The Concept of the Effective Mass Tensor in GR

The Gravitational Waves

Mirosław J. Kubiak

Zespół Szkół Technicznych, Grudziądz, Poland

Abstract: *In the paper [1] we presented the concept of the effective mass tensor (EMT) in General Relativity. In this paper we consider the concept of the EMT but in the aspect of the gravitational waves.*

keywords: *general theory of relativity; the effective mass tensor; the gravitational waves*

I. Introduction

In the Einstein's General Relativity (GR), gravitational waves are fluctuations of the gravitational fields (or the ripples in the curvature of the space-time) which are propagate as a wave and they are generated mainly by moving massive bodies. A. Einstein predicted these waves in 1916 as a consequence of the GR. According to him the gravitational waves should theoretically transport energy as a gravitational radiation. Sources of detectable gravitational waves could possibly include binary star systems composed of white dwarfs, neutron stars or black holes [2].

Although gravitational radiation has not been *directly* detected, there is *indirect* evidence for its existence. For example, the 1993 Nobel Prize in Physics (J. H. Taylor and R. A. Hulse) was awarded for measurements of the *PSR B1913+16* (also known as *PSR J1915+1606* and *PSR 1913+16* and the Hulse -Taylor pulsar) system *which suggests gravitational waves are more than mathematical anomalies* [3, 4]. Though many various gravitational wave detectors exist (a some examples of the gravitational wave detectors – see the Table I) they still remain not discovers.

In the Section II we will present the gravitational waves in the GR, in the Section III – the concept of *the effective mass density tensor* (EMDT) and the gravitational waves. In the Section IV we will compare a few the physical features concerning of the space-time curvature with the conception of the EMDT. Our conclusions we will present in the Section V.

II. The gravitational waves in the GR

As we know the Einstein's field equation has the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

where: $R_{\mu\nu}$ is the Ricci curvature tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, G is Newton's gravitational constant, c is the speed of light in the vacuum, and $T_{\mu\nu}$ is the stress-energy tensor.

Our considerations we will realize in the weak of the gravitational field, which allows us to decompose the metric tensor $g_{\mu\nu}$ into the flat Minkowski metric plus a small perturbation $h_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2}$$

where: $|h_{\mu\nu}| << 1$. We will restrict ourselves to coordinates in which $\eta_{\mu\nu}$ takes its canonical form, $\eta_{\mu\nu}$ = diag(-1, +1, +1, +1). Well-known calculation (see to [5]) give the wave equation

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}\tau_{\mu\nu} \tag{3}$$

where: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} \cdot h$ and the gauge condition $\partial_\mu \bar{h}^\mu_\lambda = 0$. The tensor $\tau_{\mu\nu}$ describes the distribution of the matter, which disturbs the gravitational field [5]. In the vacuum the eq. (3) has the form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = 0 \tag{4}$$

Equations (3 and 4) we can interpret as *the metric perturbations – the fluctuations of the space-time curvature produced by disturbing the metric tensor* $\bar{h}_{\mu\nu}$ *– propagate at the speed c as a wave in the free space*. The tensor wave eq. (4) has the solution with the form

$$\bar{h}_{\mu\nu} = A_{\mu\nu} e^{[ik(z-ct)]} \tag{5}$$

which represents a monochromatic wave of the space-time geometry propagating along the $+z$ direction with speed c and frequency kc.

III. The concept of the EMT and the gravitational waves

Let's assume that in the gravitational field

$$g_{\mu\nu} = \frac{\rho_{\mu\nu}}{\rho} \tag{6}$$

where: $\rho_{\mu\nu}$ is *the effective mass density tensor* (EMDT), ρ is *the bare mass density*. The metric we can express by

$$ds^2(g_{\mu\nu}) = ds^2(\rho_{\mu\nu}) \tag{7}$$

where: $ds^2(g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$ and $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho} dx^\mu dx^\nu$ [1].

In the weak gravitational field we can decompose of the EMDT of the body to the simple form: $\rho_{\mu\nu} = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^{*}$, where: $\rho_{\mu\nu}^{bare} = \rho \cdot \eta_{\mu\nu} = \mathrm{diag}(-\rho, +\rho, +\rho, +\rho)$ we will call *the bare mass density tensor*, $\eta_{\mu\nu}$ is the Minkowski tensor, $\rho_{\mu\nu}^{*} = \rho \cdot |h_{\mu\nu}| << 1$ is a small EMDT "perturbation" . Note that in the absence of the gravitational field the EMDT $\rho_{\mu\nu}$ becomes *the bare mass density tensor* and $\rho_{\mu\nu} \to \rho_{\mu\nu}^{bare}$.

