On The ‘Stupid’ Paper by Fromholz, Poisson and Will

by

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ABSTRACT

In their paper titled *The Schwarzschild metric: It’s the coordinates, stupid!* the Authors (Fromholz, Poisson, Will, 2013) consider the so-called ‘Schwarzschild solution’ for “*a vacuum, static spherical spacetime*” and attempt to determine a general means by which equivalent solutions can be generated. They have however, failed to obtain this means, deriving instead by their methods only one already known equivalent form, which has no physical significance.

Introduction

The fundamental issue is the vacuum state. Proper consideration of this factor demonstrates that the exposition by Fromholz, Poisson, and Will (2013) has no physical meaning and is therefore without any scientific merit. I shall therefore deal with this fundamental factor first and foremost.

(1) The Field Equations for Vacuum (in the absence of matter)

Recall that Einstein’s field equations couple his gravitational field to its material sources. Einstein’s field equations,

> “*couple the gravitational field (contained in the curvature of spacetime) with its sources*.”
>
> (Foster and Nightingale 1995)

> “*Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter.*”
>
> (Pauli 1981)

> “*Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that the metrical field (or, in mathematical language, the tensor with components* g_{ik} *) is related to the material filling the world*.”
>
> (Weyl 1952)

"*In general relativity, the stress-energy or energy-momentum tensor T^{ab} acts as the source of the gravitational field. It is related to the Einstein tensor and hence to the curvature of spacetime via the Einstein equation.*"

(McMahon 2006)

"*Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move.*"

(Carroll and Ostlie 1996)

The material sources of Einstein's gravitational field are described by the energy-momentum tensor $T_{\mu\nu}$ and the gravitational field, 'manifest' in curved spacetime geometry, is described by the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - ½Rg_{\mu\nu}$. His field equations are thus[1],

$$G_{\mu\nu} = -\kappa T_{\mu\nu} \tag{1.1}$$

wherein κ is a coupling constant. Equation (1.1) is expressed in words by following relation,

spacetime geometry = -κ x *material sources* (1.2)

Now Einstein asserted that if *material sources* = 0 his field equations become,

spacetime geometry = 0 (1.3)

which in mathematical form are, says Einstein,

$$R_{\mu\nu} = 0 \tag{1.4}$$

because in this case $R = 0$ in $G_{\mu\nu}$.

Einstein claimed that equations (1.4) describe his gravitational field "*outside*" a body such as a star, where the $T_{\mu\nu}$ vanish. However, expression (1.3) clearly shows that there are no *material sources* present whatsoever in the alleged field equations (1.4). Bearing in mind that Einstein's field equations couple his gravitational field (*spacetime geometry*) to its *material sources*, since matter is the source of his gravitational field, when we ask Einstein and his followers what then is the *material source* of the gravitational field allegedly described by equations (1.4), we are told that it is the body outside of which the alleged gravitational field exists! This is nothing but linguistic legerdemain because Einstein removes on the one hand all *material sources* from (1.2), and hence from (1.1), by setting *material* sources = 0, to get (1.3) and hence (1.4), and on the other hand, in his very next breath, reinstates the presence of a material source with the deceptive words "*outside*" a mass such as a star. Since equations (1.4) contain no *material sources* the system of nonlinear differential equations resulting from (1.4), on the assumption of a 4-dimensional pseudo-Riemannian metric with spherical symmetry, also do not contain any *material sources*, as expression (1.3) emphasizes. Consequently the so-called 'Schwarzschild solution'

[1] The Cosmological Constant is of no consequence – it can easily be included.

for equations (1.4) cannot contain a *material source*. There are simply no *material sources* in the equations (1.4) and so there is no material source in the 'Schwarzschild solution' thereto. Thus, when *material sources* = 0 there are no *material sources*. In other words, when the energy-momentum tensor $T_{\mu\nu} = 0$ there are no material sources and hence no gravitational field. Einstein managed to not only subtly deceive his readers, but also himself.

