The motivations for introducing a “new” mathematical tool within the general theory of relativity (A. Einstein’s work - GTR) have been intensively exposed in a first paper [vixra.org; 1311.0004]. This “companion” paper develops an argumentation allowing the intervening of the same tool in a pure 4D space. So far we know, it is one of the first theoretical works connecting the GTR and the Quantum Theory in a plausible manner at the Planckian limit.

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The challenge
In a 4D space, the existence of a relation like $|\bigtriangleup \cdot \Phi_{\alpha}(d^4r, (4)q) > - \{^{(4)}[P]\}_{\text{int.}} |^{(4)}q > + |^{(4)}z >} = |0 > (01)$
would generate the necessity of a gigantic calculation $F(dr) = \text{det}\{\Phi(\text{dr}) - [P]\} = |A^{\mu}_{\beta}. X' - p_{\alpha\beta}| (02)$.

Do we have to do it?
That gigantism is a sufficient argument to ask if such calculation is really justified. With the spirit evocated in [TP- vixra.org: 1308.0106], a relation like (01) is supposed to be associated with a solution of the theory of relativity [04]. Since our purpose is a development of the theory in a fully general 4D space, the unique but fundamental difference is that we do no more have to evoke the ADM procedure.

The next question is: “Do we have, somewhere in the scientific literature, a discussion justifying the existence of a relation like (01) within the context of Einstein’s work (Up to now, we shall write: “within the context of the generalized theory of relativity or GTR context?”
It took a lot of time to find the answer but the latter is spelled “yes”. And, in fact, that positive answer is the result of a confrontation between a modern method proposed by myself and an old article [03].

The extrinsic method

Let us do a “reductio ad absurdum” and a priori accept the existence of (01) within a GTR context. Provided that that context yields the cube ▽A and the target (4)q in some natural way, our problem is reduced to the discovery of a realistic pair (4)[P, (4)z]. We simply face what we have called the resolution of the (E) question in a 4D space. As mentioned previously, the discovery of the divisor, (4)[P], via an intrinsic procedure is imposing the terrific calculation of (02) and since the nature is a lazy lady, this is forcing us to develop a ruse. We have called it: the extrinsic method. The name comes from the fact that we associate a quantity, precisely a scalar "χ", and a bilinear form B, which are a priori not related to the (E) question but are needed to find the solutions.

\[ \chi^+ = <q, \triangle_{\nabla A}(dr, q) - [P]_{ext} \cdot |q > + |z > \cdot B \]

Remark: If there is a vector (4)a such that:

(04-Hypothesis)

\[ (4)q = d(4)a, \]

this is:

(05)

\[ \chi^+ = b_{\alpha\beta}. da^\alpha. (A_{\lambda\mu}^\beta. dx^\lambda. da^\mu - (p_{\mu\nu}. da^\nu + z^\nu)) \]

We also can write it:

(05-bis)

\[ 0 = b_{\alpha\beta}. (A_{\lambda\mu}^\beta. dx^\lambda - p_{\mu\nu}). da^\alpha. da^\mu + b_{\alpha\beta}. z^\beta. da^\mu - \chi^+ \]

The formalism of the previous relation suggests the essence of the extrinsic method. Indeed, let us consider some function T(a); provided the condition of continuity holds (important for a future application of that work in the set of complex numbers):

(Continuity)

\[ \partial_{\mu\alpha} T(a) = \partial_{\alpha\mu} T(a) \]

We then may recognize in the previous relation (05-bis) a kind of Taylor Mac-Laurin development for that T(a) function:

(06)

\[ T(a + da) = T(a) + \sum_{\alpha} \partial_{\alpha} T(a). da^\alpha + \frac{1}{2} \sum_{\alpha} \sum_{\beta} \partial_{\mu\alpha} T(a). da^\alpha. da^\beta + O_3 \]

For example, this is true each time we can write simultaneously:

(07)

\[ T(a + da) = T(a) \]

(08)

\[ \partial_{\alpha} T(a) = b_{\alpha\beta}. z^\beta \]

(09)

\[ \frac{1}{2} \partial_{\mu\alpha} T(a) = b_{\alpha\beta}. (A_{\lambda\mu}^\beta. dx^\lambda - p_{\mu\nu}) \]
Remark: the extrinsic method obviously applies to (approximately) periodic and continuous \(T(a)\) functions, for any kind of cube (anti-symmetric or not) and in any D space. That method may thus have many concrete applications in condensed matter contexts, in gauge lattice theories and in any situation where a periodic structure is involved.

