR $T^1_{\hat{r}}$ with the iso-radius $\hat{r}$, so T^1 and $T^1_{\hat{r}}$ are *locally iso-morphic* and are both centered on the origin $O \in X$. Here, note that in addition to being the iso-radius of T^1, $\hat{r}$ also serves as the iso-unit for Santilli's iso-multiplication [1, 2, 3, 4, 5, 22], where the iso-unit inverse $\hat{\kappa}$ is also the iso-curvature of $T^1_{\hat{r}}$.

3. Third, given the dynamic iso-topic lifting and dynamic iso-spheres of [24, 25], we furthermore define T^1's iso-radius as an iso-function in the positive-definite form
$$T^1_{\hat{r}(m)}:\ \hat{r} \equiv \hat{r}(m) \equiv ma + b \equiv \frac{1}{\hat{\kappa}(m)} > 0, \tag{5}$$

where $\hat{r}(m)$ is the *dynamic iso-radius iso-function* (or "dynamic iso-unit iso-function") with the parameter m and $\hat{\kappa}(m)$ is the corresponding *dynamic iso-curvature iso-function*, such that m is some general *mathematical* quantity while a and b are coefficients that may be customized depending on context and application (i.e. the classic "point-intercept form", etc.). Thus, eq. (3) is rewritten as

$$\vec{x}_{\hat{r}(m)} \equiv \vec{x} \times \hat{r}(m), \;\forall \vec{x} \in T^1 \rightarrow \forall \vec{x}_{\hat{r}(m)} \in T^1_{\hat{r}(m)} \tag{6}$$

so eq. (4) becomes

$$\begin{aligned} f(\hat{r}(m), T^1): &\quad T^1 &\rightarrow&\; T^1_{\hat{r}(m)} \\ f^{-1}(\hat{r}(m), T^1_{\hat{r}(m)}): &\quad T^1_{\hat{r}(m)} &\rightarrow&\; T^1. \end{aligned} \tag{7}$$

4. Fourth, given that eq. (5) is a *dynamic* iso-unit iso-function, we wish to show that $\hat{r}(m)$ is characterized by change as it's parameter m varies and takes on values from some positive-definite sequence M, such that $m \in M$ as $m \rightarrow \infty$. In [24, 25], there are *two* distinct types of dynamic iso-unit iso-functions:

   - *continuous dynamic iso-unit iso-functions*, so M may be a *continuous* sequence of positive-definite values such as, for example, the case of $M \equiv M_{\mathbb{R}+}$ for the positive-definite interval of *real numbers*

     $$M_{\mathbb{R}+} = (0, \infty_{\mathbb{R}+}), \; m \in M_{\mathbb{R}+}, \; m \rightarrow \infty_{\mathbb{R}+}; \tag{8}$$

     and

   - *discrete dynamic iso-unit iso-functions*, so M may be a *discrete* sequence of positive-definite values such as, for example, the case of $M \equiv M_{\mathbb{N}}$ for the positive-definite set of *natural numbers*

     $$M_{\mathbb{N}} = \{1, 2, 3, 4, 5, ...\}, \; m \in M_{\mathbb{N}}, \; m \rightarrow \infty_{\mathbb{N}} \tag{9}$$

     or in the case of $M \equiv M_{Fib}$ for the positive-definite set of *Fibonacci numbers*

     $$M_{Fib} = \{1, 1, 2, 3, 5, ...\}, \; m \in M_{Fib}, \; m \rightarrow \infty_{Fib}. \tag{10}$$

5. Fifth, for this introductory investigation, consider a relatively simple case and suppose that $a = 2$ and $b = 0$, where we know that the $\hat{r}(m) > 0$ and $\hat{\kappa}(m) > 0$ of eq. (5) will remain positive-definite as $m > 0$ varies and takes on values from some positive-definite sequence M, regardless of whether M is continuous or discrete. Note: the selection of $a = 2$ is arbitrary and for illustration purposes only (as long as the $\hat{r}(m) > 0$ iso-unit constraint is satisfied), but we are inspired to let $\hat{r}(m) = 2m$ for this example because it parallels the relation between the event horizon radius and mass of Schwarzschild black holes [26, 27, 28, 29, 30]. Thus, eq. (5) is rewritten as

$$T^1_{\hat{r}(m)} : \ \hat{r} \equiv \hat{r}(m) \equiv m2 + 0 \equiv 2m \equiv \frac{1}{\hat{\kappa}(m)} > 0. \tag{11}$$

