The Gravitational origin of Velocity Time Dilation

A generalization of the Lorentz Factor for comparable masses

Arindam Sinha*†

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Abstract

Does velocity time dilation (clock drift) depend on a body's velocity in the Center of Gravity Frame of the local gravitational system, rather than on relative velocities between bodies? Experiments that have measured differential clock rates have conclusively proven this to be true, and hinted at a gravitational origin of velocity time dilation. Extending this understanding, a generalized form of the velocity time dilation metric (Lorentz factor) including masses is derived. This allows prediction of velocity time dilation between bodies of any mass ratio, including comparable masses such as the Earth-Moon system. This is not possible using the Lorentz factor in its current form. The generalized form of the Lorentz factor remains consistent with results of all experiments conducted.

Keywords: Special Relativity; velocity time dilation; comparable masses; Center of Gravity sidereal frame; generalized Lorentz factor

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1 Introduction

Special Theory of Relativity (SR)[1] was conceived by Einstein on the basis of *kinematics*. One of the consequences has been that velocity time dilation is an 'all-or-none' phenomenon between bodies, since mass plays no role. The metric used, Lorentz factor ($\gamma = 1/\sqrt{1 - v^2/c^2}$), only has the parameters v the relative velocity of the bodies, and c the speed

*E-mail: arindam_sinha1@yahoo.com

of light. It therefore appears that only one of the two bodies can undergo a clock slowdown, by an amount computed based on their *relative velocity*.

Experiments over the years have indicated that in real-life situations dynamics plays its part in velocity time dilation, and clock rate difference between two bodies cannot be predicted without taking into account the gravitational background, implicitly or explicitly. Both bodies involved may undergo different amounts of measurable velocity time dilation, as would be the case in the Earth-Moon system or binary stars.

[Note: The term "time dilation" in this paper denotes "differential aging", or experimentally verified clock rate difference between two bodies, not including any reciprocal time dilation 'as seen' by the two bodies because of Doppler effects.]

For example, in Earth based experiments, small moving objects have shown clock slowdown effects, with Earth implicitly acting as the 'second body'. Given the many orders of magnitude difference in masses, the small objects practically have all the velocity, and the 'all-or-none' computation of velocity time dilation using the Lorentz factor has worked (though the mass ratio has determined which clock is time dilated in experiments).

Even clocks on Earth's surface slow down because of the Earth's rotation velocity, compared to clocks fixed to the sidereal (non-rotating) axis around which Earth rotates, as shown by the Hafele-Keating[2, 3] experiment. This sidereal axis, called the Earth Centered Inertial Frame (ECIF), is the frame from which all velocities are measured in practice for computing velocity time dilation for Earth-based experiments. This is the same velocity as used in momentum conservation and satellite orbital velocity calculations.

The inference we can draw from results of experiments like *Hafele-Keating*, *GPS satellite time dilation*[4] and *Bailey et. al. muon lifetime extension*[5] experiments is that time dilation needs to be computed based on velocities from the center of gravity of the local gravitational system, rather than any relative velocities between objects.

In this paper we examine how the Lorentz factor may be generalized, based on lessons from these experiments, to provide more intuitive predictions on velocity time dilation in a broad range of situations without an 'all-or-none' approach, and without invoking General Relativity (GR).[6]

2 Purpose of this paper

Experiments on relativity have provided indications on how the existing Special Relativity theory fits into experimental results, and how it can be extended to provide predictions for more general situations. We can obtain a more comprehensive theory that extends our knowledge and provides better predictions and understanding of physical phenomena by taking into account these lessons from experiments.

This paper shows why and how the 'principle of relativity' works in real-life situations, and modifies the interpretation of certain concepts within SR that lead to apparent paradoxes like the *twin paradox*. While the mathematical formulation of the 'principle of relativity' is well-proven, the requirement of 'reciprocality' does not stand up to experimental verification where asymmetrical clock drifts are measured. Velocities measured from a gravitationally defined single sidereal frame of reference (center of gravity frame) leads to satisfactory and intuitive explanations of physical phenomena in experiments.