Remark: Provided the local representation of the bilinear form \(B\) is invertible, the extrinsic method yields an extrinsic main divisor depending (indirectly) on the residual part of the split (01).

\[
[P]_{\text{extrinsèque}} = \lambda \Phi(a) - \frac{1}{2}. B^1 \cdot \text{Hess}_0 T(a)
\]

Remark: Provided we would have made the gigantic calculation (02) and discovered the intrinsic main divisors for (01), the extrinsic method would be now able to complete this first approach in yielding the missing part of the solutions, namely the residual part of the split (01), via the resolution of a combination of (08 and 09):

\[
\frac{1}{2}. \left( \tilde{c}_\alpha b_{\alpha \beta}. z^\beta + b_{\alpha \beta}. \tilde{c}_\mu z^\mu \right) = b_{\alpha \beta}. \left( A_{\alpha \beta} \cdot dx^\lambda - p_{\beta \mu} \right)
\]

Coming back to the end of the 19\textsuperscript{th} century

At this stage, the reader may be a little bit lost and think that we are promoting the extrinsic method instead of justifying the existence of (04). This is absolutely not the case. A mathematical problem treated by E. Christoffel in [02] is associated with the strategic Morley and Michelson experiment [01]; namely, the preservation of a generic covariant bilinear form [03, p. 390, §2, (1)]:

\[
ds^2 = a_{\alpha \beta}. dx_\alpha \cdot dx_\beta
\]

The Christoffel’s work is mentioned in [03, p. 391, § 3] and is an unavoidable help for the construction of Einstein’s work [04]. As a matter of facts, the infinitesimal elements of length within a GTR context (the \(ds^2\)) are not only a special type of bilinear forms but also solutions of the Einstein’s fields equations as demonstrated at the end of [05]. The preservation of (13) is studied via the presumption of existence of relations \(x_\alpha = \varphi_\alpha(x'_{\alpha 0}, x'_{\alpha 1}, \ldots x'_{\alpha (D - 1)})\) for \(\alpha = 0, 1, \ldots, D - 1\). An ordinary derivation yields expressions like:

\[
dx_\alpha = \frac{\partial \varphi_\alpha}{\partial x'_{\alpha \lambda}} \cdot dx'_{\lambda} + O(2)
\]

Substituting them into (13) leaves us with:

\[
ds^2 \equiv a_{\alpha \beta}. \left( \frac{\partial \varphi_\alpha}{\partial x'_{\alpha \lambda}}, dx'_{\lambda} \right) \cdot \left( \frac{\partial \varphi_\mu}{\partial x'_{\mu \lambda}}, dx'_{\lambda} \right) = \left( a_{\alpha \beta}. \frac{\partial \varphi_\alpha}{\partial x'_{\alpha \lambda}}, \frac{\partial \varphi_\mu}{\partial x'_{\mu \lambda}} \right), dx'_\lambda, dx'_\mu = a'_{\lambda \mu} \cdot dx'_\lambda \cdot dx'_\mu
\]

From which we deduce:

\[
a_{\alpha \beta}. \frac{\partial \varphi_\alpha}{\partial x'_{\alpha \lambda}} \cdot \frac{\partial \varphi_\beta}{\partial x'_{\beta \mu}} \equiv a'_{\lambda \mu}
\]

These equations (16) define a group. Actually less known or perhaps forgotten is the fact that the “differential parameters of \(ds^2\)” (see definition in [03, p. 390]) allow the discovery of new invariants. Christoffel’s brought the demonstration that the first one is the so called “G₄” [03, p. 391, (9)] and
that the latter is sufficient to find all others invariants. Pushing that demonstration further, he obtained a bilinear polar form denoted $\varphi(dr, U(r))$; see [03; p. 393, (14)], where $U(r)$ represents a function associated with the $ds^2$ in such a way preserving it. The components of the polar form are what we today call the Hessian of that function, $\text{Hess}_{_{\mathbb{L}}} U(r)$, where $(...)\text{ denotes the Christoffel's-like symbols of the connection at hand (the one preserving dU).}$

Remark: On the same vein, a bilinear covariant form can be built for Pfaff's expressions [03; at the end of p. 51]. Its components are the one of the Hessian obtained with the help of the four covariant components of the Pfaff's form.