In this example case, we will operate eq. (11) with $\hat{r}(m) = 2m$, but note that eq. (11) could be rewritten again to relate $\hat{r}$ to additional *mathematical* quantities as long as it complies with Santilli's positive-definite iso-unit constraint $\hat{r}(m) > 0$ [1, 2, 3, 4, 5, 22] for the iso-topic liftings of eqs. (6–7).

6. Sixth, in a brief side note, we recall and observe that the fundamental exterior and interior iso-duality inverse establishment [25] gives us

$$\begin{aligned} T^1_{\hat{r}(m)} &\equiv T^1_{\hat{r}_+(m)} \\ T^1_{\hat{\kappa}(m)} &\equiv T^1_{\hat{r}_-(m)} \end{aligned} \tag{12}$$

because in this context, assuming $\hat{r}(m) > 1$, the $T^1_{\hat{r}(m)} \equiv T^1_{\hat{r}_+(m)}$ of the "outer" iso-radius $\hat{r}(m) \equiv \hat{r}_+(m)$ is the *exterior iso-1-sphere IHR* that is "outside" of T^1 because $T^1_{\hat{r}_+(m)} \subset X_+$, while the $T^1_{\hat{\kappa}(m)} \equiv T^1_{\hat{r}_-(m)}$ of the "inner" iso-radius $\hat{\kappa}(m) \equiv \hat{r}_-(m)$ is the *interior iso-1-sphere IHR* that is "inside" of T^1 because $T^1_{\hat{r}_-(m)} \subset X_-$ [25]: $T^1_{\hat{r}_+(m)}$ and $T^1_{\hat{r}_-(m)}$, or equivalently $T^1_{\hat{\kappa}(m)}$ and $T^1_{\hat{r}(m)}$, are iso-dual [25] due to the fact that

$$\hat{r}_+(m) \equiv \hat{r}(m) \equiv \frac{1}{\hat{\kappa}(m)} \equiv \frac{1}{\hat{r}_-(m)}. \tag{13}$$

All of this illustrates a fundamental and important iso-duality between the iso-curvature (the iso-unit inverse) and iso-radius (the iso-unit) when T^1 is iso-topically lifted [25]. For a depiction of eqs. (12–13) see Figure 3.

7. Seventh, given eq. (11), we define the *initial iso-radius iso-state* of $T^1_{\hat{r}(m)}$ as

$$T^1_{\hat{r}(m_0)} : \ \hat{r}_0 \equiv \hat{r}(m_0) \equiv 2m_0 \equiv \frac{1}{\hat{\kappa}(m_0)} > 0, \tag{14}$$

to identify the *initial iso-1-sphere IHR iso-state* $T^1_{\hat{r}(m_0)}$, where $\hat{r}(m_0) > 0$ is the *initial iso-radius*, $\hat{\kappa}(m_0) > 0$ is the *initial iso-curvature*, and $m_0 > 0$ is the *initial quantity*, such that $m_0 \in M$, regardless of whether the positive-definite M is continuous or discrete. Therefore, for this initial case we assign $m = m_0$ for eq. (6) to establish

$$\vec{x}_{\hat{r}(m_0)} \equiv \vec{x} \times \hat{r}(m_0), \ \forall \vec{x} \in T^1 \rightarrow \forall \vec{x}_{\hat{r}(m_0)} \in T^1_{\hat{r}(m_0)} \tag{15}$$

so eq. (7) becomes

$$\begin{aligned} f(\hat{r}(m_0), T^1) : \quad & T^1 \quad \rightarrow \ T^1_{\hat{r}(m_0)} \\ f^{-1}(\hat{r}(m_0), T^1_{\hat{r}(m_0)}) : \quad & T^1_{\hat{r}(m_0)} \ \rightarrow \ T^1. \end{aligned} \tag{16}$$

