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A microscopic Interpretation of the SM Higgs Mechanism

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Abstract

A model is presented where the Higgs mechanism of the Standard Model is deduced from the alignment of a strongly correlated fermion system in an internal space with A_4 symmetry. The ground state is constructed and its energy calculated. Finally, it is claimed that the model may be derived from a field theory in 6+1 dimensions.

1 Introduction

The Higgs sector of the Standard Model (SM) of elementary particles and the associated spontaneous symmetry breaking (SSB) show a strong similarity with the Landau-Ginzburg describtion of superconductivity as well as with the linear sigma model of pion physics, and it has long been speculated, that just as in those cases an underlying microscopic pairing interation may be at work in the SM. One option put forward already in 1979 is that the Higgs particle may be composed of 'techniquarks' U and D[1, 2], in a similar way in which pions are composed of up- and down-quarks u and d, and a technicolor QCD-like theory was suggested for the underlying dynamics. The main drawback of such technicolor models, in particular in their 'extended' form, is the appearance of unwanted flavor changing neutral currents (FCNC)[8].

The starting point of the present approach is an isospin doublet $\psi = (U, D)$ of Dirac fermions just as in technicolor models, however without a technicolor quantum number and, to avoid FCNCs, without a direct interaction to quarks and leptons. Rather we shall assume that the pairing mechanism is due to exchange interactions and strong correlations of fermions, effects which in many body physics are known to be responsible for SSB in superconductors and (anti)ferromagnets. In contrast to solid state physics we do not consider these effects in physical space, but attribute them to arise from an independent dynamics which is active in the internal spaces. To be concrete, we assume the existence of a non-relativistic real internal 3-dimensional space R^3 with rotational SO(3)-symmetry for which the doublet $\psi = (U, D)$ serves as an (internal) Pauli spinor with an initial internal SU(2) spin symmetry. The geometrical picture is that the world is a fiber bundle over Minkowski space with fibers given by the R^3 spaces, and that within these fibers physical processes take place. We further assume that at high temperatures there is a symmetric state in which the internal spins are distributed randomly in the fibers, giving rise to a local SU(2) symmetry of the Lagrangian, local in the sense that on each site in each fiber the spins may be rotated independently. With respect to Lorentz symmetry both U and D can appear as lefthanded or righthanded objects, so that one may in fact consider separately a $SU(2)_L$ for the lefthanded and $SU(2)_R$ for the righthanded objects.

To recapitulate, the Standard Model SSB is triggered by the Higgs field H, a doublet under $SU(2)_L$ which via a symmetry breaking potential

$$V(H) = -\mu^2 H^+ H + \lambda(H^+ H)^2 \tag{1}$$

acquires a non-vanishing vacuum expectation value $\langle H^+ H \rangle = \frac{\mu^2}{2\lambda}$. More in detail the Higgs doublet can be parametrized as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\pi_x - i\pi_y) \\ \sigma - i\pi_z \end{pmatrix} \tag{2}$$

so that

$$V(H) = -\frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) + \frac{1}{4}\lambda(\sigma^2 + \vec{\pi}^2)^2 \tag{3}$$

with minimum at

$$\Lambda_F^2 := \langle \sigma^2 \rangle = \frac{\mu^2}{\lambda} \tag{4}$$

which is often called the Fermi scale. Note that σ is a real scalar field, while $\vec{\pi} = (\pi_x, \pi_y, \pi_z)$ is an axial vector field which can be interpreted as the longitudinal components of the afterwards massive W/Z bosons. In the framework of our model $\vec{\pi}$ can be identified with the internal chiral spin vector, and x, y, z are the coordinates of the internal 3-dimensional R^3 space.

Although π-condensates could be conceivable, in particle physics it turns out that the vev is attributed to the σ field alone, i.e.

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \langle \sigma \rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \Lambda_F \end{pmatrix} \tag{5}$$

The shifting relation $\sigma = \Lambda_F + \phi$ defines the physical Higgs particle ϕ, whose tree level mass can easily be shown to be $m_\phi = \sqrt{2}\mu$. The values $\Lambda_F = 246$ GeV and $m_\phi = 124$ GeV fix the Higgs potential completely.

2 Symmetry Breaking in an A_4 model

In ref. [4] it was shown that the internal (spin and vibrational) excitation spectrum of the Shubnikov group $A_4 + S(S_4 - A_4)$[15, 17, 16] yields the correct multiplet