Remark: At the end, let us also remark that similar relations than (13)-(16) can be written if we manage contra-variant components (the $x^\alpha$ instead of the $x_\alpha$); in fact, if we work inside the dual space.

Discussion
Because of the experimental results exposed in [01], the GTR is practically obliged to accept and to start with the existence of an invariant: the so-called $ds^2$. In order to extend the discussion developed for the 3D spatial part of our universe to any 4D space, our first reflex has been to explore the formalism of a relation like (01) with the secret hope to identify it with a $ds^2$. Since it can easily be demonstrated that that relation is, in general, not a conoid (a polynomial of degree 2) but a polynomial of degree 4, our first hope has been rapidly discarded.

Scrutinizing the literature, we were lucky enough to discover the E. Cotton's work (see reference [03], in French language). The latter contains the essence of the solution of our problem in that sense that it proves that one can always find a bilinear polar form associated with a given invariant bilinear form.

The most famous and pedagogical example based on that work is the GTR [04] because of the existence of a natural invariant (the $ds^2$) [01]. The lecture of [03] explains that the set of covariant expressions associated with that $ds^2$ contains many elements, the degree of which being any one, eventually two (like in [03; p. 393, end of the page after (16)] or three (like in [03; p. 394, (17) and (18)], in any case depending on the function associated with the $ds^2$ at hand. Of the highest interest for us is the existence of a bilinear polar form denoted $\varphi(dr, U(r))$; see [03; p. 393, (14)] obtained in following the recommendations exposed in [03; pp. 389-394].

As a matter of facts, the scalar associated with the extrinsic method always is a conoid, whatever the dimension of the space we are working with is. So that, although we know that splits obtained by that way will automatically be approximated solutions of the so-called (E) question, this will give us a pragmatic possibility to work in any 4D context.

The challenge of our approach is the introduction of some ad hoc extended (eventually exterior = built with the help of an anti-symmetric cube) product relating important informations. Consequently, we may ask our self if the scalar associated with any extended product having a split (up to now: the so-called "associated scalar") is an acceptable representation of a polar bilinear form associated with an important fact in physics. The invariance $ds^2$ is of course one of these unavoidable informations on which the GTR is built. But as we shall demonstrate it soon, that $ds^2$ is not the only fundamental invariant bilinear form in physics.

Mathematically, the challenge can now be reduced to a simple comparison between two conoids. The first one is the scalar associated with the split of the pair $(q, dr)$. 
Please note attentively that it is the “inverse” pair of the one, \((dr, q)\), which we have introduced above with the notion of extrinsic method:

\[
\chi = <dr, \Box_\mathfrak{A}(q, dr)> - [[Q]_{ext}. |dr > + |t >]_{B}
\]

Here again \(\nabla A\) is some (but a priori any) cube defining the extended product locally, \(([Q], t)\) is the generic split of \((q, dr)\), \(r\) is a 4D “position” (a moment in a local chronology, a spatial position), \(q\) is any vector in the 4D space and \(B\) is any bilinear form. And the second one writes:

\[
dU = <dr, Hess\{,\}; U(r). dr >_{id4} with Hess\{,\}; U(r) = \frac{\partial^2 U(r)}{\partial \alpha \partial \beta} - \{^\alpha _\gamma ^\beta \} \cdot \frac{\partial U(r)}{\partial \gamma}
\]

Here the symbol \(\{^\alpha _\gamma ^\beta \}\) denotes some local Christoffel’s-like connection and is not necessarily the Levi-Civita one. In fact that connection is depending on the preserved bilinear form; for example, within the GTR, the preserved bilinear form is of course the \(ds^2\) and the symbol “Hess” denotes the Hessian matrix for the \(U(r)\) function relatively to that Christoffel’s-like connection. Strictly speaking, we propose an identification yielding three generic relations:

\[
(17)
\]

\[
\chi \approx dU
\]

\[
<dr, t >_{B} \approx 0
\]

\[
<dr, |\Box_\mathfrak{A}(q, dr)> - [[Q]_{ext}. |dr >]_{B} \approx <dr, Hess\{,\}; U(r). dr >_{id4}
\]

