A generic formula of 2-Poulet numbers and also a method to obtain sequences of n-Poulet numbers

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Abstract. In this paper I present a formula based on 2-Poulet numbers which seems to conduct always to a prime, a square of prime or a semiprime, a conjecture that this formula is generic for 2-Poulet numbers, and, in case that the conjecture doesn't hold, I present another utility for this formula, namely to generate sequences of n-Poulet numbers.

Conjecture:

Any 2-Poulet number P can be written at least in one way as P = (q*2^a*3^b*5^c ± 1)*2^n + 1, where q is a prime, a square of prime or a semiprime, a, b, c are non-negative integers and n is non-null positive integer.

In other words, there always exist a number q = ((P – 1)/2^n ± 1)/(2^a*3*b*5^c), where P is a 2-Poulet number, a, b, c are non-negative integers and n is non-null positive integer, such that q is a prime, a square of prime or a semiprime.

Note: In this paper I will consider the number 1 to be a prime (not to repeat the formulation: q is prime, square of prime, semiprime or is equal to number 1).

Verifying the conjecture:
[for the first ten 2-Poulet numbers, only for a restrictive version of the conjecture, considering just the formula P = (q*2^a*3^b*5^c + 1)*2^n + 1]

For P = 341, we have:
q = ((341 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 13^2;
q = ((341 – 1)/2^2 – 1)/(2^2*3^1*5^0) = 7.
For P = 1387, we have:
q = ((1387 – 1)/2^1 – 1)/(2^2*3^0*5^0) = 173.
For P = 2047, we have:
q = ((2047 – 1)/2^1 – 1)/(2^1*3^0*5^0) = 7*73.
For P = 2701, we have:
q = ((2701 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 19*71;
q = ((2701 – 1)/2^2 – 1)/(2^1*3^0*5^0) = 337.
For P = 3277, we have:
q = ((3277 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 1637;
q = ((3277 – 1)/2^2 – 1)/(2^1*3^0*5^0) = 409.

For P = 4033, we have:
q = ((4033 - 1)/2^1 - 1)/(2^0*3^0*5^1) = 13*31;
q = ((4033 - 1)/2^2 - 1)/(2^0*3^0*5^0) = 19*53;
q = ((4033 - 1)/2^3 - 1)/(2^0*3^0*5^0) = 503;
q = ((4033 - 1)/2^4 - 1)/(2^0*3^0*5^0) = 251;
q = ((4033 - 1)/2^5 - 1)/(2^0*3^0*5^3) = 1;
q = ((4033 - 1)/2^6 - 1)/(2^1*3^0*5^0) = 31.
For P = 4369, we have:
q = ((4369 - 1)/2^1 - 1)/(2^0*3^0*5^0) = 37*59;
q = ((4369 - 1)/2^2 - 1)/(2^0*3^0*5^0) = 1091;
q = ((4369 - 1)/2^3 - 1)/(2^0*3^0*5^1) = 109;
((4369 - 1)/2^4 - 1)/(2^0*3^0*5^0) = 251;
For P = 4681, we have:
q = ((4681 - 1)/2^1 - 1)/(2^0*3^0*5^0) = 2339;
q = ((4681 - 1)/2^2 - 1)/(2^0*3^0*5^0) = 7*167;
q = ((4681 - 1)/2^3 - 1)/(2^3*3^0*5^0) = 73;
For P = 5461, we have:
q = ((2701 - 1)/2^1 - 1)/(2^0*3^0*5^0) = 2729;
q = ((2701 - 1)/2^2 - 1)/(2^2*3^0*5^0) = 11*31.
For P = 7957, we have:
q = ((7957 - 1)/2^1 - 1)/(2^0*3^0*5^1) = 37*43;
q = ((7957 - 1)/2^2 - 1)/(2^2*3^0*5^0) = 7*71.

Verifying the conjecture:
(for seven greater consecutive 2-Poulet numbers)

For P = 27657600833, we have:
q = ((27657600833 - 1)/2^4 - 1)/(2^0*3^1*5^0) = 653*882389;
q = ((27657600833 - 1)/2^1 + 1)/(2^0*3^1*5^0) = 22433*205483;
q = ((27657600833 - 1)/2^2 + 1)/(2^0*3^0*5^0) = 6914400209.
q = ((27657600833 - 1)/2^4 + 1)/(2^0*3^0*5^0) = 6911*250123.
q = ((27657600833 - 1)/2^6 + 1)/(2^1*3^0*5^0) = 8093*26699.
For P = 27667059281, we have:
q = ((27667059281 - 1)/2^1 - 1)/(2^0*3^0*5^0) = 103*134306113;
q = ((27667059281 - 1)/2^4 + 1)/(2^1*3^0*5^0) = 864595603.
For P = 27675991081, we have:
q = ((27675991081 - 1)/2^1 - 1)/(2^0*3^0*5^0) = 10169*680401;
q = ((27675991081 - 1)/2^2 - 1)/(2^2*3^0*5^0) = 1109*779869.
For P = 27681232903, we have:
q = ((27681232903 - 1)/2^1 - 1)/(2^1*3^0*5^2) = 276812329;
q = ((27681232903 - 1)/2^2 + 1)/(2^4*3^0*5^0) = 67*807083.
For P = 27685810639, we have:
q = ((27685810639 - 1)/2^1 + 1)/(2^3*3^0*5^1) = 4740721.
For P = 27686175193, we have:
q = ((27686175193 - 1)/2^1 - 1)/(2^0*3^0*5^1) = 20208887;
q = ((27686175193 - 1)/2^1 + 1)/(2^0*3^0*5^0) = 2837*4879481;
q = ((27686175193 - 1)/2^3 + 1)/(2^2*3^0*5^2) = 113*306263.
For P = 27702689701, we have:
q = ((27702689701 - 1)/2^2 - 1)/(2^3*3^0*5^0) = 11*78700823;
q = ((27702689701 - 1)/2^1 + 1)/(2^0*3^0*5^0) = 15971*867281;
q = ((27702689701 - 1)/2^2 + 1)/(2^1*3^0*5^0) = 199*17401187.

Comment:

If the Conjecture doesn’t hold, it may be considered a more premissive version: Any 2-Poulet number P can be written at least in one way as P = (q*2^a*3^b*5^c ± 1)*2^n + 1, where q is a prime, a square of prime or a semiprime and a, b, c, n are non-negative integers.
In this case we have, for instance for P = 27686175193, q = ((27686175193 – 1)/2^0 - 1)/(2^0*3^0*5^0) = 27686175191 which is prime.

Comment:

If the Conjecture doesn’t hold, it has anyhow at least one utility: it’s a method for finding sequences of Poulet numbers (not only 2-Poulet numbers).

Taking, for instance, q = 223*r, where r is prime, we have the sequence of Poulet numbers P defined as P = (223*r + 1)*2^n + 1, with the first three terms {41041, 10261, 52633}, obtained for the following values of (r,n): {23,1), (23,3), (59,2)}.

Taking, for instance, q = 29*r, where r is prime, we have the sequence of Poulet numbers P defined as P = (29*r + 1)*3^n + 1, with the first term 2701, obtained for the following value of (r,n): (31,1).

Taking, for instance, q = 37*r, where r is prime, we have the sequence of Poulet numbers P defined as P = (37*r + 1)*5^n + 1, with the first term 561, obtained for the following value of (r,n): (3,5).

Taking, for instance, q = 13^2, we have the sequence of Poulet numbers P defined as P = (13^2 + 1)*2^n + 1, with the first term 341, obtained for the following value of n: 1.