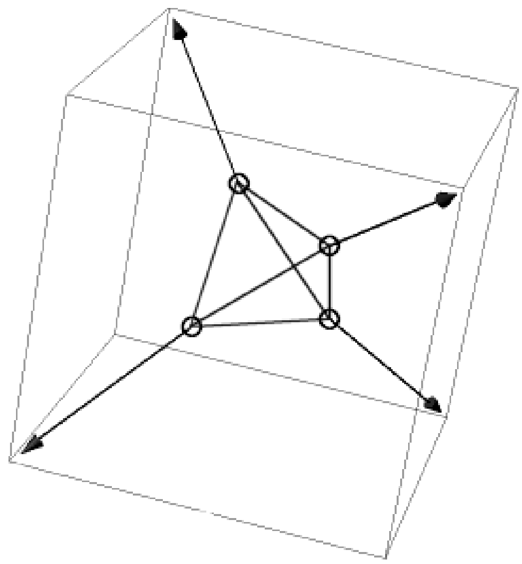

Figure 1: The local ground state of the model, living in a 3-dimensional internal R^3 space (called the 'fiber'). Shown are the corner points (small circles) of the internal tetrahedron, which can be represented by their coordinate vectors $\vec{r}_i$. The origin of coordinates is taken to be the center of the tetrahedron, and is identical to the base point of the fiber in Minkowski space. On each corner point $i = 1, 2, 3, 4$ there is a chiral spin vector $\vec{\pi}_i$, pointing in the same radial direction as $\vec{r}_i$. (Note that the spin vectors are shown but not the coordinate vectors $\vec{r}_i$.) The tetrahedron itself has the tetrahedral group S_4 as point group symmetry. However due to the pseudovector property of the spin vectors the whole system has the Shubnikov point symmetry $A_4+Q(S_4-A_4)$[15], where Q is the internal parity operation and A_4 is the subgroup of S_4 which does not contain reflections. The Shubnikov group is chiral, the configuration with opposite chirality being given when the 4 spin vectors would point inwards instead of outwards. Before the formation of the chiral tetrahedron the internal spins U and D, which according to eq. (7) are the building blocks of the spin vectors $\vec{\pi}_i$, can freely rotate and thus there is an internal spin SU(2) symmetry group, which however is broken to $A_4 + Q(S_4 - A_4)$ when the chiral tetrahedron is formed.

structure of all 24 quark and lepton states of the 3 families

$$
\begin{aligned}
&A(\nu_e) + A'(\nu_\mu) + A''(\nu_\tau) + T(d) + T(s) + T(b) + \\
&\quad A_s(e) + A'_s(\mu) + A''_s(\tau) + T_s(u) + T_s(c) + T_s(t)
\end{aligned} \tag{6}
$$

where A, A', A'' and T are singlet and triplet representations of A_4 and the index s denotes genuine representations of the Shubnikov group[15].

This discovery has led to the main assumption of the present paper, namely that fig. 1 should be taken as the *local* ground state of the model. In other words, it is assumed that in each of the 3-dimensional internal R^3 fibers there is a discrete tetrahedral structure and that the internal dynamics is such that spin vectors arrange themselves according to this internal tetrahedral symmetry, as depicted in fig. 1. Shown are the corner points of the internal tetrahedron and on each corner point $i = 1, 2, 3, 4$ the chiral spin vector $\vec{\pi}_i$, pointing in the same radial direction as the coordinate vector $\vec{r}_i$. (The $\vec{r}_i$ are not shown in the figure, and the precise mathematical definition of the chiral spin vectors $\vec{\pi}_i$ will be given later in eq. (7).) The tetrahedron itself has the tetrahedral group S_4 as point group symmetry. However, due to the pseudovector property of the internal spin vectors the whole system loses its reflection symmetries and obtains instead the Shubnikov symmetry group $A_4 + Q(S_4 - A_4)$[15, 17, 16], where Q is the internal parity operation and A_4 is the subgroup of S_4 which does not contain reflections. Note that Q itself does not belong to the Shubnikov group, and so internal parity is violated by the ground state fig. 1. One can rephrase this by stating that the ground state and its symmetry are chiral with respect to the internal coordinates, the configuration with opposite chirality being given when the 4 spin vectors would point inwards instead of outwards.

In section 1 it was argued that the internal R^3 spaces are nonrelativistic, at rest (no boosts allowed, because they are fixed to their base point in Minkowski space) and rotationally invariant, with an internal rotational SO(3) and a corresponding spin SU(2) under which the fundamental spinor $\psi = (U, D)$ transforms. Due to this symmetry at high temperatures each of the vectors $\vec{\pi}_i$ can freely rotate in the internal space. This symmetry, however, is valid only before the formation of the internal tetrahedron and is broken to $A_4 + Q(S_4 - A_4)$ when the tetrahedron is formed and the $\vec{\pi}_i$ are fixed to their position in fig. 1. In the language of many-particle physics fig. 1 is a frustrated antiferromagnet configuration[11], because the spin vectors try to avoid each other as far as possible, but do not achieve to form a completely anti-parallel configuration.

Note that this breaking as yet has nothing to do with the spontaneous breaking of $SU(2)_L$, but is dictated by the internal dynamics which leads to the formation of one tetrahedral 'molecule'. Rather it can be related to the breaking of the so called 'custodial' SU(2) to be defined below.

As shown in the next section, the breaking of internal parity Q is accompanied by a breaking of parity in external Minkowski space. The point is that assuming a universal field theory for 1+3+3 dimensions a connection will be established between the internal and external parity operations. Any particle with chiral interactions in the internal space will experience an internal polarization due to the chiral structure in fig. 1, and this polarization will be accompanied by a corresponding chiral interaction of the particle in the base space, an effect which will be used to explain the $V - A$ structure of the weak interactions (for details see the next section).

Within the formalism of section 3 the simultaneous violation of internal and external parity will show up in the simultaneous appearance of $\vec{\tau}$ and γ_5 in eqs. (7) and (40), where $\vec{\tau}$ denotes the triplet of internal Pauli matrices and $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$ the γ_5-matrix in Minkowski space. These quantities are representatives of parity violating behavior in their respective spaces (internal R^3 and Minkowski space), because γ_5 gives it a pseudoscalar behavior in Minkowski space and $\vec{\tau}$ a pseudovector behavior in the internal space. They are the building blocks for the chiral spin vectors, which will now be constructed. Namely, one chooses to define

$$\vec{\pi} = \frac{1}{\Lambda^2}(\bar{\psi} i\gamma_5 \vec{\tau}\psi) = \frac{2}{\Lambda^2}\begin{pmatrix} -\operatorname{Im}[\bar{U}_L D_R + \bar{D}_L U_R] \\ \operatorname{Re}[\bar{U}_L D_R + \bar{D}_L U_R] \\ -\operatorname{Im}[\bar{U}_L U_R + \bar{D}_R D_L] \end{pmatrix} \tag{7}$$

where Λ at this point is just a mass scale to keep the dimensions right. To make the list of components of the Higgs doublet eq. (2) complete we write

$$\sigma = \frac{1}{\Lambda^2}(\bar{\psi}\psi) = \frac{1}{\Lambda^2}[\bar{U}U + \bar{D}D] = \frac{2}{\Lambda^2}\operatorname{Re}[\bar{U}_L U_R + \bar{D}_R D_L] \tag{8}$$