For the pedagogy, let us believe that they hold. What do they mean? Answer: the associated scalar is identified with the variation of the \(U(r)\) function (in extenso: \(\chi \approx dU\)) and is now supposed to be an invariant (in extenso the new invariant associated with the initial one; for example and said before, within the GTR, the initial one is the \(ds^2\)).

Let us remark that a “force” time “a distance” is an energy (or, equivalently, a work in thermodynamic) and that there exists a well-known uncertainty principle initiated by Heisenberg [12]. One of the concrete applications of that principle concerns the pair (time, energy). This uncertainty principle also stipulates the existence of a limit at the Planckian scale. We are now ready to imagine another development for our theory at that scale. Indeed, if the extended product at hand has the units [time x force], we may write at the exact and invariant Planckian limit:

\[
(19)
\]

\[
\text{Invariant} = \frac{h}{4\pi} = [\chi] = [dU] = [\text{time x energy}] = [(\text{time x force}) x \text{length}]
\]

And the next three relations follow from (18-1, 2, 3):

\[
(20-1, 2, 3)
\]

\[
\frac{h}{4\pi} = \chi = dU
\]

\[
<dr, t >_{B} \approx 0
\]

\[
\forall dr : B. \{\lambda \Phi(q) - [Q]_{ext}\} = Hess\{,\}; U(r)
\]

**The connection involved here does perhaps preserve the \(ds^2\) but certainly preserves \(h/4\pi\).**
Theoretically, these three relations can also do the expected work; namely: “To allow the introduction of some extended product within the GTR in a 4D context if the force involved in that product takes care of the results obtained within the general theory of relativity (the invariance of the $ds^2$ and the principle of covariance).” This is the reason why we argue that our way of thinking represents a real scoop for and a revolutionary progress for the understanding of physics. With that innovative approach, the long standing incompatibility between quantum field theory and A. Einstein’s work may soon belong to the past.

**Example**

**A new expression for the EM field tensor**

For formal and evident reasons, we shall test this way of thinking with the simplest illustration which we ever can imagine, namely a simplified version of the Lorentz Einstein Law (LEL), sometimes also called the covariant version of the Lorentz law. How does look this law like? The full and complete version for a particle with mass $m$ and charge $q$ moving in an EM field is usually written [13. p. 106, (20.4)]:

$$m \cdot \left( \frac{du}{dt} + \bigtriangledown \Gamma(u, u) \right) = q \cdot [F_{\alpha\beta}] \cdot u$$

where $\bigtriangledown \Gamma$ represents the Levi-Civita connection and $[F_{\alpha\beta}]$ represents the $(1, 1)$ up down mixed EM field tensor

Per definition, we write the so-called simplified version of it:

(21)

$$m \cdot (du + \bigtriangledown \Gamma(u, dr)) = q \cdot [F_{\alpha\beta}] \cdot dr$$

This is evidently an extended product with split; its units are $[\text{time x force}]$. For non-massless particles ($m \neq 0$) we may envisage an application of the demonstration made previously in writing:

(22-1, 2, 3, 4)

$$\bigtriangledown A = \bigtriangledown \Gamma$$

$$q = u$$

$$[Q]_{\text{ext}} = \frac{q}{m} \cdot [F_{\alpha\beta}]$$

$$t = - du$$

All actors are perfectly identified and we don’t have to look for them or for their interpretation. All is as if that simplified expression (24) would have yield the answer to a special case of the (E) question concerning the product $\bigtriangledown \Gamma(u, dr)$; we shall call it the “gravitational part” of the simplified versus of the LEL.