8. Eighth, suppose that the quantity m_0 undergoes a change that is characterized by

$$\delta_m : \ m_0 \rightarrow m_1, \tag{17}$$

which causes

$$\delta_{\hat{r}(m)} : \ \hat{r}(m_0) \rightarrow \hat{r}(m_1), \tag{18}$$

such that

$$m_0 = m_1 - \Delta_m. \tag{19}$$

Thus, a second version of eq. (14) is written to define the *final iso-radius iso-state* of $T^1_{\hat{r}(m)}$ as

$$T^1_{\hat{r}(m_1)} : \ \hat{r}_1 \equiv \hat{r}(m_1) \equiv 2m_1 \equiv \frac{1}{\hat{\kappa}(m_1)} > 0, \tag{20}$$

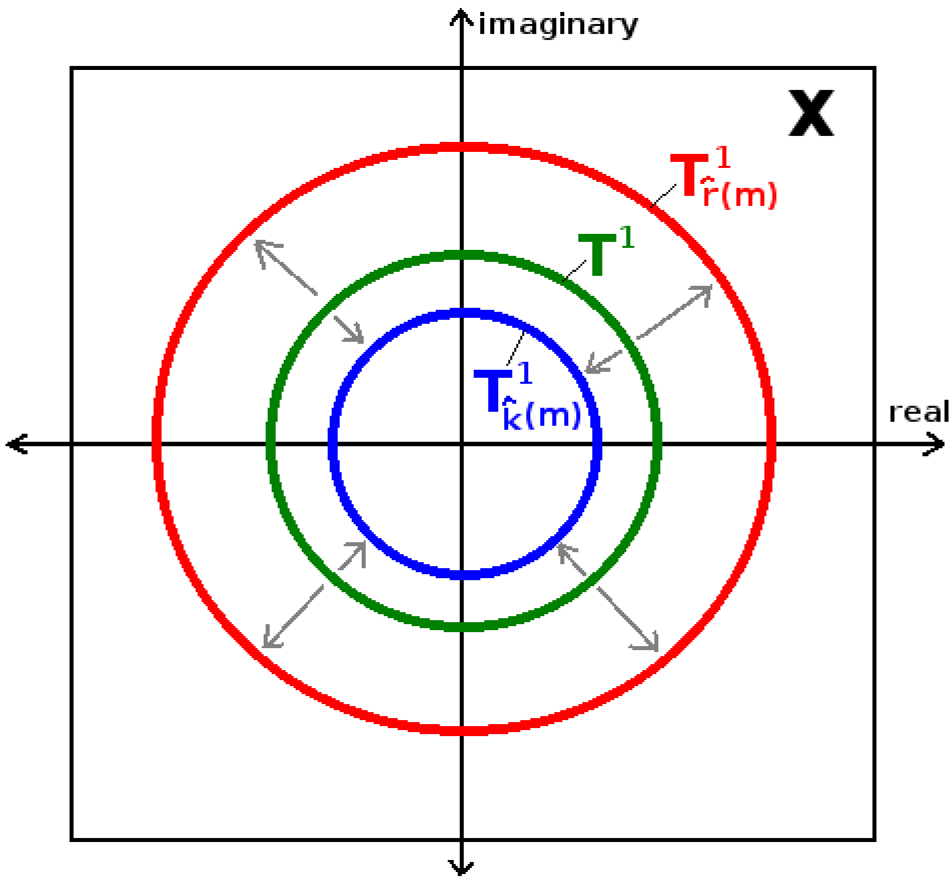

Fig. 3: When the 1-sphere IHR $T^1 \subset X$ of radius $r = 1$ is iso-topically lifted to the iso-1-sphere IHR $T^1_{\hat{r}(m)} = T^1_{\hat{r}_+(m)} \subset X_+$ of iso-radius $\hat{r}(m) = \hat{r}_+(m) > r$ and iso-curvature $\hat{\kappa}(m) = \frac{1}{\hat{r}(m)} = \hat{r}_-(m) < r$, there also exists the iso-1-sphere IHR $T^1_{\hat{\kappa}(m)} = T^1_{\hat{r}_-(m)} \subset X_-$ of iso-radius $\hat{r}_-(m)$ and iso-curvature $\hat{r}_+(m)$, such that $T^1_{\hat{r}_+(m)}$ and $T^1_{\hat{r}_-(m)}$ are iso-dual inverses [25].