When combined to the Higgs potential eq.(3), the theory is invariant under $SU(2)_L \times SU(2)_R \times U(1)$ transformations, where the charge of the $U(1)$-transformations $\psi \rightarrow$

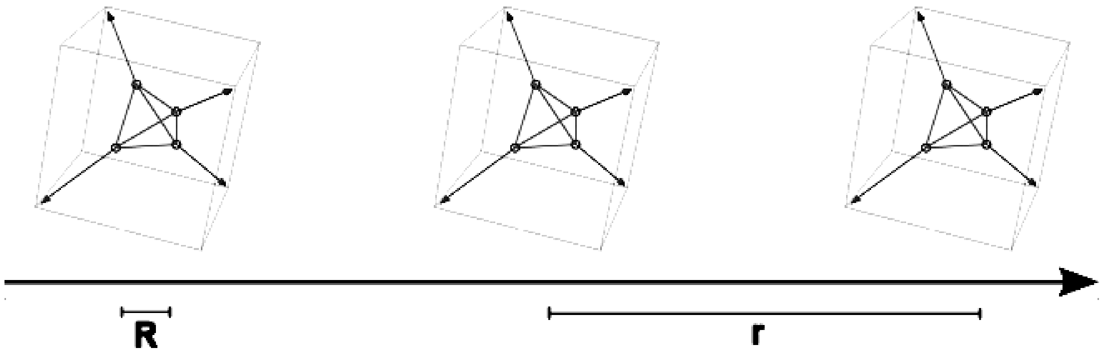

Figure 2: The global ground state of the model after SSB consists of an aligned system of chiral tetrahedrons over Minkowski space (the latter is represented by the long arrow). R is the magnitude of a tetrahedron and r the distance between two of them. Associated to the 2 length scales R and r are the energy scales Λ_R and Λ_r, as defined in the text. Before the SSB the chiral tetrahedrons are oriented randomly (not shown) and there is a corresponding local $SO(3)_L$ symmetry, because each rigid tetrahedron can be rotated freely and independently from the others. After the SSB the coordinate- as well as the spin-vectors of all tetrahedrons are aligned (as shown in the figure). Unfortunately, this picture is only symbolic, not only because R is initially a length scale defined in internal space but also because the condensate $\langle \bar{U}U + \bar{D}D \rangle$ responsible for the SSB does not define a direction in internal R^3 space and breaks the covering group $SU(2)_L$ of $SO(3)_L$ rather than the group itself.

$e^{i\alpha}\psi$ can be identified with the internal fermion number. The vev of the σ field

$$\langle \sigma \rangle = \frac{1}{\Lambda^2} \langle \bar{\psi}\psi \rangle \tag{9}$$

breaks this symmetry to $SU(2)_D \times U(1)$, where $SU(2)_D$ is the diagonal so called 'custodial' SU(2) group. In the framework of the present model it can be identified with the internal spin SU(2) introduced before, and one concludes that although it is a symmetry of the Higgs potential it is not a symmetry of the system as a whole, because it is broken by the formation of the internal tetrahedron.

Inserting (7) and (8) in (3), the bilinear term $\sim H^+H$ of the Higgs potential has precisely the form of a 4-fermion interaction as appears in the Nambu-Jona-Lasinio

(NJL) Lagrangian[9]

$$L_{NJL} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + \frac{1}{\Lambda_R^2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \tag{10}$$

where m denotes the bare mass of the fundamental fermions $\psi = (U, D)$ and Λ_R^{-2} the NJL-coupling which for dimensional reasons is written in terms of a new scale Λ_R. In technicolor theories this scale is usually interpreted as the mass of a heavy vector boson exchanged between the techniquarks, and is running due to renormalization group effects. Introducing a vev $\langle\bar{\psi}\psi\rangle$ a comparison between (10) and (1), i.e.

$$\mu^2 H^+ H = \frac{1}{\Lambda_R^2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \tag{11}$$

fixes the unknown energy scale Λ in eqs. (7)-(9) in terms of Λ_F and Λ_R. Renormalization effects even allow to derive a gap equation for the mass of the fundamental fermion. For consistency reasons, at low energies $\sim \Lambda_F$ all scales involved $\Lambda \sim \mu \sim \Lambda_R$ must then be of the same order $\mathrm{O}(\Lambda_F)$.

In contrast, at high energies, where there is no condensate and no symmetry breaking potential ($V > 0 \rightarrow L < 0$), the NJL coupling Λ_R^{-2} must be small and negative, the scale Λ_R roughly corresponding to the extension of an internal tetrahedron, cf. fig. 2. In that regime it is thus a repulsive potential and leads to the antiferromagnetic configuration fig. 1. If one is looking closely, one can identify the $\vec{\pi}\vec{\pi}$ term in the original Higgs potential eq. (3) together with (7) as a sort of an internal Heisenberg spin-spin interaction. Such an interaction takes the form

$$H_H = -J\sum_{i\neq j} \vec{\pi}_i \vec{\pi}_j \tag{12}$$

where the sum is over sites i of a given discrete structure and J is the coupling derived from an exchange integral in internal space. $J > 0$ accounts for ferromagnetic attraction and $J < 0$ for antiferromagnetic repulsion. The appearance of a large exchange integral is a quantum effect due the Pauli principle and explains the phenomenon of magnetism in solid state physics. In the present model J is the internal exchange integral defined by integrating over internal coordinates (cf. section 3 for details).