The logic explained in this paper also provides intuitive answers to some questions that cannot be satisfactorily resolved within the existing SR framework:

- Why time dilation between Earth-clocks and plane-clocks cannot be computed directly from their relative velocities using the SR formula in Hafele-Keating experiment. Why the velocity of 'westward planes' is effectively a 'reduction of velocity' compared to Earth clocks
- Why small objects moving near Earth would have their clocks slow down (rather than Earth clocks slowing down relatively), although the reciprocal relative velocity is the same
- Why GPS satellites experience clock slowdown compared to Earth clocks (eliminating gravitational time dilation), but not *among* GPS satellites, in spite of the satellites having proper relative velocities with regard to one another

The extension of the Lorentz Factor derived in this paper also provides us the ability to predict outcomes that cannot be directly derived from existing SR formulation:

- Velocity time dilation computation between comparable masses like the Earth-Moon system, or unequal binary stars. Both bodies in such a situation must undergo 'some' velocity time dilation, and the clock drift can be computed from the 'difference'
- Why equal binary stars would not have any relative clock drift, and by how much their clocks would slow down compared to a coordinate non-moving clock

The way out of some of the above difficulties has been to use the Schwarzschild metric from GR, but note that the Schwarzschild metric[7, 8] has the implicit assumption of velocities being measured from the center of gravity of the system. Further, there is no exact solution in GR when two comparable massive bodies are considered, and it is nearly impossible to apply in multi-body situations.

Most importantly, the experimental results indicate a gravitational origin of velocity time dilation, which deserves further investigation.

3 The Lorentz Factor in experiments

At present, computations based on the Lorentz factor allow prediction of velocity time dilation (differential clock rates) only in two mass ratio situations:

a. **Disparity between masses of two bodies is many orders of magnitude:** For example, the clock of a small object moving near Earth slows down as predicted by SR and computed by the Lorentz factor. Since the mass of the small object is negligible compared to Earth, the latter may be considered stationary. In an analogy to the well-known *twin paradox*, the small objects may be considered the traveling twin.

b. **Two bodies have identical mass:** For example, binary stars of equal masses would not have any clock rate difference between themselves because of the symmetry of the situation. In terms of the *twin paradox*, the stars may be considered to be twins traveling at equal velocities but in different directions.

Since mass does not feature in the Lorentz factor, it cannot predict velocity time dilation between masses that are comparable but unequal, such as the Earth and the Moon (or unequal binary stars). However, leveraging the lessons from experiments, we can generalize the Lorentz factor to predict velocity time dilation in such situations by taking the mass ratio into account.

The experiments we will consider are the *Hafele-Keating* experiment, *GPS satellite time dilation* and *Bailey et. al.* muon lifetime extension experiment.

The common thread among all these experiments is that 'small objects' (compared to Earth) are considered to have the complete velocity, and their clocks have been proven to slow down as computed using the Lorentz factor. If we were to consider the Sun-Earth situation though, in contrast, it is Earth clocks which would slow down compared to the Sun clocks because of velocity time dilation, as the Earth would have all the velocity.

It is clear that velocities used to compute differential clock rates are measured from a sidereal (non-rotating) frame through the Center of Gravity (CG) of the local gravitational system. In these experiments, the CG practically coincides with the CG of Earth because of the small moving masses involved. Therefore, the velocities are measured from the ECIF.

The following explanations from the three experiments help establish this fact.

Hafele-Keating Experiment: In this experiment, clocks on 'eastward bound' planes were found to be slower than Earth stationary clocks, which were in turn slower than clocks on 'westward bound' planes. In this case, the westward bound planes were actually *traveling slower than the Earth stationary clocks* because of Earth's rotational velocity in the ECIF.