to identify the *final iso-1-sphere IHR iso-state* $T^1_{\hat{r}(m_1)}$, where $\hat{r}(m_1) > 0$ is the *final iso-radius*, $\hat{\kappa}(m_1) > 0$ is the *final iso-curvature*, and $m_1 > 0$ is the *final quantity*, such that $m_1 \in M$, regardless of whether the positive-definite M is continuous or discrete. Therefore, for this final case we assign $m = m_1$ for eq. (6) to establish

$$\vec{x}_{\hat{r}(m_1)} \equiv \vec{x} \times \hat{r}(m_1),\ \forall \vec{x} \in T^1 \to \forall \vec{x}_{\hat{r}(m_1)} \in T^1_{\hat{r}(m_1)} \tag{21}$$

so eq. (16) becomes

$$\begin{aligned} f(\hat{r}(m_1), T^1):& \quad T^1 \quad \to \ T^1_{\hat{r}(m_1)} \\ f^{-1}(\hat{r}(m_1), T^1_{\hat{r}(m1)}):& \quad T^1_{\hat{r}(m_1)} \ \to \ T^1. \end{aligned} \tag{22}$$

9. Ninth, given the impact of eqs. (17–22), the initial iso-1-sphere IHR iso-state $T^1_{\hat{r}(m_0)}$ (of initial iso-radius $\hat{r}(m_0)$) is iso-topically lifted to the final iso-1-sphere IHR iso-state $T^1_{\hat{r}(m_1)}$ (of final iso-radius $\hat{r}(m_1)$) via

$$\vec{x}_{\hat{r}(m_1)} \equiv \vec{x}_{\hat{r}(m_0)} \times \frac{\hat{r}(m_1)}{\hat{r}(m_0)},\ \forall \vec{x}_{\hat{r}(m_0)} \in T^1_{\hat{r}(m_0)} \to \forall \vec{x}_{\hat{r}(m_1)} \in T^1_{\hat{r}(m_1)} \tag{23}$$

for the iso-transition and its inverse

$$\begin{aligned} f(\tfrac{\hat{r}(m_1)}{\hat{r}(m_0)}, T^1_{\hat{r}(m_0)}):& \quad T^1_{\hat{r}(m_0)} \ \to \ T^1_{\hat{r}(m_1)} \\ f^{-1}(\tfrac{\hat{r}(m_1)}{\hat{r}(m_0)}, T^1_{\hat{r}(m_1)}):& \quad T^1_{\hat{r}(m_1)} \ \to \ T^1_{\hat{r}(m_0)} \end{aligned} \tag{24}$$

with the iso-radius ratio $\frac{\hat{r}(m_1)}{\hat{r}(m_0)}$ and the corresponding iso-curvature ratio $\frac{\hat{r}(m_0)}{\hat{r}(m_1)}$ characterize the iso-transition to establish that T^1, $T^1_{\hat{r}(m_0)}$, and $T^1_{\hat{r}(m_1)}$ are indeed locally iso-morphic—see Figure 4.

10. Tenth, we note that the iso-transition between $T^1_{\hat{r}(m_0)}$ and $T^1_{\hat{r}(m_1)}$ depends on Δ_m and complies with the trichotomy:

    - **Case $\Delta_m < 0$:** $T^1_{\hat{r}(m_0)}$ is *de-magnified* to become $T^1_{\hat{r}(m_1)}$ via the iso-topic lifting $T^1_{\hat{r}(m_0)} \to T^1_{\hat{r}(m_1)}$ because $m_1 < m_0$ so $\hat{r}(m_1) < \hat{r}(m_0)$.