Comparing (12) with (3) one can identify $J = \mu^2/2$, i.e. there is attraction in the SSB regime of energies $\sim \Lambda_F$, where $\mu^2 > 0$. For high energies (small distances)

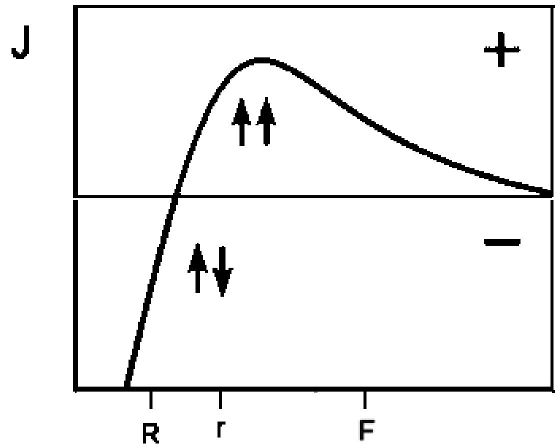

Figure 3: Bethe-Slater curve: the exchange integral J as a function of the distance between 2 internal spin vectors. If the spin vectors lie within one tetrahedron, their distance is small $\sim R$ and according to the figure J is negative. This corresponds to antiferromagnetic behavior and leads to the formation of the frustrated structure fig. 1 with symmetry $A_4 + Q(S_4 - A_4)$, because the spin vectors try to avoid each other as far as possible. In contrast, if the internal spin vectors belong to different tetrahedrons, their distance is large, of order r, and J is positive. This corresponds to ferromagnetic behavior. At distances of the order of the Fermi scale, in the picture denoted by F, one is still in the ferromagnetic regime. Note that in ordinary magnetism the Bethe-Slater curve is used to understand the magnetic behavior of metals. Elements like Fe and Co are characterized by large lattice spacings and corresponding large distances between spin vectors, much larger than the extension of the electron wave function. In these cases one has $J > 0$ and a ferromagnetic behavior. On the other hand, antiferromagnets like Cr and Mn are characterized by small lattice spacings and corresponding small distances between spin vectors, typically not much larger than the extension of the electron wave function. In these cases $J < 0$ and therefore antiferromagnetic behavior.

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where the SSB gets lost, the potential is repulsive with $J < 0$ and leads to the internally frustrated antiferromagnetic configuration of fig. 1. Such an energy dependence of the exchange integral is very well known from the theory of magnetism and is given by the so-called Bethe-Slater curve depicted in fig. 3. The antiferromagnetic repulsion at short distances follows from the Coulomb force which governs the exchange integral J. (Note again that in the model discussed here all quantities are defined and to be taken in the internal space, not in real space.)

In the present case one can even calculate the energy for the local vacuum state fig. 1 and prove that it is a local minimum. To see this, just consider the products $\vec{\pi}_i\vec{\pi}_j = |\vec{\pi}|^2 \cos\alpha_{ij}$ where $|\vec{\pi}|^2$ is the length of the spin vectors and α_{ij} the angle between them $(i,j = 1,2,3,4)$. It can then easily be seen that for the configuration fig. 1 one gets the same energy as for the ideal antiferromagnetic configuration where 2 spin vector show in the +z and the other 2 in the -z direction, namely $\sum_{i\neq j=1}^{4} \vec{\pi}_i\vec{\pi}_j = 2|\vec{\pi}|^2$, while all other configurations give larger values.

When the distances become larger and the energy is lowered towards the Fermi scale, J changes sign due to the Bethe-Slater effect shown in fig. 3, and would in principle lead to an attractive 'ferromagnetic' interaction between 2 distinct tetrahedrons fig. 2, so that an alignment of spin vectors of these tetrahedrons would occur, induced by the last term $\sim \vec{\pi}\vec{\pi}$ in the NJL-Lagrangean eq. (10). A useful order parameter for magnetic systems in such a situation is the total magnetization, in our case the sum of all internal chiral spin vectors over internal and Minkowski space. Unfortunately, in the present case the total internal spin vector is not suitable to use. The point is simply that for a single local ground state configuration fig. 1 the 'magnetization' vanishes:

$$\vec{\pi} = \sum_{i=1}^{4} \vec{\pi}_i = 0 \tag{13}$$

(in agreement with eq. (5)), i.e. effectively there is no internal magnetic interaction between 2 tetrahedrons. In other words, looked at from the distance the internal 'magnetic field' of a single internal tetrahedron cannot be perceived. This is the deeper reason why there are no chiral π-condensates in the Standard Model. One has to search for another order parameter, and that is how the Higgs doublet H comes into play. According to eq.(2) H contains besides the chiral spinvector $\vec{\pi}$ the scalar field σ, and it is this quantity which carries the condensate and should be used as the order parameter, cf. eq.(5).