That equations (1.4) contain no material sources and so do not describe a gravitational field "*outside*" a body such as a star is reaffirmed by the static homogeneous cosmological solutions. There are only three possible static homogeneous universes in General Relativity: (1) Einstein's 'cylindrical world'; (2) de Sitter's empty world; (3) empty Minkowski spacetime. In Einstein's cylindrical world $T_{\mu\nu} \neq 0$; in de Sitter's empty world $T_{\mu\nu} = 0$; in Minkowski spacetime $T_{\mu\nu} = 0$ since there is no matter present in the metric, which is given by,

$$ds^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{1.5}$$

Now de Sitter's empty world satisfies the 'field equations',

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{1.6}$$

wherein Λ is the so-called 'cosmological constant'. There are no material sources present in (1.6) because $T_{\mu\nu} = 0$: which is precisely why de Sitter's empty universe is empty. Although (1.6) contains no material sources because $T_{\mu\nu} = 0$ there, Einstein alleges (1.4) contains a material source even though $T_{\mu\nu} = 0$ there as well. Thus, $T_{\mu\nu} = 0$ both includes and excludes material sources. This is impossible. Equations (1.4) contain no material sources by mathematical construction just as equations (1.5) and (1.6) contain no material sources by mathematical construction. Consequently the 'Schwarzschild solution' for equations (1.4) contains no material source; it does not describe any gravitational field; it is physically meaningless. This is a consequence of the fact that $R_{\mu\nu} = 0$ is physically meaningless. Einstein's claim that (1.4) describes a gravitational field is patently false. Furthermore, all experiment attests that gravitation is an interaction between two or more bodies. General Relativity cannot account for the simple experimental fact that two stationary suspended bodies approach one another upon release.

Since $R_{\mu\nu} = 0$ is physically meaningless because it contains no matter by mathematical construction, it follows that everything expounded in the paper by Fromholz, Poisson, and Will (2013) is physically meaningless. No further analysis of their paper is therefore really necessary. However, they commit all the very same additional errors in both mathematics and physics committed by all proponents of the black hole, and so it is worth while to expose these additional errors to accentuate their wild imaginings.

(2) Empty Static Spherical Spacetime

In the 3rd paragraph of § **I. INTRODUCTION** of their paper these Authors say, "*In linearized general relativity, for example, the use of coordinates defined by the Lorenz gauge leads to a simple wave equation.*"

What these Authors don't realise is that even if it is assumed, as they do, that linearization of Einstein's field equations is admissible (which they have not proven or even addressed), the "*simple wave equation*" they refer to is in fact coordinate dependent. Consequently the speed of propagation of the 'gravitational waves' they allude to is coordinate dependent. In other words, the speed of propagation of Einstein's gravitational waves can be given any speed one likes, by a simple change of coordinates. Einstein merely wished his gravitational waves to propagate at speed c (light in vacuum) and so he deliberately chose a set of coordinates to make it so. The wave equation obtained from the linearised field equations is not unique at all. The speed of propagation of Einstein's gravitational waves is not deduced from Einstein's linearised form of his field equations; it is set by hypothesis and a set of coordinates deliberately chosen to satisfy the hypothesis. The arbitrariness of the speed of propagation of Einstein's gravitational waves was pointed out by Eddington in 1923 (NOTE: $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$) :

$$\text{“}\frac{\partial^2 h_{\mu\nu}}{\partial t^2} - \frac{\partial^2 h_{\mu\nu}}{\partial x^2} - \frac{\partial^2 h_{\mu\nu}}{\partial y^2} - \frac{\partial^2 h_{\mu\nu}}{\partial z^2} = 0$$

... showing that the deviations of the gravitational potentials are propagated as waves with unit velocity, i.e. the velocity of light. But it must be remembered that this representation of the propagation, though always permissible, is not unique. ... All the coordinate-systems differ from Galilean coordinates by small quantities of the first order. The potentials $g_{\mu\nu}$ *pertain not only to the gravitational influence which is objective reality, but also to the coordinate-system which we select arbitrarily. We can 'propagate' coordinate-changes with the speed of thought, and these may be mixed up at will with the more dilatory propagation discussed above. There does not seem to be any way of distinguishing a physical and a conventional part in the changes of the* $g_{\mu\nu}$.