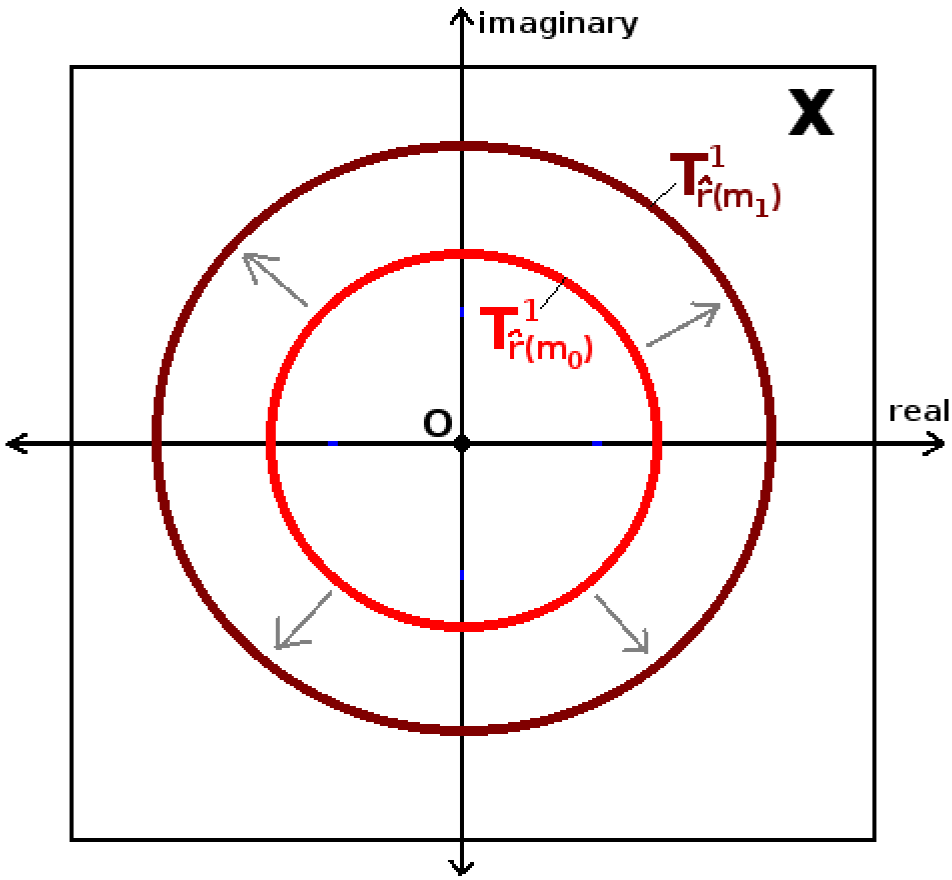

Fig. 4: For the iso-transition $T^1_{\hat{r}(m_0)} \to T^1_{\hat{r}(m_1)}$ of Δ_m, the initial iso-1-sphere IHR iso-state $T^1_{\hat{r}(m_0)}$ is iso-topically lifted to the final iso-1-sphere IHR iso-state $T^1_{\hat{r}(m_1)}$, where $T^1_{\hat{r}(m_0)}$, and $T^1_{\hat{r}(m_1)}$ are indeed locally iso-morphic.

- **Case $\Delta_m = 0$:** $T^1_{\hat{r}(m_0)}$ is equivalent to $T^1_{\hat{r}(m_1)}$ via the iso-topic lifting $T^1_{\hat{r}(m_0)} \to T^1_{\hat{r}(m_1)}$ because $m_1 = m_0$ so $\hat{r}(m_1) = \hat{r}(m_0)$.
- **Case $\Delta_m > 0$:** $T^1_{\hat{r}(m_0)}$ is *magnified* to become $T^1_{\hat{r}(m_1)}$ via the iso-topic lifting $T^1_{\hat{r}(m_0)} \to T^1_{\hat{r}(m_1)}$ because $m_1 > m_0$ so $\hat{r}(m_1) > \hat{r}(m_0)$.