Strictly speaking one must distinguish the chiral spin vector $\vec{\pi}$ for the local internal ground state in fig. 1, which sums up to zero, from the fields $\vec{\pi}$ in the Higgs doublet, which can be interpreted as the longitudinal modes of the W/Z bosons. Conceptually, they are related to each other in the same way as the vacuum condensate $\langle\sigma\rangle$ is related to the Higgs field ϕ. While the spin vectors can be defined for one tetrahedron alone (just as in ferromagnetism the spin vector $f^+\vec{\tau}f$ can be defined for one electron alone), the bound states, when formed, turn out to be extended objects over many tetrahedrons over Minkowski space.

To summarize the situation, the breaking of the internal symmetries consists in 2 steps:

- The formation of a tetrahedron due to an internal interaction within one single internal space. This interaction is 'antiferromagnetic' and leads to a 'frustrated' configuration, because the spin vectors try to avoid each other but do not achieve to form a completely anti-parallel configuration. Nevertheless, the frustrated tetrahedron breaks internal spin SU(2) as well as internal parity to the Shubnikov group $A_4 + Q(S_4 - A_4)$. This symmetry breaking however is *not* spontaneous but arises from the arrangement of a single 'molecule' due to the internal antiferromagnetic exchange interaction which avoids parallel spin states. (How this kind of internal magnetism can be understood from a more fundamental higher dimensional theory will be explained in section 3.) The local ground state thus is a chiral configuration, i.e. it violates both internal and external parity, and the whole system is left $SU(2)_L$-symmetric - where the precise definition of the group $SU(2)_L$ is as follows:

- Before the SSB each local tetrahedral ground state can rotate independently of the others, i.e. it can freely rotate over its base point in Minkowski space, and this rotational symmetry of the rigid chiral spin vector system corresponds to a $SO(3)_L$ symmetry group, whose covering group defines $SU(2)_L$. It is in fact a local symmetry, because the rotation can be different for tetrahedrons over different base points. The SSB breaking of this group and the corresponding Higgs mechanism consist in the alignment of *all* rigid tetrahedrons over

Minkowski space, cf. fig. 2. In other words, fig. 2 shows the *global* ground state of the universe after the SSB. Unfortunately, this picture is only symbolic, because the condensate $\langle \bar{U}U + \bar{D}D \rangle$ responsible for the SSB does not define a direction in internal R^3 space and breaks the covering group $SU(2)_L$ of $SO(3)_L$ rather than the group itself.

Corresponding to this scenario 2 new energy scales may be introduced: one is the magnitude of the tetrahedron Λ_R, which determines the strength of the NJL-coupling at small distances, and the other the average distance Λ_r between 2 tetrahedrons in Minkowski space (cf. fig. 2). One can also associate these scales to 2 temperatures $T_R > T_r$. Cooling down the universe from big bang temperatures, at about T_R the rigid tetrahedrons are formed in the internal fibers. Afterwards, at T_r, the tetrahedrons are aligned over finite domains of Minkowski space. In this regime of distances the NJL-coupling becomes positive, increases (an effect which in technicolor theories is attributed to the renormalization group, but which I associate to the Bethe-Slater behavior of the internal exchange integral), and finally, at the Fermi temperature, reaches its maximum value, when the local domains combine to extend to the whole universe. In this domain picture the Fermi scale extends over many tetrahedrons, and it is well possible that an additional long range correlation is at work here, similar to the role phonons play in superconductivity.

The expert reader will find that the presented formulas (7)-(11) are similar to what one gets in simple technicolor models[8]. It should be noted, however, that there are 2 important differences: first, there is no need of a (techni)color quantum number here, because my model is a strongly correlated fermion system in the sense of solid state physics and the bound states are formed by these correlations instead of by (techni)color forces. Secondly, the fermions U and D do not interact directly with quarks and leptons[4], and so the model does not have problems with FCNCs. Finally, in technicolor theories the value of the condensate is usually assumed to be

$$\langle \bar{\psi}\psi \rangle \sim \Lambda_F^3 \tag{14}$$

In other words, the extension of the condensates (and of the Higgs particle) is of the order of the Fermi scale. Such a value of the condensate is also appropriate in the present model, although the interpretation is somewhat different (see above).

It is interesting to note that at energies $\leq \Lambda_F$ the microscopic tetrahedral structure does not shine up at all in the Higgs system. The only effect of the tetrahedral structure at low energies are the multiplets of the vibrational and spin excitations eq. (6), to be interpreted as the observed quark and lepton spectrum.

I repeat that the internal spin transformations are local in the sense, that the tetrahedrons can be rotated *independently* over different points of the base space (Minkowski space). One has here a fiber space, where each fiber has a discrete crystalline structure. Connections can be defined over the fibers, which give rise to the gauge fields. While the photon is a story of its own to be discussed in the next section, the explicit construction of the $SU(2)_L$ gauge fields may easily be sketched, because it is quite similar to the construction of the Higgs doublet. In fact they are also bound states of the fundamental fermions U and D and differ from the chiral internal spin vector field $\vec{\pi}$ only by their Lorentz behavior:

$$\vec{W} = \frac{1}{\Lambda^2}(\bar{\psi}\gamma_\mu(1-\gamma_5)\vec{\tau}\psi) \tag{15}$$

Due to the appearance of the factor $\vec{\tau}$ they are polarized in internal space by the internal chiral vacuum fig. 1, and, as shown in the next section, this polarization will carry down to give a handedness $\sim 1-\gamma_5$ in Minkowski space, providing the chiral nature of the weak interactions.

