On an iterative operation on positive composite integers which probably always conducts to a prime

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Abstract. By playing with one of my favorite class of numbers, Poulet numbers, and one of my favorite operation, concatenation, I raised to myself few questions that seem interesting, worthy to share. I also conjectured that, reiterating a certain operation which will be defined, eventually for every Poulet number it will be find a corresponding prime. Then I extrapolated the conjecture for all composite positive integers.

Conjecture 1:

For any Poulet number P is defined the following operation which always, eventually, leads to a prime number:

Let P be a Poulet number, $P = p_1*p_2*...*p_n$, where $p_1 \leq p_2 \leq p_3 \leq ... \leq p_n$ are the prime factors of P (not distinct, it can be seen from definition: for instance for the only two Poulet numbers non-squarefree known, the squares of Wieferich primes, we have $P = p_1*p_2$, where $p_1 = p_2$; the reason for writing the number P in this way instead writing $P = p_1$^2 it will be seen further).

Then we consider the number $Q_1 = p_1p_2...p_n$, obtained by concatenation of the numbers that form the ordered set $\{p_1, p_2, p_3, ...,p_n\}$.

(Example: P = 561 = 3*11*17; then Q_1 = 31117)

Then we have the following posibilities: Q is a prime or a composite number; if it is composite (it doesn't matter if is squarefree or not) we reiterate the operation until is obtained a prime number.

(Example: Q_1 = 31117 = 29*29*37; then Q_2 = 292937 = 457*641; Q_3 = 457641 = 3*3*50849; Q_4 = 3350849 = 131*25579; Q_5 = 13125579 = 3*4375193; finally, the number Q_6 = 34375193 is prime)

Note:

Our conjecture is that, reiterating the operation above, from every Poulet number is obtained, eventually, a prime.

Verifying the conjecture for the first few Poulet numbers (beside 561 which was given above as an example):

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P = 341 = 11*31;
:    1131 = 3*13*29;
:    31329 = 3*3*59*59;
:    335959 = 13*43*601;
:    1343601 = 3*3*3*7*7109;
:    33377109 = 3*607*18329;
:    360718329 = 3*120239443;
:    3120239443 = 523*5966041;
:    5235966041 = 419*12496339;
:    41912496339 = 3*7*1995833159;
:    371995833159 = 3*3*19*1459*1491031;
:    331914591491031 = 3*3*709*145949*356399;
:    33709145949356399 = 2011*16762379885309;
:    201116762379885309 = 3*17*1357927*2904033817;
:    3171357927290403381 7 = 13*337*7238890498266157;
:    13337723889049826615 7 = 3*13*53*6452696608151827 1;
:    31353645269660815182 71 = 3037*10323887148390126 83;
:    30371032388714839012 683 = 3*3*2879*16260571*72084117943;
:    33287916260571720841 17943 = 15765766319*211140490015097;
:    15765766319211140490 015097 = 575063*27415720224064390319;
:    57506327415720224064 390319 = 32869*1749561210128699506051
:    32869174956121012869 9506051 =
     23*61*1283*1597*30391*3762309931337;
:    23611283159730391376 2309931337 =
     3*509*13381*1155558620550901420 1651;
:    35091338111555586205 509014201651 is a prime number.

P = 645 = 3*5*43;
:    3543 = 3*1181;
:    31181 is a prime number.

P = 1105 = 5*13*17;
:    51317 = 7*7331;
:    77331 = 3*149*173;
:    3149173 is a prime number.

P = 1387 = 19*73;
:    1973 is a prime number.

P = 1729 = 7*13*19;
:    71319 = 3*23773;
:    323773 = 199*1627;
:    1991627 = 11*331*547;
:    11331547 = 29*390743;
:    29390743 is a prime number.

P = 1905 = 3*5*127;
:    35127 = 3*3*3*1301;
:    3331301 is a prime number.
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P = 2047 = 23*89;
: 2389 is a prime number.

Verifying the conjecture for the two squares of Wieferich primes (because they represent a special case):

P = 1194649 = 1093*1093;
: 10931093 = 73*137*1093;
: 731371093 = 17*223*192923;
: 17223192923 = 2089*8244707;
: 20898244707 = 3*11483606643;
: 311483606643 = 3*3*11*11*286027187;
: 3311286027187 = 3*110370428675729;
: 3110370428675729 = 21977*141528435577;
: 21977141528435577 = 3*11*17351*38382455519;
: 3111735138382455519 = 3*11*11*113899*164113*458599;
: 3111111389916411345859 = 359*8666048439878583136l;
: 3598666048439878583136l = 162523*22142503205330l907;
: 1625232214250320533019O7 is a prime number.

P = 1194649 = 3511*3511;
: 35113511 = 73*137*3511;
: 731373511 = 11*66488501;
: 1166488501 = 53*2687*8191;
: 5326878191 = 653*8157547;
: 6538157547 = 3*67*32528147;
: 36732528147 = 3*7*37*47274811;
: 373747274811 = 3*3*41527474979;
: 3341527474979 is a prime number.

Note:

The numbers P = 1387 = 19*73 and P = 2047 = 23*89 conducted to a prime from the first step: 1973 and 2389 are both primes. These two 2-Poulet numbers have in common the fact that, in both cases, $p_2 = 4*p_1 - 3$; indeed, 73 = 19*4 - 3 and 89 = 4*23 - 3. Another such 2-Poulet number is P = 13747 = 59*233; 59233 is also a prime number.

Conjecture 2:

For any composite positive integer, the operation defined above, always, eventually, leads to a prime number; so, we have the function f defined on the set of composite positive integers with values in the set of prime numbers; the first five values of f are:
: f(4) = 211;
: f(6) = 23;
: f(8) = 3331113965338635107;
: f(9) = 311;
: f(10) = 773.