Correlation between the Earth's Magnetic Field and the Gravitational Mass of the Outer Core

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The theory accepted today for the origin of the Earth's magnetic field is based on *convection currents* created in the Earth's outer core due to the rotational motion of the planet Earth around its own axis. In this work, we show that the origin of the Earth's magnetic field is related to the *gravitational mass* of the outer core.

Key words: Quantum Gravity, Gravitational Mass, Gravitational Mass of Earth's Outer Core, Earth's Magnetic Field.

1. Introduction

The Earth's interior is divided into 5 layers: the crust, upper mantle, lower mantle, outer core, and inner core [1]. Seismic measurements show that the inner core is a solid sphere with a radius of 1,221.5 km, and that the outer core is a liquid spherical crust (*plasma*) around the inner core, with an external radius of 3,840.0 km, and density of 12,581.5 kg.m^{-3} [2]. Thus, *the inertial mass of the outer core* is $2.88\times10^{24}kg$. The outer core is composed mainly of liquid iron (85 %) and nickel (5 %) with the rest made up of a number of other elements [3].

The temperature of the *inner core* can be estimated by considering both the theoretical and the experimentally demonstrated constraints on the melting temperature of impure iron at the pressure which iron is under at the boundary of the inner core (about 330 GPa). These considerations suggest that its temperature is about 5,700 K [4]. The pressure in the Earth's inner core is slightly higher than it is at the boundary between the outer and inner cores: it ranges from about 330 to 360GPa [5].

Currently, the theory accepted for the origin of the Earth's geomagnetic field is based on *convection currents* created in the Earth's outer core due to the rotational motion of the planet Earth around its own axis.

Here we show that the origin of the Earth's magnetic field is related to the *gravitational mass* of the outer core.

2. Theory

The origin of the Earth's geomagnetic field can be described in the framework of Quantum Gravity.

The quantization of gravity shows that the *gravitational mass* m_g and the *inertial mass* m_i are correlated by means of the following factor [6]:

$$\chi=\frac{m_g}{m_{i0}}=\left\{1-2\left[\sqrt{1+\left(\frac{\Delta p}{m_{i0}c}\right)^2}-1\right]\right\} \qquad (1)$$

where m_{i0} is the *rest* inertial mass of the particle and Δp is the variation in the particle's *kinetic momentum*; c is the speed of light.

In general, the *momentum* variation Δp is expressed by $\Delta p = F\Delta t$ where F is the applied force during a time interval Δt. Note that there is no restriction concerning the *nature* of the force F, i.e., it can be mechanical, electromagnetic, etc.

For example, we can look on the *momentum* variation Δp as due to absorption or emission of *electromagnetic energy*. In this case, it was shown previously that the expression of χ, in the particular case of *incident radiation on a heterogeneous matter*(powder, dust, clouds, *heterogeneous plasmas* *, etc), can be expressed by the following expression [7]:

$$\chi=\frac{m_g}{m_{i0}}=\left\{1-2\left[\sqrt{1+\frac{n^6S_m^4\phi_m^4\mu\sigma P^2}{4\pi\rho^2c^2f^3}}-1\right]\right\} \qquad (2)$$

where f and P are respectively the frequency and the power of the incident radiation; n is the number of atoms per unit of volume; μ,σ and ρ are respectively, the

* *Heterogeneous plasma* is a mixture of *different ions*, while *Homogeneous plasma* is composed of a single ion specie.

magnetic permeability, the *electrical conductivity* and the *density* of the mean.

In the case of the *free electrons* of the outer core plasma, the variable ϕ_m refers to the average "diameter" of these particles; $S_m = \frac{1}{4}\pi\phi_m^2$ is the geometric cross-section of the particle. When the particles are atoms its "diameters" are well-known. In the case of electrons, their "diameters" can be calculated starting from the *Compton sized electron*, which predicts that the electron's radius is $R_e = 3.862\times10^{-13}m$, and the standardized result recently obtained of $R_e = 5.156\times10^{-13}m$ [8]. Based on these values, the average value is $R_e = 4.509\times10^{-13}m$. Consequently, we can assume that the electron's "diameter" is

$$\phi_m = 9.018\times10^{-13}m \qquad (3)$$

On the other hand, by considering that the outer core plasma is composed mainly of liquid iron, the values of n, μ, σ, ρ and are given by