"*The statement that in the relativity theory gravitational waves are propagated with the speed of light has, I believe, been based entirely upon the foregoing investigation; but it will be seen that it is only true in a very conventional sense. If coordinates are chosen so as to satisfy a certain condition which has no very clear geometrical importance, the speed is that of light; if the coordinates are slightly different the speed is altogether different from that of light. The result stands or falls by the choice of coordinates and, so far as can be judged, the coordinates here used were purposely introduced in order to obtain the simplification which results from representing the propagation as occurring with the speed of light. The argument thus follows a vicious circle.*"

(Eddington 1960)

These facts are not altered by the Landau-Lifshitz linearised formulation used by these Authors, viz.,

$$\mathfrak{g}^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} \tag{2.1}$$

In relation to eq. (2.1), in paragraph 7 of § **I. INTRODUCTION** of their paper we find the following assertions,

> "*This approach has found its fullest utility in the program of calculating the orbital evolution of inspiralling binaries of compact objects (black holes or neutron stars) and the gravitational waves emitted, …* "

However, eq. (2.1) can only apply to alleged weak gravitational fields, what these Authors call "*a weak-field approximation*" (paragraph 6 of § **I. INTRODUCTION**). But all alleged black holes, neutron stars, and "*inspiriling binaries*" thereof are not weak fields. Furthermore, all black hole universes are obtained not from the linearised field equations but from the nonlinear field equations. In the case of the so-called 'Schwarzschild solution' the nonlinear 'field' equations are $R_{\mu\nu} = 0$. All alleged solutions to Einstein's field equations for black hole universes pertain to nonlinear field equations. All alleged big bang universes also pertain to nonlinear field equations. Neither Einstein's linearised field equations nor the linearised form (2.1) generates the black hole or the big bang universes. Therefore, the application of linearised field equations used by these Authors, to "*the orbital evolution of inspiralling binaries of compact objects (black holes or neutron stars) and the gravitational waves emitted*", be it in the form of eq. (2.1) above or in the form $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$, is invalid. They apply a linear system to describe alleged phenomena related to entities that have been obtained from the nonlinear field equations. One cannot rightly impose linear relations upon entities obtained from nonlinear relations where the linear relations themselves are modifications of the nonlinear relations that supposedly generated the nonlinear entities in the first place.

The black holes the Authors refer to are supposedly in a big bang universe, since this is their general dogma. However, all alleged black hole universes (Crothers 2013a, 2013b),

(a) are spatially infinite
(b) are eternal
(c) contain only one mass
(d) are not expanding
(e) are asymptotically flat or asymptotically curved.

All alleged big bang universes

(a) are either spatially finite (one case) or spatially infinite (two cases)
(b) are of finite age
(c) contain radiation and many masses, including multiple black holes (some of which are primordial)
(d) are expanding
(e) are not asymptotically anything.

A simple comparison of the two lists above immediately reveals that the black hole universes and the big bang universes are defined very differently. The defining features of the black hole universe contradict the defining features of the big bang universe. Consequently the black hole and the big bang are mutually exclusive – they cannot coexist. This fact alone completely invalidates the entire contents of their paper.