---

1 This relation is actually yet under study. It results from a strict application of ideas exposed by A. Einstein (the covariance principle) and not from a mathematical demonstration. Some authors argue that one should incorporate the effects of retarded potentials into that formula (see, e. g.: [14]).
The scalar associated with that extended product and some bilinear form B preserves $\hbar/2\pi$ if we can simultaneously write:

$$\frac{\hbar}{2\pi} = \chi = dU$$

$$<dr, du>_{\theta} \equiv 0$$

$$\forall dr : B. \{r\Phi(u) - \frac{q}{m} [F^\alpha_\beta]\} = \text{Hess}_{\{\cdot\}} U(r)$$

The first relation results from the identification between the associated scalar and the bilinear polar form suggested by the Christoffel's work. The second relation may be interpreted as a relation describing the orthogonality between the speed and the proper acceleration of the particle. We must also absolutely remark at this stage that if B would have been chosen in coincidence with the local metric, $G$, knowing the invariance of $ds^2 = <u, u>_G$, we would have obtained (23-2). Consequently, our approach is totally compatible with the GTR and completes it. The Planckian limit appears to be the limit value of the polar bilinear form associated with the preservation of the $ds^2$. Seeking for simplicity we shall suppose that the bilinear form B is invertible. For the special case $B = G$, this is yielding a new theoretical expression for the EM field tensor:

$$(24) \quad \frac{q}{m} [F^\alpha_\beta] = r \Phi(u) - G^{-1} \cdot \text{Hess}_{\{\cdot\}} U(r)$$

### Special properties

Supposing that the local metric tensor is the right tool for raising and lowering the indices and subscripts of the EM field tensor (the relations given in [13; p. 77, (14.28 and 29)] hold true), we can write:

$$(25) \quad \frac{q}{m} [F^\alpha_\beta] = G \cdot r \Phi(u) - Hess_{\{\cdot\}} U(r)$$

The $(2, 0)$ versus of the EM field tensor is an anti-symmetric matrix. Its transposition yields:

$$(26) \quad \frac{q}{m} [F^\alpha_\beta] = G \cdot r \Phi(u) - Hess_{\{\cdot\}} U(r) = \frac{q}{m} [F^\alpha_\beta]$$

From which we get:

$$(27) \quad \frac{2q}{m} [F^\alpha_\beta] = \{G \cdot r \Phi(u) - r \Phi(u)^\dagger \cdot G^\dagger\} + \{\text{Hess}_{\{\cdot\}} U(r) - Hess_{\{\cdot\}} U(r)\}$$

That formulation makes it clear that, in general, the EM field tensor depends on the speed and on the position of the particle. But we meet two kinds of situations:

1°) the function $U(r)$ is a continuous one:

$$(28) \quad \frac{2q}{m} [F^\alpha_\beta] = G \cdot r \Phi(u) - r \Phi(u)^\dagger \cdot G^\dagger$$

We already had have the opportunity to demonstrate that if $u = A$ where the latter denotes the EM potential vector and if $A$ is parallel transported in the Levi-Civita connection, then the classical part of the EM field tensor vanishes and we are left with the Yang Mills (bilinear) terms only [TP-NSMGBv16].
2°) the function $U(r)$ is not a continuous one: we must work with (27). And, in this special case, the non-“speed-dependent” part of the EM field, in extenso its “potential” part is a sort of whirl operator acting in a 4D world. The existence of such situations should not afraid us. Let us just remember the fact that the energy of quantum devices has a discontinuous spectrum.

**Remark:** An equation like (28) has already been explored in [TP-NSMGBv16]; with its help, we were able to find the law for the supra-conductors of type I again.

**The Lagrangian of the field for a scholar case**

Let us work with a symmetric metric and suppose that $[F^\alpha_\beta] = G^{-1} \cdot [F^{\alpha\beta}]$. G. Let us write the simplest Lagrangian:

(29) \[
\mathcal{L} = \text{Trace} \left( [F_{\alpha\beta}] \cdot [F^{\alpha\beta}] \right) = \text{Trace} \left( (G \cdot [F^\alpha_\beta]) \cdot ([F^\alpha_\beta] \cdot G^{-1}) \right)
\]

Seeking once more time for simplicity in writing all these equations, we shall adopt the conventions:

(30-1, 2, 3) $H = \text{Hess} (\cdot ; U(r))$, $r\Phi(u) = \Phi$ and $F = \frac{q}{m} \cdot [F^\alpha_\beta]$.