- $n = N_0\rho_{outer}/A = 1.078\times10^{25}\rho_{outer}$; ($N_0 = 6.022\times10^{26}\,atoms/kmole$ is the Avogadro's number; A is the iron molar mass $A = 55.845kg/kmole$).
- $\mu_{outer} = \mu_0$ (Above the Curie Temperature, the material is paramagnetic. Since the *Curie temperature* for Iron is 768 °C and it's melting point as 1538 °C (1811K), then *for liquid Iron*, $\mu_r = 1$).
- $\sigma_{outer} \cong 1\times10^6 S/m$ [9]
- $\rho_{outer} = 12{,}581.5kg.m^{-3}$ [10]

Substitution of these values into Eq. (2) gives

$$\chi = \frac{m_{g(outercore)}}{m_{i0(outercore)}} =$$

$$= \left\{1 - 2\left[\sqrt{1 + 4.793\times10^3\frac{P^2}{f^3}} - 1\right]\right\} \qquad (4)$$

The inner core with the temperature of 5,700K works as a black body. The density D of the black body radiation can be expressed by the *Planck's radiation law* i.e.,

$$\frac{D}{f} = \frac{2hf^3}{c^2\left(e^{hf/kT} - 1\right)}$$

where $k = 1.38\times10^{-23}j/K$ is the Boltzmann's constant; f is given by the Wien's law $\left(\lambda = 2.886\times10^{-3}/T\right)$, i.e., $f/T = c/2.886\times10^{-3}$; T is the *black body temperature*. Thus, the Equation above can be rewritten as follows:

$$\frac{D^2}{f^3} = 1.232\times10^{-49}T^5 \qquad (5)$$

Since $D = P/S$, then Eq. (3) can be rewritten as follows

$$\frac{P^2}{f^3} = 1.232\times10^{-49}T^5S^2 = 2.606\times10^{-4} \qquad (6)$$

where $S = 4\pi r_{innercore}^2 = 1.875\times10^{13}m^2$ is the surface area of the inner core, and $T = 5{,}700K$ its temperature.

Substitution of Eq. (6) into Eq. (4) yields

$$\chi = \frac{m_{g(outercore)}}{m_{i0(outercore)}} = 6.295\times10^{-4} \qquad (7)$$

Therefore, while the inertial mass of the outer core is $m_{i0(outercore)} = 2.888\times10^{24}kg$, its *gravitational mass* is

$$m_{g(outercore)} = \chi\; m_{i0(outercore)} = 1.818\times10^{21}kg \qquad (8)$$

The *quantization of gravity* leads to the following expression for the *electric charge*, *q*, [6]:

$$q = \pm\sqrt{4\pi\varepsilon_0 G}\; m_{g(imaginary)}\; i \qquad (9)$$

where

$$m_{g(imaginary)} = \chi_{imaginary} m_{i0(imaginary)} =$$

$$= \chi_{imaginary}\left(\frac{2}{\sqrt{3}}m_{i0(real)}\; i\right)$$

However,

$$\chi_{imaginary} = \frac{m_{g(imaginary)}}{m_{i0(imaginary)}} = \frac{m_{g(real)}\; i}{m_{i0(real)}\; i} = \chi_{real}$$

Therefore we can write that

$$m_{g(imaginary)} = \chi_{real}\left(\frac{2}{\sqrt{3}}m_{i0(real)}\; i\right) \qquad (10)$$

Substitution of this expression into Eq. (9) gives

$$q = \pm\sqrt{\tfrac{16}{3}\pi\varepsilon_0 G}\ \chi\ m_{i0} \qquad (11)$$

In the *Earth's outer core*, we have

$$q^- = -\sqrt{\tfrac{16}{3}\pi\varepsilon_0 G}\ \chi\ m_{i0(outercore)} \qquad (12)$$

$$q^+ = +\sqrt{\tfrac{16}{3}\pi\varepsilon_0 G}\ \chi\ m_{i0(outercore)} \qquad (13)$$

Thus, $q^+ + q^- = 0$, and

$$q_{total} = |q^+| + |q^-| = 2\sqrt{\tfrac{16}{3}\pi\varepsilon_0 G}\ \chi\ m_{i0(outercore)} =$$

$$= 3.617\times 10^{11} C \qquad (14)$$

The rotational motion of this electric charge produces the Earth's magnetic field (See Fig.1), whose *intensity at the Earth's center* can be expressed by

$$B = \frac{\mu_{r(innercore)}\mu_0 I}{2R} = \frac{\mu_{r(innercore)}\mu_0 q_{total}\,\omega}{2R} \qquad (15)$$

where $\omega = 7.29\times 10^{-5} rad/s$ is the Earth's angular velocity around its axis. Figure1, shows the length $2R$, which can be expressed by $2R = r_{innercore}/k$.