11. Finally, given the new and developing framework of [26, 27, 28, 29, 30] that characterizes the effective *physical* state of black holes for an emission or absorption transition, we are motivated to define the effective *iso-mathematical* state of dynamic iso-1-sphere IHRs (which are also spherically-symmetric objects) for a transition from $T^1_{\hat{r}(m_0)}$ to $T^1_{\hat{r}(m_1)}$. Therefore, given the *physical black hole effective radius* definition from eq. (5) of [29], we implement the dynamic iso-topic lifting of [24, 25] and define the *(iso-mathematical) effective dynamic iso-1-sphere IHR iso-radius* as

$$T^1_{\hat{r}(m_0)} \to T^1_{\hat{r}(m_1)} : \hat{r}_E \equiv \hat{r}_E(m_0, m_1) \equiv 2m_E(m_0, m_1) \equiv \frac{1}{\hat{\kappa}_E(m_0, m_1)} > 0, \tag{25}$$

where $\hat{\kappa}_E(m_0, m_1)$ is the *effective dynamic iso-1-sphere IHR iso-curvature* and inverse of the iso-unit, such that the *effective iso-1-sphere IHR quantity* is defined as

$$m_E(m_0, m_1) \equiv \frac{m_0 + m_1}{2}, \tag{26}$$

which is simply the *average* of $T^1_{\hat{r}(m_0)}$'s initial quantity m_0 and $T^1_{\hat{r}(m_1)}$'s final quantity m_1. Thereafter, we define the *effective dynamic iso-1-sphere IHR* as

$$T_{\hat{r}_E} \equiv T_{\hat{r}_E(m_0,m_1)} \equiv \{\vec{x} \in X : |\vec{x}| = \hat{r}_E(m_0, m_1)\}, \tag{27}$$

where $T_{\hat{r}_E(m_0,m_1)} \subset X$ is centered on the origin $O \in X$; $T_{\hat{r}_E(m_0,m_1)}$ is the multiplicative group of all non-zero complex numbers with the effective iso-amplitude-radius $\hat{r}_E(m_0, m_1)$—see Figure 5.

At this point, we've assembled a preliminary construction of the effective iso-radius encoding platform for representing dynamic iso-sphere IHR [25] iso-states and iso-transitions for the IHR topology [22, 31, 32, 33].