3 The background Scenery: QED in 7 and 8 dimensions

The interested reader may worry, what kind of dynamical framework can account for the phenomena described in the last section. The correlations between the internal ($\vec{\tau}$) and external (γ_5) axial structures are so intriguing that one is tempted to look for a unifying higher dimensional model which comprises all the necessary features. An immediate suggestion is a 6+1 dimensional space R^{6+1} with SO(6,1) symmetry which fibers out to give the R^3 fibers discussed in the last section. This fibration may be associated with the formation of the tetrahedrons (fig. 1) at scale Λ_R, which span a 3-dimensional subspace of R^{6+1} and may in fact be used to define what a fiber is. Namely, the fibers can be *defined* to be spanned by the tetrahedrons, while everything orthogonal is called Minkowski space. One then has

$$SO(6,1) \to SO(3,1) \times SO(3) \tag{16}$$

While the base space is to be identified with physical Minkowski space and its SO(3,1) Lorentz group, a kind of non-relativistic physics with rotational SO(3) will take place in the fibers.

As to the dynamics I suggest to consider 6+1-dimensional QED ('QED7') broken down to the above fiber space, and want to show how this splits into ordinary QED4 plus a non-relativistic electrodynamics in the fibers, i.e. an interaction which to some extent can be discussed within the so-called NRQED framework[12, 13]. A major difference as compared to ordinary NRQED will be the appearance of a chiral factor γ_5 which prevents the whole problem from being fully factorizable and bends the fibers down to the base space and furthermore relates internal and external chiralities.

The main ingredients of QED7 are a SO(6,1) spinor field (the QED7-'electron') and a massless vector field (the QED7-'photon'). As for the fermion there is a single 8-dimensional spinor representation in SO(6,1) decomposing as[10]

$$8 \to (1,2,2) + (2,1,2) \tag{17}$$

under the fibration $SO(6,1) \to SO(3,1) \times SO(3)$. Here representations of $SO(3,1) \times SO(3)$ are denoted by a set of 3 numbers (a,b,c), where (a,b) are representations of the Lorentz group and c is the dimension of a $SO(3)$-representation. For example, c=2 corresponds to a non-relativistic Pauli spinor in internal space, whose 2 spin orientations are identified with the SU(2) flavors U and D introduced in the last section. It should be noted that (1,2,2) and (2,1,2) are complex conjugate with respect to each other, so one is the antiparticle representation of the other.

The QED7-photon transforms according to the fundamental 7-dimensional representation of SO(6,1) and decomposes as[10]

$$7 \to (2,2,1) + (1,1,3) \tag{18}$$

i.e. into an ordinary QED4 photon which is a singlet under internal spin SU(2) plus an internal 3-dimensional vector potential to describe the internal photon.

The Lagrangian of QED7 resembles that of QED4

$$L_{QED7} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\Gamma_\mu D^\mu - m)\psi \tag{19}$$

where μ and ν run from 0 to 6, ψ is the 8-dimensional spinor of eq. (17), Γ_μ are the Dirac matrices in 6+1 dimensions and $D^\mu = \partial^\mu + ieA^\mu$ is the covariant derivative containing the 7-vector multiplet A^μ of eq. (18).

To make the decomposition $SO(6,1) \to SO(3,1) \times SO(3)$ explicit one should decompose the corresponding 6+1 dimensional Dirac algebra. In fact the Dirac matrices of SO(6,1) are 8×8 matrices and can be built up as tensor products of Pauli matrices[18]

$$\Gamma_0 = \tau_1 \otimes \tau_0 \otimes \tau_0 \tag{20}$$
$$\Gamma_1 = i\tau_2 \otimes \tau_0 \otimes \tau_0 \tag{21}$$
$$\Gamma_2 = i\tau_3 \otimes \tau_1 \otimes \tau_0 \tag{22}$$
$$\Gamma_3 = i\tau_3 \otimes \tau_2 \otimes \tau_0 \tag{23}$$
$$\Gamma_4 = \underline{i\tau_3 \otimes \tau_3} \otimes \tau_1 \tag{24}$$
$$\Gamma_5 = \underline{i\tau_3 \otimes \tau_3} \otimes \tau_2 \tag{25}$$
$$\Gamma_6 = \underline{i\tau_3 \otimes \tau_3} \otimes \tau_3 \tag{26}$$

where the first 2 columns on the rhs correspond to Lorentz SO(3,1) and the last column to internal SO(3). τ_0 is the 2-dimensional unit matrix, so that the first 4 Γ-matrices $\Gamma_{0,1,2,3} = \gamma_{0,1,2,3} \otimes \tau_0$ give a set of Dirac matrices in Minkowski space. The last 3 Γ-matrices have the form $\Gamma_{4,5,6} = i\tau_3 \otimes \tau_3 \otimes \tau_{1,2,3}$, i.e. they are proportional to $\vec{\tau}$ in the internal space part. The interesting point to note here is the appearance of a common prefactor $i\tau_3 \otimes \tau_3$ in $\Gamma_{4,5,6}$, which due to $\tau_1\tau_2 = i\tau_3$ is nothing else than the matrix $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$ in Minkowski space. We thus have $\Gamma_{4,5,6} = \gamma_5 \otimes \tau_{1,2,3}$ and this will in fact lead to the anticipated appearance of products of the form $\gamma_5\vec{\tau}$ in the internal interactions, which is responsible for the structure of the NJL-Lagrangian eq. (10). As will shortly be seen, this makes sure that internal parity violating effects from the A_4 symmetry structure are passed down to Minkowski space. In more mathematical terms it can be related to the structure and existence of octonion multiplication, when 6+1 spacetime is assumed to be spanned by the octonion units I,J,K,L,IL,JL and KL[6, 7, 3].