The temperature of the inner core can be estimated by considering both the theoretical and the experimentally demonstrated constraints on the melting temperature of impure iron at the pressure which iron is under at the boundary of the inner core (~330 GPa). These considerations suggest that its temperature is about 5,700 K [4]. The pressure in the Earth's inner core is slightly higher than it is at the boundary between the outer and inner cores: it ranges from about 330 to 360 GPa [5]. Iron can be solid at such high temperatures only because *its melting temperature increases dramatically* (and also the *Curie temperature*) *at pressures of that magnitude* (see the Clausius–Clapeyron relation) [11]. *This means that the inner core have ferromagnetic properties*. The inner core is believed to consist of a nickel-iron alloy known as *NiFe* [12]. Typical relative magnetic permeability of nickel-iron alloys are: 50,000 (78.5% Ni-Fe), 17,000 (49% Ni-Fe), 7,000 (45% Ni-Fe) [13]. Note that the value of the relative magnetic permeability (μ_r) *decreases* with the reduction of the Ni percentage in the alloy. The Ni percentage in the inner core is very low (6%) [12].

For short coils there is an effective relative permeability defined as $\mu_{r(eff)} = \mu_r / 1 + (\mu_r - 1)N_m$ where N_m is the *demagnetizing factor*. For very long coils we can take $\mu_{r(eff)} \cong \mu_r$ [14]. In the case of the Earth's core, due to its very large dimensions, it can be considered as *a very long coil*. Thus, we can assume $\mu_{r(eff)} \cong \mu_r$. Thus, the Eq. (15) can be rewritten as follows

$$B_{core} \cong \frac{\mu_r k \mu_0 q_{total}\,\omega}{r_{innercore}} = \frac{33.118\mu_r k}{r_{innercore}} =$$

$$= 2.711\times 10^{-5}\mu_r k \qquad (16)$$

In order to calculate the intensity of the Earth's magnetic field at *outer core* and at the *Earth's surface*, we can use the well-known relation:

$$B = \frac{\mu_r\mu_0 I R^2}{2\left(R^2+x^2\right)^{3/2}} = \left(\frac{\mu_r\mu_0 I}{2R}\right)\frac{R^3}{\left(R^2+x^2\right)^{3/2}} =$$

$$= \left(\frac{\mu_r k\mu_0 I}{r_{innercore}}\right)\frac{R^3}{\left(R^2+x^2\right)^{3/2}} =$$

$$= B_{core}\frac{R^3}{\left(R^2+x^2\right)^{3/2}} \qquad (17)$$

which reduces to Eq. (15) for $x = 0$.

It is rather difficult to determine the boundary between the outer and the inner core since this boundary is not as sharp as the separating line between the core and the mantle. Seismologists presume that instead of a boundary there is a transition layer whose thickness is about 100 km. [15]. This is the so-called *Lehman zone*, which separates the outer and the inner core at a depth of about 5000 to 5100 km [16]. Thus, in order to calculate the intensity of the Earth's magnetic field at the *outer core* we will take the average value of 5050 km, i.e., we will assume that outer core begins at $x = (6{,}378km - 5{,}050km) = 1{,}328km \cong 1.1 r_{innercore}$

Then, at this region, Eq. (17) gives

$$B_{outercore} = B_{core}\left(\frac{\frac{r_{innercore}}{2k}}{\sqrt{\left(\frac{r_{innercore}}{2k}\right)^2 + \left(1.1\ r_{innercore}\right)^2}}\right)^3 =$$

$$= B_{core}\left(\frac{1}{\sqrt{1+4.84k^2}}\right)^3 = \frac{2.711\times10^{-5}\mu_r k}{\left(\sqrt{1+4.84k^2}\right)^3} \quad (18)$$