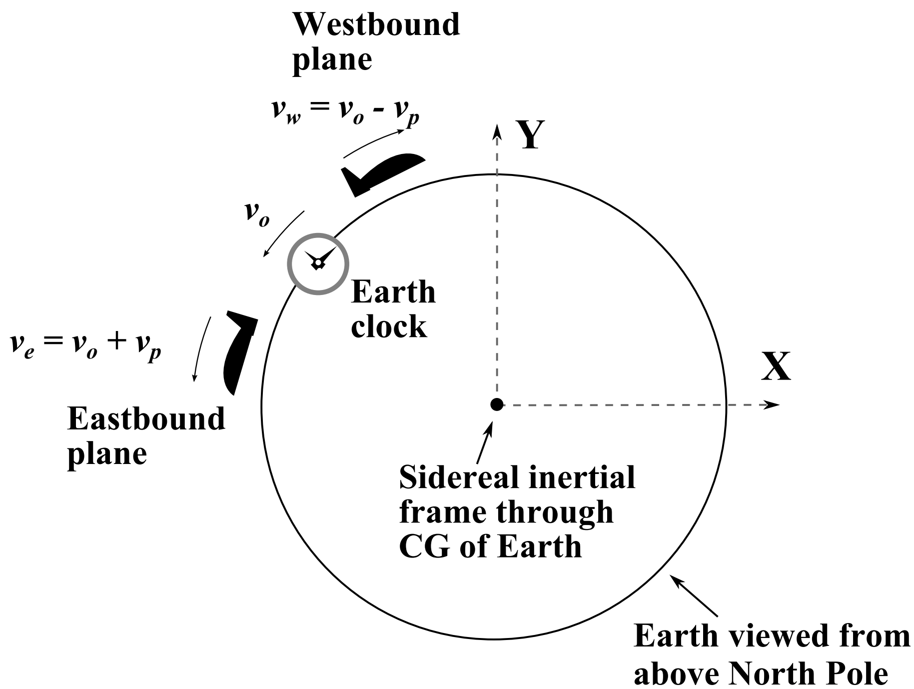

Figure 1: Planes travelling in opposite directions in Hafele-Keating experiment

In Fig. 1, the velocity of the Earth clock in this experiment is v_o, the rotational velocity of Earth as computed from the Earth CG sidereal frame. The velocity of the Eastbound plane (v_e) and Westbound plane (v_w) are found by vector addition of the plane velocities (v_p) to the Earth rotational velocity v_o.

The experiment's measured velocity time dilation (clock drift) agrees with Lorentz factors computed using the above velocities measured from Earth's sidereal frame, and **not based on any relative velocities** between the planes and the Earth clock. The *Earth clock* effectively has a *higher velocity than the westbound plane* because of this. The *velocity* of the *westbound* plane *reduces its velocity time dilation*, while that of the *eastbound* plane *increases its velocity time dilation*.

GPS Satellites: Clocks in GPS satellites run slower (ignoring gravitational time dilation) compared to Earth-based clocks, but do not show any clock drift among different satellites, in spite of moving in six different planes.

As seen in Fig. 2, the satellites travel at an orbital velocity computed based on the mass of Earth (M) and the distance from the CG of Earth (R). Although the satellites have **proper relative velocities** with respect to each other, their **time dilations (clock drift) are not affected by that**. The only thing that *determines their time dilation* compared to Earth clocks is the Lorentz factor computed using their *velocity around the Earth CG*.

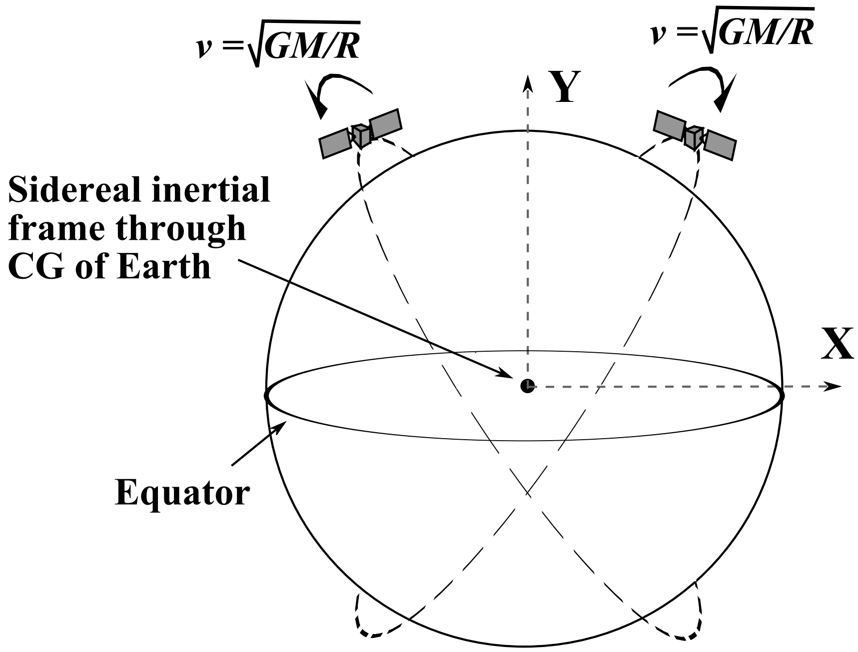

Figure 2: GPS satellites in orbit around Earth

Bailey Experiment: In this experiment, it is of course the muon whose 'clock' slows down because of its velocity in the lab or Earth frame.

4 Generalized Lorentz Factor formula

How would the clock rate difference change with the comparative mass ratio in a two body situation?

Existing SR does not provide a solution for this. Nor is there an exact solution in GR for such a scenario. The Schwarzschild metric may be used in a complex iterative way between two bodies to provide an approximate answer. There is no real solution in GR for a system of more than two bodies.