With these conventions, the new relation (24) writes $F = \Phi - G^{-1} \cdot H$. For $m = q = 1$, the scholar Lagrangian of that theory is:

\[
\mathcal{L} = \text{Trace} \left( (G \cdot F) \cdot (F \cdot G^{-1}) \right)
\]

\[
= \text{Trace} \left( ([G \cdot (\Phi - G^{-1} \cdot H)] \cdot \{(\Phi - G^{-1} \cdot H) \cdot G^{-1}\}) \right)
\]

\[
= \text{Trace} \left( (G \cdot (\Phi - H)) \cdot \{(\Phi - G^{-1} \cdot H) \cdot G^{-1}\} \right)
\]

\[
= \text{Trace} \left( G \cdot (\Phi^2 \cdot G^{-1} - H \cdot \Phi \cdot G^{-1} - G \cdot \Phi \cdot G^{-1} - H \cdot G^{-1} + H \cdot G^{-1} \cdot H \cdot G^{-1}) \right)
\]

For the very special case where:

(31) $H = G$

This simplifies:

(32) $\mathcal{L} = \text{Trace} \left( G \cdot (\Phi^2 \cdot G^{-1} - 2 \cdot G \cdot \Phi \cdot G^{-1} + \text{Id}_4) \right)$

Considering a concept of “adjoint representation” related to the local metric tensor, this is also:

(33) $\mathcal{L} = \text{Trace} \left( (\Phi^2)^* - 2 \cdot \Phi^* + \text{Id}_4^* \right) = \text{Trace} \left( (\Phi^* - \text{Id}_4^*)^2 \right) = \text{Trace} \left( (r\Phi(u) - \text{Id}_4)^* \right)^2$

That result can be reached quicker if we invoke the associativity of the matrix product and the concept of adjoint representation introduced previously. For the pedagogy, note that the trace of the matrix $r\Phi(u) - \text{Id}_4$ is [13; p. 89, (17.5)]:

\[
\sum \alpha \frac{\partial \ln \sqrt{|g|}}{\partial x^\alpha} \cdot \frac{dx^\alpha}{dt} - 4 = \frac{d\ln \sqrt{|g|}}{dt} \cdot 4 = \text{Trace} \left( r\Phi(u) - \text{Id}_4 \right)
\]

Amazingly, in that theory, the Lagrangian of the EM field only depends on the difference between the trivial split of the gravitational part of the (simplified) LEL and the identity matrix. The absence of difference between them corresponds to what we have called the smallest natural computer in previous works [TP-NSMGBv16] and to a field with a vanishing Lagrangian. Since the energy in vacuum may be very small but not null we also may infer that the difference is never totally zero.
Furthermore, intuitively, we are pushed to look for relations between that expression, (33), and some results exposed in [15, p. 2, after (4)]; but we shall develop this intuition later.

Provisory conclusion
Putting the previous considerations all together, it becomes now more and more evident that we do have good reasons to evoke the existence of a relation like (01), in extenso of and extended product within a GTR context. Why all this could be useful? I shall also develop that item; for now, please see [11]. Indeed, quite pioneering, but in coherence with the units appearing there and because the condition (23-1) is really minimizing the scalar associated with the extrinsic split of the pair (q, dr), we can envisage both the preservation of a scalar $\chi$ related to the Heisenberg' uncertainty principle (HUP) for the pair (energy, time) and the one of $ds^2$. This idea has been explored in [TP-005-2].
III. Bibliography

Extern contributions


[02] Über die Transformationen der homogenen Differenzialausdrücke zweites Grades: E.B. Christoffel, in Journal für die reine und angewandte Mathematik, (pp. 46 - 70), Berlin 1826; 03.01.1869.

[03] Cotton, E.: Sur les variétés à trois dimensions; annales de la faculté des sciences de Toulouse, 2ème série, tome 1, n°4 (1899), p. 385-438 ; please visit: [www.numdam.org/item?id=AFST_1899_2_1_4_385_0].


[12] Please visit the following addresses by yourself: [http://www.aip.org/history/heisenberg/p08a.htm] and, for a visual explanation: [http://www.youtube.com/watch?v=a8FTl2qMutA]


Personal contributions