In order to calculate the value of k we can consider the Earth's magnetic field as produced by a solenoid with $N = 1$ (See Fig.1), and apply the expression of B for the solenoid, i.e.,

$$B = \mu\frac{N}{l}i = \mu\frac{1}{\pi\ r}i = \mu\frac{k}{\pi\ R}i = \mu\frac{2k}{2\pi\ R}i =$$

$$= \frac{\mu\ i}{2R}\left(\frac{2k}{\pi}\right) = \frac{\mu\ i}{2R} \quad (19)$$

whence we see that $2k/\pi = 1$. Thus, the value of k is

$$k = \frac{\pi}{2} \quad (20)$$

The average magnetic field strength in the Earth's *outer core* was measured to be $2.5\times10^{-3}T$ [17]. Thus, Eq. (18) yields

$$B_{outercore} = \frac{2.711\times10^{-5}\mu_r k}{\left(\sqrt{1+4.84k^2}\right)^3} = 2.5\times10^{-3} \quad (21)$$

whence we obtain the value of μ_r, i.e.,

$$\mu_r \cong 2734 \quad (22)$$

At the Earth's surface $x = r_{earth} = 5.221\ r_{innercore}$. Then, Eq. (17) gives

$$B_{surface} = B_{core}\left(\frac{\frac{r_{innercore}}{2k}}{\sqrt{\left(\frac{r_{innercore}}{2k}\right)^2 + \left(5.221\ r_{innercore}\right)^2}}\right)^3 =$$

$$\cong \frac{B_{core}}{\left(\sqrt{1+109.04k^2}\right)^3} = \frac{0.1164}{\left(\sqrt{1+109.04k^2}\right)^3} =$$

$$\cong 2.6\times10^{-5}T \quad (23)$$

The magnetized rocks in the crust and in the upper mantle of the Earth increase this value, in such way that on the Earth's surface the intensity of magnetic field varies in the range of $2.6\times10^{-5}T$ - $6.5\times10^{-5}T$ [18].

Since $2R = r_{innercore}/k$ and $k = \pi/2$ (Eq.19) then we obtain

$$R \cong 0.318 r_{innercore} \cong 380km \quad (24)$$

This is the radius of the *innermost inner core* of the Earth (See Fig. 1). Based on an extensive seismic data set, Ishii, M. and Dziewonski, A.M. [19] have proposed in 2002 the existence of an *innermost inner core*, with a radius of ~300 km, which exhibits a distinct transverse isotropy relative to the bulk inner core.

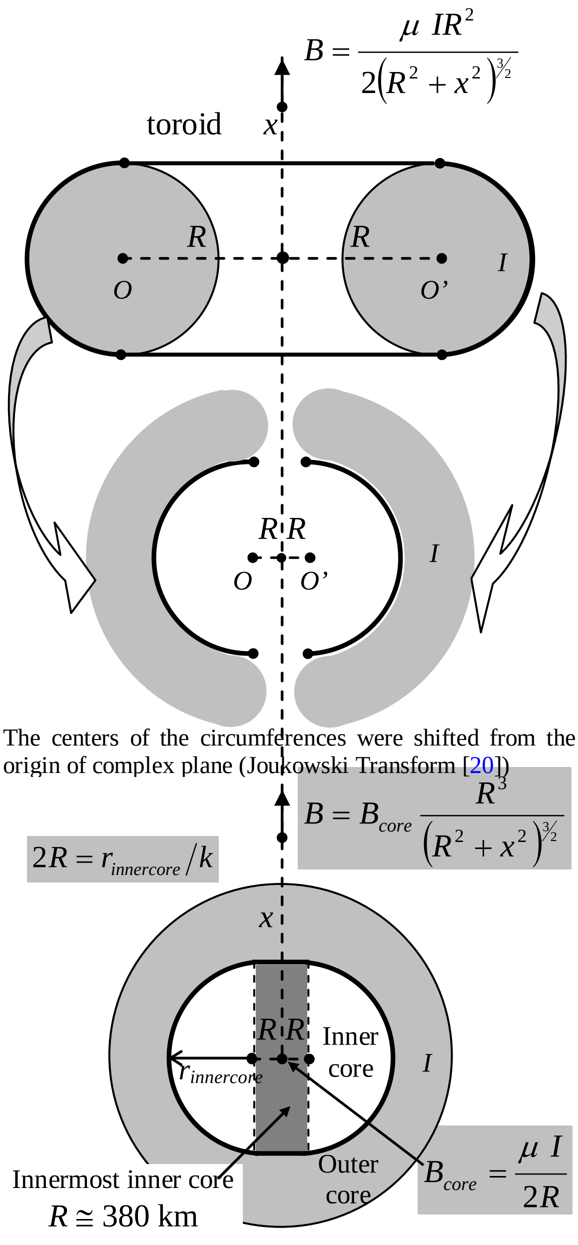

Fig. 1 – Similarity between the magnetic field produced by a toroid and the Earth's magnetic field.

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