We derive below a more general form of the Lorentz factor which will help us predict velocity time dilation in comparable mass situations like the Earth-Moon system or even for multi-body gravitational systems. The approximate answer from GR ultimately converges to this generalized Lorentz factor. Moreover, the interpretation of the GR answer supports the more direct logic of our derivation.

Let M and m be two masses in circular orbit around their common center of gravity (barycenter) with orbital velocities v_M and v_m respectively. The momentum conservation equation is:

$$Mv_M = mv_m \tag{1}$$

If v is the *relative velocity* of the two objects around a sidereal axis through the center of gravity, we can write each velocity in terms of v:

$$v = v_M + v_m \tag{2}$$

$$v_M = \left(\frac{m}{M+m}\right) v \tag{3}$$

$$v_m = \left(\frac{M}{M+m}\right) v \tag{4}$$

The time dilation factors for M and m may then be modified as:

$$\gamma_M = \frac{1}{\sqrt{1 - \frac{{v_M}^2}{c^2}}} = \frac{1}{\sqrt{1 - \left(\frac{m}{M+m}\right)^2 \times \frac{v^2}{c^2}}} \tag{5}$$

$$\gamma_m = \frac{1}{\sqrt{1 - \frac{{v_m}^2}{c^2}}} = \frac{1}{\sqrt{1 - \left(\frac{M}{M+m}\right)^2 \times \frac{v^2}{c^2}}} \tag{6}$$

This is the *generalized form of the Lorentz factor*, including the mass ratio. This allows us to predict clock rate differences between bodies even if their masses are not many orders of magnitude different (or exactly equal).

Note that if Earth is M in the experiments referred to earlier, and the moving objects (plane, satellite, muon) are m, we have $m \ll M$, such that $v_M \cong 0$. In these cases, the time dilation factor of m reduces in the limit to the current Lorentz factor $\gamma_m = 1/\sqrt{1 - v^2/c^2}$, and that of Earth is $\gamma_M \cong 1$ (no time dilation), as seen in the experiments. *Thus, the generalized Lorentz factor remains consistent with existing experiments.*

We may write ${v_m}^2$ and ${v_M}^2$ in terms of M, m, G (Gravitational constant) and R (distance between M and m) as:

$${v_M}^2 = \frac{Gm^2}{R\,(M+m)} \tag{7}$$

$${v_m}^2 = \frac{GM^2}{R\,(M+m)} \tag{8}$$

The generalized Lorentz factors in (5) and (6) may then also be written as:

$$\gamma_M = \frac{1}{\sqrt{1 - \frac{{v_M}^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{Gm^2}{R(M+m)c^2}}} \tag{9}$$

$$\gamma_m = \frac{1}{\sqrt{1 - \frac{v_m{}^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{GM^2}{R(M+m)c^2}}} \tag{10}$$

Therefore, we can compute the velocity time dilations of both bodies from their masses and the distance between them.

Let us apply these formulations to different situations and see how this helps explain naturally the velocity time dilations that may be expected in each case (without any confusions as to which twin is moving, in an analogy to the *twin paradox*).

5 Application to different situations

We will consider the different situations below using a real example in each case.

5.1 M is much larger than m ($M \gg m$)

Taking M as the Earth, this applies to all the three experiments mentioned, with small objects m moving near Earth. As explained above, the time dilations will be computed using $\gamma_m = 1/\sqrt{1 - v^2/c^2}$.

- In **Hafele-Keating**, the clocks on the planes and Earth-stationary clocks all have time dilations given by $\gamma_{m_i} = 1/\sqrt{1 - v_i{}^2/c^2}$, where v_i stands for velocities of the clocks (m_i) in the sidereal inertial frame at the CG of Earth. The difference of these computed clock rates have been verified in the experiment.
- For **GPS**, the small rotational velocity of Earth may be ignored. Clocks on Earth will not have any velocity time dilation (i.e. will run at the same rate as a 'coordinate clock' at rest), while the time dilation of the satellites will be given by the formula $\gamma_m = 1/\sqrt{1 - v^2/c^2}$. Here, v is the rotational velocity of satellites around Earth's CG, and is the same for all satellites, even though traveling in six different planes.
- Muons in the **Bailey et. al.** experiment have near light-speed velocities, and any velocity of Earth is immaterial. We obtain the time dilation (lifetime extension) factor using the metric $\gamma_m = 1/\sqrt{1 - v^2/c^2}$, where v is the velocity of the muons in the muon ring.