The wave equation (eq. 3) has now the form

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\frac{\overline{\rho}_{\mu\nu}^{*}}{\rho} = -\frac{16\pi G}{c^4}\tau_{\mu\nu} \tag{10}$$

where: $\overline{\rho}_{\mu\nu}^{*} = \rho_{\mu\nu}^{*} - \frac{1}{2}\rho \cdot \eta_{\mu\nu}$ and the gauge condition $\partial_{\mu}\overline{\rho}_{\sigma}^{\mu} = 0$. In the vacuum the eq. (10) has the form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\frac{\overline{\rho}_{\mu\nu}^{*}}{\rho} = 0 \tag{11}$$

Equations (10) and (11) we can interpret as *the small perturbation of the effective mass density* $\frac{\overline{\rho}_{\mu\nu}^{*}}{\rho}$, *which propagates with the speed c as a wave.* The tensor wave eq. (11) has the solution with the form

$$\frac{\overline{\rho}_{\mu\nu}^{*}}{\rho} = A_{\mu\nu} e^{[ik(z-ct)]} \tag{12}$$

which represents *a monochromatic wave in the effective mass density* propagating along the +*z* direction with speed *c* and frequency *kc*.

In the GR the gravitational waves are the ripples in the curvature of the space-time that propagate as a wave. The concept of the EMDT predict that the gravitational waves are *perturbation in the effective mass density* that propagate as a wave. And although in both cases the gravitational waves are propagated with the speed of light, it **the way of their detecting should be different**. (see to Table I).

IV. The space-time curvature vs. the effective mass density tensor

Let's compare a few physical features concerning of the space-time curvature with the physical features of the EMDT discussed in this paper. The results of this comparison are presented in Table I below.

Table. I. The space-time curvature vs. the effective mass density tensor.

The space-time curvature	The effective mass density tensor
The metric tensor $g_{\mu\nu}$	*The effective mass density tensor* $\rho_{\mu\nu} = \rho \cdot g_{\mu\nu}$

The weak field approximation $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$ The weakness of the gravitational field is expressed as ability to decompose the metric tensor into the flat Minkowski metric tensor plus a small “perturbation” tensor, $\lvert h_{\mu\nu} \rvert << 1$.	*The weak field approximation* $$\rho_{\mu\nu} = \rho(\eta_{\mu\nu} + h_{\mu\nu}) = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^{*}$$ The weakness of the gravitational field is expressed as ability to decompose the EMDT to *the bare mass density tensor* plus a small “perturbation” of the EMDT tensor $\rho_{\mu\nu}^{*} = \rho \cdot \lvert h_{\mu\nu} \rvert << 1$.
The gravitational waves in the weak field $$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}\tau_{\mu\nu}$$	*The gravitational waves in the weak field* $$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\frac{\bar{\rho}_{\mu\nu}^{*}}{\rho} = -\frac{16\pi G}{c^4}\tau_{\mu\nu}$$
The solution The tensor wave eq. (4) has the solution with the form $\bar{h}_{\mu\nu} = A_{\mu\nu}e^{[ik(z-ct)]}$ which represents a monochromatic wave of space-time geometry propagating along the +*z* direction with speed *c* and frequency *kc*.	*The solution* The tensor wave eq. (11) has the solution with the form $\frac{\bar{\rho}_{\mu\nu}^{*}}{\rho} = A_{\mu\nu}e^{[ik(z-ct)]}$ which represents a monochromatic wave of the effective mass density propagating along the +*z* direction with speed *c* and frequency *kc*.
The physical interpretation of the wave equation The gravitational waves are ripples in the curvature of the space-time that propagate as a wave with the speed *c* as a wave.	*The physical interpretation of the wave equation* The perturbation in the effective mass density $\frac{\bar{\rho}_{\mu\nu}^{*}}{\rho}$, which propagates through the space-time with the speed *c* as a wave.
What we measure in the detector? Contemporary detector can measure the dimensionless amplitude $\bar{h}_{\mu\nu} = \frac{\Delta L}{L}$, which is generated by the gravitational waves.	*What should we measure in the detector?* Designed detector should measure the dimensionless amplitude $\frac{\bar{\rho}_{\mu\nu}^{*}}{\rho} = \frac{\Delta\rho^{*}}{\rho}$, which is generated by the gravitational waves.
Some examples of the gravitational wave detectors: 1. Ground based: *GEO 600, LIGO, Virgo* [6]. 2. Space-based: *LISA* [7]. 3. Pulsar Timing Arrays [8].	*The gravitational wave detector* ?
The results of search Though many various gravitational wave detectors exist the gravitational waves still remain not discover.	*The results of search* Waiting for discovery.

V. Conclusion

In the GR the gravitational waves are the ripples in the curvature of the space-time that propagate as a wave. The concept of the EMDT predict that the gravitational waves are *the perturbation in the effective mass density* that propagate as a wave.

And although in both cases the gravitational waves are propagated with the speed of light, it the way of their detecting should be different, what open a new ways to searches of the gravitational waves.

Reference

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