Six conjectures and the generic formulas for two subsets of Poulet numbers

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Abstract. I was following an interesting "track", i.e. the pairs of primes [p,q] that apparently can form strictly Carmichael numbers of the form p*q*(n*(q – 1) + p), like for instance [23,67] and [41,241], when I observed that also all the Poulet numbers P which have the numbers p = 30*k + 23 and q = 90*k + 67 respectively p = 30*k + 11 and q = 180*k + 61 as prime factors can be written as P = p*q*(n*(q – 1) + p) and I made few conjectures.

The generic formula for Poulet numbers which have two prime factors of the form 30k + 23 and 90k + 67

Conjecture 1:

Any Poulet numbers P which have the numbers p = 23 and q = 67 as prime factors can be written as P = p*q*(n*(q – 1) + p) = 3*p^3*(3*n + 1) – p^2*(15*n + 2) + 6*p*n, where n non-null positive integer (we took q = 3*p – 2).

Verifying the conjecture (for the first few such Poulet numbers):

: For n = 1 we have the Poulet number P = 137149 = 23*67*89;
: For n = 3 we have the Poulet number (also Carmichael number) P = 340561 = 13*17*23*67;
: For n = 9 we have the Poulet number P = 950797 = 23*67*617;
: For n = 10 we have the Poulet number P = 1052503 = 23*67*683;
: For n = 13 we have the Poulet number P = 1357621 = 23*67*881.

Comment:

This formula is important for determining sequences of Poulet numbers; in their case there is not an instrument for obtaining such formulas as there is the Korselt's criterion in the case of Carmichael numbers. See also the sequence A182515 that I submitted to OEIS.

Note:

The formula P = p*q*(n*(q – 1) + p) is not a pattern for any Poulet numbers which have two prime factors of the form p and q = 3*p – 2; for instance, for [p,q] = [7,19] and Carmichael numbers 1729 and 63973 the formula doesn’t apply.

Conjecture 2:

Any Poulet numbers P which have the numbers p = 30*k + 23 and q = 90*k + 67, where k non-negative integer, as prime factors can be written as P = 3*p^3*(3*n + 1) – p^2*(15*n + 2) + 6*p*n, where n non-null positive integer.

Note:

As it can be seen, the formula from above it is not anymore derived from and equivalent to the formula P = p*q*(n*(q – 1) + p), equivalence that exists only in the case of the Conjecture 1.

Verifying the conjecture for p = 53 and q = 157 (for the first few such Poulet numbers):

: For n = 3 we have the Poulet number (also Carmichael number) P = 4335241 = 53*157*521;

: For n = 10 we have the Poulet number P = 13421773 = 53*157*1613;

: For n = 13 we have the Poulet number (also Carmichael number) P = 17316001 = 53*157*2081.

Verifying the conjecture for p = 113 and q = 337 (for the first few such Poulet numbers):

: For n = 1 we have the Poulet number (also Carmichael number) P = 17098369 = 113*337*449;

: For n = 7 we have the Poulet number (also Carmichael number) P = 93869665 = 5*17*29*113*337;

: For n = 13 we have the Poulet number P = 170640961 = 113*337*4481.

Note:

It is notable how easily we found Poulet numbers with this formula, for at least three values of n from n = 1 to n = 13, for any of the three pairs of primes considered: [23,67], [53,157], [113,337].

Conjecture 3:

There is an infinity of Poulet numbers which have the numbers p = 30*k + 23 and q = 90*k + 67, where k non-negative integer, as prime factors (implicitly there is an infinity of pairs of primes of the form [30*k + 23, 90*k + 67]).

The generic formula for Poulet numbers which have two prime factors of the form 30k + 11 and 180k + 61

Conjecture 4:

Any Poulet numbers P which have the numbers p = 11 and q = 61 as prime factors can be written as P = p*q*(n*(q – 1) + p) = 6*p^3*(6*n + 1) – p^2*(66*n + 5) + 30*p*n, where n non-null positive integer (we took q = 6*p – 5).

Verifying the conjecture (for the first such Poulet number):

: For n = 21 we have the Poulet number (also Carmichael number) P = 852841 = 11*31*41*61.

Note:

The formula P = p*q*(n*(q – 1) + p) is not a pattern for any Poulet numbers which have two prime factors of the form p and q = 6*p – 5; for instance, for [p,q] = [7,37] and Carmichael number 63973 = 7*13*19*37 the formula doesn't apply.

Conjecture 5:

Any Poulet numbers P which have the numbers p = 30*k + 11 and q = 180*k + 61, where k non-negative integer, as prime factors can be written as P = 6*p^3*(6*n + 1) – p^2*(66*n + 5) + 30*p*n, where n non-null positive integer.

Verifying the conjecture for p = 41 and q = 241 (for the first few such Poulet numbers):

: For n = 2 we have the Poulet number (also Carmichael number) P = 5148001 = 41*241*521;
: For n = 3 we have the Poulet number (also Carmichael number) P = 7519441 = 41*241*761;
: For n = 4 we have the Poulet number (also Carmichael number) P = 9890881 = 7*11*13*41*241;
: For n = 5 we have the Poulet number (also Carmichael number) P = 12262321 = 17*41*73*241.

Conjecture 6:

There is an infinity of Poulet numbers which have the numbers p = 30*k + 11 and q = 180*k + 61, where k non-negative integer, as prime factors (implicitly there is an infinity of pairs of primes of the form [30*k + 11, 180*k + 61]).