Now that we have established products $\gamma_5 \times \vec{\tau}$ in the couplings, we can try to derive the parity violation of the weak interaction. In principle, the presence of such a coupling corresponds already to a parity violating behavior, both in internal and Minkowski space. However, for this to actually become perceivable, an additional appropriate 'chiral situation' has to be provided, again both in internal and Minkowski space. In Minkowski space this can be achieved, for example, by using polarized beams or if there is a second vertex with a γ_5-coupling in the diagram. An analogous requirement must be met in the internal space. In other words, a configuration with a handedness must be present, in order to pick up a non-vanishing contribution from the coupling, and this in the case at hand is given by the local chiral ground state structure fig. 1. In fact, the non-relativistic circumstances of the internal R^3 space make it a similar situation as one has in optical activity of molecules, where in addition to a circularly polarized photon there must be a handed molecule in order to produce a non-vanishing effect.

Writing $A^\mu = (\tilde{A}^{\mu=0-3}, \vec{C})$ in eq. (19) one can make explicit the separation of ordinary QED and the internal interaction

$$\bar{\psi}\Gamma_\mu A^\mu \psi = \bar{\psi}\gamma_\mu \tilde{A}^\mu \psi + \bar{\psi}\gamma_5 \vec{\tau}\vec{C}\psi \tag{27}$$

The second term looks quite promising, because it corresponds to a chiral interaction in Minkowski space. Unfortunately, it is of order of the electromagnetic coupling and not large enough to explain the Bethe-Slater effect fig. 3. On the other hand we know since the time of Heisenberg[14], that ordinary magnetism is purely an effect of the Coulomb interaction plus the Pauli principle, which lead to the exchange integral J. What is therefore missing in the above equation, is an internal scalar potential C_0 to provide for the Coulomb force.

To introduce such a field we restart by considering one more dimension, namely a space with SO(6,2) symmetry group instead of SO(6,1)and decompose it as

$$SO(6,2) \to SO(3,1) \times SO(3,1) \to SO(3,1) \times SO(3) \tag{28}$$

i.e. we allow for a separate dynamics and time evolution within the fibers. Afterwards however (second arrow in eq. (28)), the fibers are fixed to their base point in Minkowski space and become non-relativistic at rest (no boosts allowed) with symmetry group SO(3).

Introducing this preface step does not make much of a difference concerning the fermions, but it affects the internal photon in the desired way. To see this, one should remember that SO(6,2) possesses three 8-dimensional spinor representations. Two of these are Weyl representations (8_L and 8_R) which build up a 16-dimensional Dirac field, just as in 3+1 dimensions a Dirac fermion can be built from two 2-dimensional Weyl representations. The third 8-dimensional spinor representation 8_V coincides with the fundamental 8-dimensional vector representation of SO(6,2). The fact that these SO(6,2) representations appear in 3 inequivalent forms is known as triality[5], a characteristic property of this group, which makes it very special indeed and again goes back to the existence of the division algebra of octonions. When one decomposes these representations according to (28) one obtains[10]

$$8_L \rightarrow (1,2,1,2)+(2,1,2,1) \rightarrow (1,2,2)+(2,1,2) \tag{29}$$

$$8_R \rightarrow (1,2,2,1)+(2,1,1,2) \rightarrow (1,2,2)+(2,1,2) \tag{30}$$

$$8_V \rightarrow (2,2,1,1)+(1,1,2,2) \rightarrow (2,2,1)+(1,1,3)+(1,1,1) \tag{31}$$

Here representations of $SO(3,1)\times SO(3,1)$ are denoted by (a,b,c,d), where the first 2 numbers (a,b) stand for representations of the Lorentz group, and (c,d) characterize representations of the internal SO(3,1). While (29) and (30) yield the same structure as eq. (17) in the limit eq. (16), the expression (31) does not agree with eq. (18), but according to eq. (31) contains the desired singlet field (1,1,1). Note that (2,2,1,1) resp. (2,2,1) denotes the ordinary photon, while (1,1,2,2)→ (1,1,3)+(1,1,1) is the (internally) non-relativistic decomposition of the internal 'photon'.

One may also construct the SO(6,2) Dirac spinor as the sum of 8_L and 8_R

$$8_L+8_R \rightarrow (1,2,1,2)+(2,1,1,2)+(1,2,2,1)+(2,1,2,1) = (\underline{12+21},\underline{12+21})$$

where the underlined expression is a short form which makes it easy to understand, that it decomposes into a Dirac fermion in Minkowski space times a Dirac fermion in the internal space. This object should be chosen as the 'QED8-electron' and enters the QED8-Lagrangian which formally has the same structure as eq. (19) with the indices now running from 0 to 7.

The Dirac matrices of SO(6,2) entering the QED8-Lagrangian will be called G_μ and consist of 16×16 Matrices which can be written as tensor products of the form

$$G_0 = \tau_1 \otimes \tau_0 \otimes \tau_0 \otimes \tau_0 \tag{32}$$

$$G_1 = i\tau_2 \otimes \tau_0 \otimes \tau_0 \otimes \tau_0 \tag{33}$$

$$G_2 = i\tau_3 \otimes \tau_1 \otimes \tau_0 \otimes \tau_0 \tag{34}$$

$$G_3 = i\tau_3 \otimes \tau_2 \otimes \tau_0 \otimes \tau_0 \tag{35}$$

$$G_4 = \underline{\tau_3 \otimes \tau_3} \otimes \tau_1 \otimes \tau_0 \tag{36}$$

$$G_5 = \underline{i\tau_3 \otimes \tau_3} \otimes \tau_2 \otimes \tau_0 \tag{37}$$

$$G_6 = \underline{i\tau_3 \otimes \tau_3} \otimes \tau_3 \otimes \tau_1 \tag{38}$$

$$G_7 = \underline{i\tau_3 \otimes \tau_3} \otimes \tau_3 \otimes \tau_2 \tag{39}$$

where the first 2 columns on the rhs correspond to the Lorentz group and the last 2 to the internal SO(3,1). Looking closely at these equations one understands that $G_{0,1,2,3}$ yield the ordinary Dirac matrices in Minkowski space (up to trivial factors $\tau_0 \otimes \tau_0$), and $G_{4,5,6,7}$ yield Dirac matrices in the internal space - however with a prefactor $\gamma_5 \sim \tau_3 \otimes \tau_3$.