The inference we can draw from these observations is that, for an independent gravitationally bound system, the CG represents the local sidereal inertial rest frame. Therefore the local rest frame is not static in a Universal sense (i.e. fixed with regard to distant stars or a Universal ether), but is *universally local, coinciding with the local CG*, much like the local constancy of the speed of light in GR and related to it.

This may be understood as a manifestation of Mach's Principle, taking into account not just the gravitational effect of the distant mass of the Universe, but including the gravitational effect of the local large masses and the free fall velocity of the gravitationally bound system. Gravitational effects of all other mass of the Universe are common to all bodies in the gravitationally bound system, and therefore only the velocities within the system matter from the perspective of velocity time dilation. In spite of the velocity with regard to the fixed stars, the sidereal nature of this inertial frame appears to be maintained as seen in these experiments.

This explains why Earth-based experiments exhibit local Lorentz Invariance (LLI). For Earth and small objects nearby, the local inertial rest frame practically coincides with the CG of Earth. Therefore, Earth-based experiments, like those of the Hughes Drever[9, 10] type, show strong agreement with LLI and the 'principle of relativity'.

5.2 M is much smaller than m ($M \ll m$)

Again taking M as the Earth, we may consider the Sun as m. In this case, it is Earth which gets the complete time dilation because of its velocity around the Sun, given the Sun's overwhelmingly larger mass. Clocks on Earth will run slower compared to a coordinate clock by a factor of $\gamma_M = 1/\sqrt{1 - v^2/c^2}$, where v is the orbital velocity of Earth in the sidereal frame through the CG of Sun (around which the Sun rotates).

5.3 M is the same as m ($M = m$)

Binary stars of equal mass exemplify this situation. From the symmetry of the situation, we do not expect any clock rate difference between the two stars, but their clocks both run equally slower than coordinate clocks. The time dilation factor of either will be given by:

$$\gamma_M = \gamma_m = \frac{1}{\sqrt{1 - \left(\frac{M}{M+M}\right)^2 \times \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{4c^2}}} = \frac{1}{\sqrt{1 - \frac{GM}{2Rc^2}}} \tag{11}$$

where v is the *relative velocity* between the stars (double the orbital velocity).

Note that there is no way of arriving at this value using the current Lorentz factor. While the current Lorentz factor can show that they will not have any differential clock rates between themselves because of identical velocities, the correct difference of their clock rates from a coordinate clock cannot be computed.

5.4 M is somewhat greater than m ($M > m$)

The Earth-Moon system is an example of this (as are unequal sized binaries). Since the mass of the Moon ($m = 7.35 \times 10^{22} kg$) is not negligible, the CG of the system cannot be considered to coincide with the CG of Earth ($M = 5.97 \times 10^{24} kg$). We have to compute their time dilation factors separately. Taking both as point masses, this may be done using known values $v_M = 12.6 m/s$, $v_m = 1,023 m/s$ and $c = 299,792,458 m/s$ from (5) and (6). The difference of γ_M and γ_m will give the difference of clock rates between Earth and Moon clocks.

Given the small velocities involved, the clock slowdown for Earth (0.08 $nanoseconds/day$) and Moon (503 $nanoseconds/day$) compared to a coordinate clock are very small, but even the Earth's time dilation is not zero.

5.5 Multiple bodies with total mass M ($M = \Sigma m_i$)

When there are multiple bodies in a gravitationally bound system (e.g. the Solar System), the velocity time dilation of each body may be found using the equation $\gamma_{m_i} = 1/\sqrt{1 - {v_i}^2/c^2}$, where m_i is the mass of the i^{th} body, and v_i is its velocity with respect to the CG of the system.

6 Conclusions

Experimentally verified velocity time dilation depends on gravity, at least in the definition of the local inertial frame. Velocities of objects are determined from this local sidereal CG inertial frame for computing time dilation. The arbitrary designation of 'stationary' and 'moving' states in the twin paradox, implicit in all resolutions of twin paradox within the SR framework to explain differential clock rates, is unambiguously resolved in real life situations by the local gravitational inertial frame.

We derived a generalized Lorentz factor in this paper based on the lessons from actual experiments such that velocity time dilations even between comparable masses may be predicted. This generalized Lorentz factor is valid for existing experiments as well.

An important question that arises from these considerations is whether velocity time dilation is in fact a gravitational phenomenon. This needs to be investigated further to create a more comprehensive theory of relativity.*

*(This is discussed in detail in another paper by the author on the overall theory including gravitational time dilation: Relativity and the Gravitational Potential of the Universe)

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