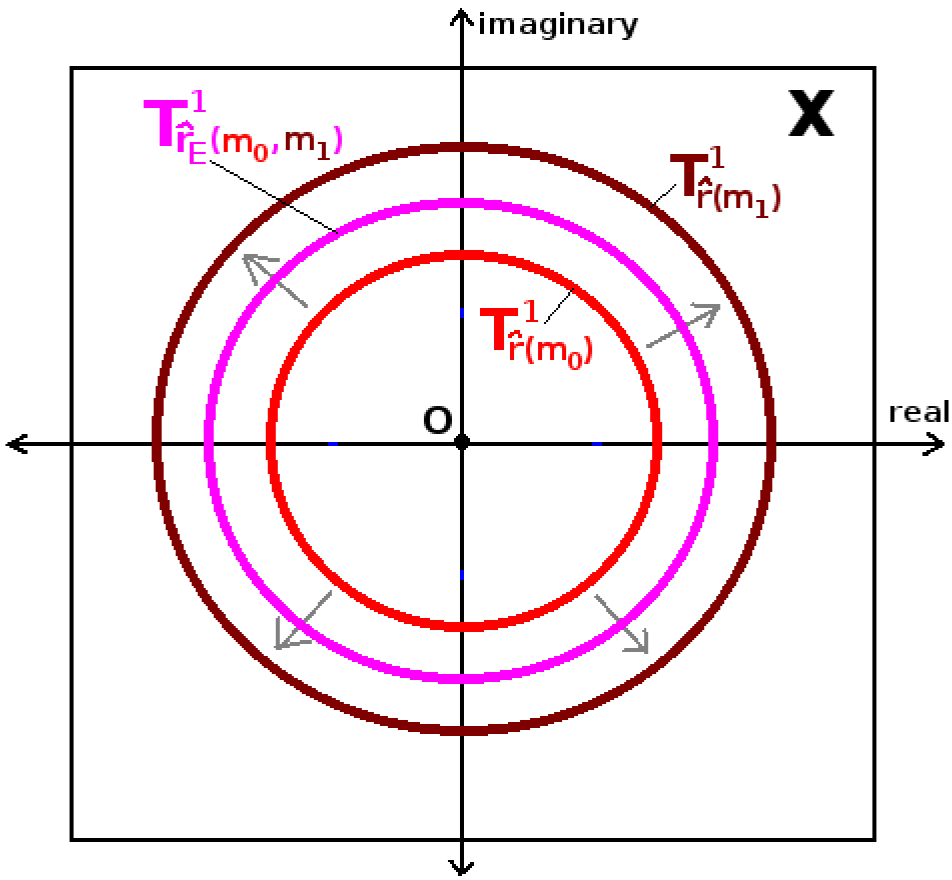

Fig. 5: For the iso-transition $T^1_{\hat{r}(m_0)} \to T^1_{\hat{r}(m_1)}$ of Δ_m, $T^1_{\hat{r}(m_0)}$ is the initial iso-1-sphere IHR iso-state, $T^1_{\hat{r}(m_1)}$ is the final iso-1-sphere IHR iso-state, and $T_{\hat{r}_E(m_0,m_1)}$ is the characteristic effective iso-1-sphere IHR iso-state with the effective iso-1-sphere IHR iso-radius $\hat{r}_E(m_0,m_1)$ and the effective iso-1-sphere IHR iso-curvature $\hat{\kappa}_E(m_0,m_1)$.

3 Conclusion

In this work, we merged pertinent aspects from Inopin's IHR topology [22, 31, 32, 33], Santilli's iso-topic liftings [1, 2, 3, 4, 5] and Corda's effective radius [26, 27, 28, 29, 30] into a single iso-mathematical model of effective dynamic iso-sphere IHRs [25] with effective iso-radii. More specifically, we successfully assembled the effective iso-radius for the dynamic iso-1-sphere IHR [25] "base case" in the IHR topology [22, 31, 32, 33] and introduced the corresponding notion of effective iso-state to begin encoding the iso-transition between two distinct iso-states. For this, the procedure and step-by-step developing results were presented in Section 2, which apply to both continuous and discrete dynamic iso-sphere IHRs. Also, we demonstrated that all of these outcomes comply with the exterior and interior IHR inverse iso-duality [25]. To recapitulate the final results more precisely, we defined—for the dynamic iso-sphere IHR $T^1_{\hat{r}(m)}$—the effective iso-radius $\hat{r}_E(m_0, m_1)$ as the average of the initial iso-radius $\hat{r}(m_0)$ and the final iso-radius $\hat{r}(m_1)$ in eqs. (25–26), which correspond to the initial dynamic iso-sphere IHR $T^1_{\hat{r}(m_0)}$ and the final dynamic iso-sphere IHR $T^1_{\hat{r}(m_1)}$, respectively. Ultimately, we deployed these constructs to propose the new iso-geometrical, iso-topological class of effective dynamic iso-sphere IHRs for a generalized encoding methodology.

The results, constructions, and implications of this preliminary investigation are significant because they exemplify alternative modes of cutting-edge iso-mathematics research that facilitate a heightened quantifiable characterization of dynamic iso-sphere IHRs [25] in terms of effective iso-states for iso-transitions with iso-duality. Hence, given that iso-sphere IHRs are equipped with topological deformation order parameters [32, 33, 22], a next logical step of this analysis could be to implement iso-topic liftings [1, 2, 3, 4, 5] for the order parameters and then topologically incorporate these "iso-deformations" into the existing effective iso-state definition. From there, we may build on this platform and continue to develop the framework by exploring and assessing the frontiers of iso-, geno-, and hypermathematics [1, 2, 3, 4, 5]. One of the questions that comes to mind is this: *do effective dynamic iso-sphere IHRs equipped with effective iso-radii have direct application to chemistry and/or physics such as, for example, the non-strictly thermal, non-strictly continuous energy spectrum of black holes*

[26, 27, 28, 29, 30]? Our hypothesis is: *yes.* Hence, in order to test this conjecture via the scientific method, this developing class of effective dynamic iso-sphere IHRs must be subjected to further development, scrutiny, collaboration, and hard work in order to advance it for future application in the disciplines of iso-mathematics, physics, and science in general.

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