Application Ricci flows for the analysis of the event horizon

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This paper analyses the quantum fluctuations of the event horizon. Based on the method of Ricci flow are given topological properties of the event horizon and displays the law of loss of mass of the black hole with the emission of Hawking.
1. Quantum fluctuations of the gravitational radius.

Possible manifestations of quantum nature of physical fields and particles are fully applicable when considering the quantum effects in black holes. Qualitative assessment of the size of fluctuation processes in black holes: you can use simple reasoning. Suppose that in a region of space-time with the characteristic size $L$ occurred fluctuation of the metric and its value $\langle g \rangle$ on the value $\delta g$. The curvature in this field will change to the value of $\frac{\delta g}{L^2 g}$, and the values of the action $S$ for the gravitational field experiences a change of order

$$\delta S \sim \frac{\delta g}{g} \cdot L^2 \cdot \frac{c^2}{\hbar} \quad (1.1)$$

The possibility of such fluctuations are significant only in the case when $\delta S \sim \hbar$. Therefore, fluctuations in the metric values in the space-time region of $L$ we obtain the following estimation

$$\frac{\delta g}{g} \sim \frac{L_p^2}{L^2} \quad (1.2)$$

Where the square of the Planck length $L_p^2 = \frac{G\hbar}{c^3}$ ≈ 2.56 $10^{-70}$ sm$^2$. Thus described by quantum-gravitational fluctuations lead to a peculiar quantum «jitter» event horizon. For a spherical black hole with mass $M$ of the amplitude of oscillations $\delta r$ gravitational radius is based on (1.2) the following form:

$$\delta r \sim \frac{L_p^2}{r_g} \quad (1.3)$$

The amplitude fluctuations $\delta r$ is extremely small for black holes with the size of the $r_g >> L_p$. But do not need discard this effect, because it leads to interesting results. Under the influence of quantum fluctuations in the vacuum at the event horizon surface appear fluctuations $\delta r$ of the Schwarzschild radius $r_g$. You can define them as deviations from the mean gravitational radius

$$u(t, r, \theta, \varphi) = r_g - \langle r_g \rangle \quad (1.4)$$

Fluctuations $u(t, r, \theta, \varphi)$ on the surface of the black hole must not disappear, because are the quantum fluctuations of the vacuum on the Schwarzschild sphere (1.3).
2. Effect flow Ricci on a area of the event horizon

Select size zone $\Delta x$ where appear quantum fluctuation (1.2) on the horizon of events:

$$\frac{\delta g_{ik}}{g_{ik}} = - \frac{2 \cdot \delta r_g}{r_g} ; \ i, k = 1, 2 \quad (2.1)$$

$g_{ik}$ - metric event horizon. $r_g$ - the radius of the event horizon.

This fluctuation can be increased, if to use on zone $\Delta x$ the Ricci flow:

$$\frac{\partial g_{ik}}{\partial \tau} = -2R_{ik} \quad (2.2)$$

$\tau$ - valid parameter with respect to which the entropy Ricci is growing.

For physical phenomena this option $\tau$ should be associated with the time $t$:

$$\frac{\partial \tau}{K} = \frac{\partial t}{T} \quad (2.3)$$

$K, T$ - constants of the system of values $\tau, t$.

In as a constant $T$ take time period of quantum fluctuations on the horizon of events:

$$T = \frac{2 \pi r_g}{c}.$$ 

Then the Ricci flow equation (2.2) will have the following form:

$$\frac{\partial g_{ik}}{\partial t} = - \frac{c \cdot K \cdot R_{ik}}{\pi r_g} \quad (2.4)$$

Using (2.1), we obtain the rate of growth of quantum fluctuations on the horizon of events:

$$\frac{\partial r_g}{\partial t} = \frac{c K}{2\pi} \cdot R \quad (2.5)$$

$$R = g^{ik}R_{ik}$$
Formula (2.5) determines the rate of growth of the quantum fluctuations under the influence of the Ricci flow on a plot of $\Delta x$ event horizon.

For the spherical surface of the event horizon Ricci scalar is defined as:

$$R = g^{ik}R_{ik} = 2k_{gauss} = \frac{2}{r_g^2} = \frac{c^4}{2(GM)^2} \quad (2.6)$$

Formula (2.5) becomes the final form:

$$\frac{\partial r_g}{\partial t} = \frac{c^5 K}{4\pi(GM)^2} \quad (2.7)$$

She (2.7) lets you warp metric on the event horizon, but in the process of deformation education possible «singularities» - points, at which the curvature tends to infinity, and deformation cannot continue. To resolve this problem, apply surgical operations. This approach to the singularity of the Ricci flow stopped and produce «surgery» - emit small connected component or cut out of the neck (i.e. nested subvariety $L_H \times S^2$), and the obtained holes stick with balloons:

![Diagram of surgical operations](image1)

Under the influence of the Ricci flow with surgery, the event horizon, as two-dimensional sphere emits torpedo tubes $L_H \times S^2$:

![Diagram of torpedo tubes](image2)

In the limiting case of these tubes can be considered as strings if the radius of $R_H \rightarrow 0$ tube small compared to the length $L_H$:

![Diagram of strings](image3)

Perhaps, torpedo tubes $L_H \times S^2$ particles are released from the surface of the event horizon. This effect can be interpreted as the Hawking radiation of a black hole. Indeed, according to the
formula (2.7) the radius of a black hole on a plot of x event horizon increases, but taking him for a report level, the average radius of Schwarzschild decreases and consequently the mass of the black hole:

\[ \frac{\partial < r_g >}{\partial t} = - \frac{\partial r_g}{\partial t} = - \frac{c^5 K}{4\pi (GM)^2} \]  

Carved tube \( L_H \times S^2 \) is a membrane and for her to define the action string theory:

\[ I = -q \int \sqrt{-h} \, d\sigma^1 d\sigma^2 d\tau \]  

\[ h = \text{det}(h_{ik}); \ i, k = 0,1,2 \]

\( q \) - tension membrane.

Note that for the membrane, unlike strings, action (2.8) is conformally invariant. This circumstance is extremely complicates the analysis of the quantum theory of relativistic membrane.

However, if the membrane is present in the form of a tube \( L_H \times S^2 \) constant radius \( R_{H=\text{const}} \ (S=4\pi R_H^2=\text{const}) \), then the action will be:

\[ d\sigma^2 = d\varphi \cdot R_H \]

\[ I = -q R_H \int \sqrt{-h} \, d\varphi \, d\sigma^1 d\tau \]  

Consider tube \( L_H \times S^2 \) as a cylindrical surface:

\[ \int_0^{2\pi} \sqrt{-h} \, d\varphi \, d\sigma^1 d\tau \rightarrow 2\pi \int \sqrt{-g} \, d\sigma^1 d\tau \]

\[ g = \text{det}(g_{\mu\nu}); \ \mu, \nu = 0,1 \]

Get action in the following form:

\[ I = -2\pi q R_H \cdot \int \sqrt{-g} \, d\sigma^1 d\tau \]  

In such a record form (2.9) tube-membrane \( L_H \times S^2 \) is determined by for a relativistic string. Perhaps a string is a consequence of the theory of membranes in the form of tubes. To explore this issue need further studies.


Berlin, 2006).