Taking the non-relativistic limit in the internal fibers we end up with the γ-matrices discussed after eq. (26). Furthermore, there is an internal scalar potential $C_0 = A_4$ in addition to the internal vector potential $\vec{C} = A_{\mu=5,6,7}$. What we have achieved then is that we can apply the ordinary NRQED machinery to the internal spaces (except for the appearance of factors of γ_5 in the interactions). For example, the internal dynamics will be governed by the (slightly modified) Pauli Lagrangian

$$L_{2f} = \bar{\psi}\{i\partial_t + \frac{1}{2m}\vec{D}^2 + \frac{e}{2m}\gamma_5\vec{\tau}\vec{B}\}\psi \tag{40}$$

where m and e are the mass and charge of the fundamental fermion $\psi = (U, D)$. $D_t = -\partial_t + ieC_0$ and $\vec{D} = -\vec{\nabla} + ie\vec{C}$ are covariant derivatives in the internal dimensions. $\vec{B}$ is the internal magnetic field strength of the internal photon, C_0 its desired scalar and $\vec{C}$ its vector potential. The Pauli Lagrangian may be considered as the leading order NRQED contribution to the 2-fermion interactions of internal NRQED. There is also a leading 4-fermion Lagrangian[12, 13] which contains the terms arising in the NJL-Lagrangian (10):

$$L_{4f} = \frac{d_s}{m^2}(\bar{\psi}\psi)^2 + \frac{d_v}{m^2}(\bar{\psi}\gamma_5\vec{\tau}\psi)^2 \tag{41}$$

with $d_s, d_v = O(\alpha)$ being couplings of the effective NRQED field theory. They are obviously too small to account for the internal magnetic effects discussed in the last section. Strong correlations are needed to explain the formation of the internal tetrahedral structure. As discussed earlier, these naturally arise as exchange contributions via the (internal) Pauli principle.

4 Conclusions

In this paper the SM Higgs mechanism has been analyzed on the basis of dynamics taking place in a 3-dimensional internal space with a chiral tetrahedral structure. It was shown that weak parity violation can be completely understood from interactions within one single tetrahedron and has no spontaneous character. In contrast, the breaking of $SU(2)_L$ is spontaneous and due to an alignment of *all* internal fibers over the whole of Minkowski space.

Since this is a rather unusual approach it may seem hard to understand where it comes from and to where it will lead, in particular because I have mostly restricted myself to the symmetry breaking aspects and did not consider other questions[3, 4]. In fact, there is a certain physical picture in my mind where our universe resembles a huge crystal of molecules, each 'molecule' of tetrahedral form like in fig. 1, and arranged in such a way that certain symmetries are (spontaneously) broken. For such a model to be consistent, a 6+1 dimensional space time has been introduced, i.e. the 'molecules' extend to 3 internal dimensions which are orthogonal to physical space. The strong correlations within this system provide the Higgs particle and the weak vector bosons as bound states. Furthermore, as shown in ref. [4], internal spin and vibrational excitations can be interpreted as the quark and lepton spectrum. Then, it happens that an excitation in one molecule is able to excite an excitation in the neighbouring internal space and thus can travel as a quasi-particle through Minkowski space with a certain wave vector $\vec{k}$ which is to be interpreted as the physical momentum of the particle.

In ferromagnets with Pauli spinors $f = (f_\uparrow, f_\downarrow)$ the appropriate order parameter is the sum of the spin vectors

$$\vec{S} = f^+ \vec{\tau} f \tag{42}$$

whereas in superconductivity the condensate of Cooper pairs

$$< f_\uparrow f_\downarrow > \tag{43}$$

determines the order of the system. The phenomena of high energy physics are such that the relevant quantity is the (internal + relativistic) generalization of (43), but not that of (42). In section 2 an intriguing explanation was found for this fact. Furthermore it was shown that the chiral spin vectors eq. (7) play an important role in the dynamics of the system. They are not only essential ingredients of the Higgs doublet and the NJL Lagrangian, but in the internal spaces they interact in an 'antiferromagnetic' way, thereby determining the local ground state of the system.

I have finished this paper leaving a lot of open questions. For example, the mixing of the photon and Z-boson has not been worked out. Then there is the question, whether the fundamental fermions U and D are in principle observable or whether they act just as a sort of background fields for the physical excitations. Thirdly, it is unclear, whether the condensate $\langle\bar\psi\psi\rangle = \langle\bar U U + \bar D D\rangle$ is really $SU(2)_D$-symmetric or whether $\langle\bar U U\rangle \neq \langle\bar D D\rangle$. This is a well justified question in view of the fact that the chiral tetrahedron breaks $SU(2)_D$. And finally, there is the question, which force keeps the original tetrahedron (the circles in fig. 1) together.

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