Furthermore, since all alleged black hole universes and all alleged big bang universes are obtained from the nonlinear field equations, the Principle of Superposition is not valid in relation to them. One cannot therefore superpose any alleged black hole universe upon any alleged big bang universe or upon any other alleged black hole universe. Similarly one cannot superpose any alleged big bang universe upon any alleged black hole universe or upon any other alleged big bang universe. One cannot superpose any matter and radiation onto any black hole universe or big bang universe in order to get stars and galaxies and accretion discs and jets and planets, multiple black holes, black hole binaries, etc. To do so violates the mathematical structure of General Relativity. However, superposition is precisely how the astrophysical scientists have generated their big bang universe with its multiple black holes and black hole binaries and stars and galaxies etc. It is easy to express this mathematically. Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be solutions to Einstein's field equations. It does not matter if $\boldsymbol{X}$ and $\boldsymbol{Y}$ are the same or different. Let a and b be scalars. Then the linear combination $a\boldsymbol{X} + b\boldsymbol{Y}$ is not a solution to Einstein's field equations, because General Relativity is nonlinear. To amplify further, let $\boldsymbol{X}$ be an alleged black hole solution to Einstein's field equations and let $\boldsymbol{Y}$ be an alleged big bang solution to Einstein's field equations. Then the linear combination (i.e. superposition) $\boldsymbol{X} + \boldsymbol{Y}$ is not a solution to Einstein's field equations, because General Relativity is nonlinear. Indeed, in this particular case $\boldsymbol{X}$ and $\boldsymbol{Y}$ relate to completely different sets of Einstein's field equations and so they bear no relation to one another whatsoever.

These Authors (paragraph 7 of § **I. INTRODUCTION**) also invoke

> "*numerical relativity, which then describes the final inspiral and merger of the two compact bodies.*"

However, they have no well-defined problem in General Relativity to apply numerical methods to. They first superpose black hole universes with one another and with some big bang universe and then attempt to use numerical methods to describe mergers of compact bodies such as black holes and neutron stars. Such numerical methods do not create well-posed configurations of masses in General Relativity. Not only are there no known sets of Einstein's field equations for two or more masses and hence no solutions thereto, there is in fact no existence theorem by which it can even be asserted that Einstein's field equations contain latent solutions for two or more masses, let alone for two or more black holes in a big bang universe.

In paragraph 9 of § **I. INTRODUCTION** of the subject paper we find the following,

> "*Furthermore, although one solution of the LL equations yields the Schwarzschild metric in the so-called harmonic radial coordinate r_H, related to the standard Schwarzschild coordinate r_S by $r_H = r_S - M$, where M is the mass of the object, …* "

This raises two issues: (1) the identity of r_S and r_H, (2) the identity of M as mass. These Authors call r_S the "*Schwarzschild coordinate*" and $r_H = r_S - M$ the "*harmonic* *radial coordinate*". Let us note what r_S is often called in the literature. It has been variously and vaguely called "*the distance*", "*the radius*", "*the radius of a 2-sphere*", "*the coordinate radius*", "*the radial coordinate*", "*the radial space coordinate*", "*the Schwarzschild radial coordinate*", "*the areal radius*", "*the reduced circumference*", "*the shortest distance a ray of light must travel to the centre*", and even "*a gauge choice: it defines the coordinate r*". In the particular case of $r = 2m = 2GM/c^2$ it is invariably referred to as the "*Schwarzschild radius*" or the "*gravitational radius*". That r_S goes by so many different identities attests to the confusion of the proponents of the black hole. They don't know what it is. However, they always treat it as the radius, and this is clear from the fact that they always call $r = 2m = 2GM/c^2$ the "*Schwarzschild radius*" or the "*gravitational radius*", which is just the alleged radius of a black hole even horizon.

However, none of these various and vague concepts of what r is are correct because the irrefutable geometrical fact is that r is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section, and as such it is neither a radius nor a distance in the metric. To see this let us examine the Hilbert metric, incorrectly attributed to Schwarzschild, which we will express using the signature (-, +, +, +) as used by these Authors [see their eq. (4)].

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{2.2}$$

$$0 \le r$$

These Authors write r_S in place of r in the metric (2.2) above. Now in the final paragraph of § **I. INTRODUCTION,** these Authors say this,

> "*We use units in which G* = *c* = 1"

This is also done in Hilbert's metric (2.2) above. Setting $G = c = 1$ is very deceptive. Let us therefore write Hilbert's metric with c and G explicitly so that nothing is hidden,

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{2.3}$$

$$0 \le r$$

In the first paragraph of § **II. THE "TEXTBOOK" SOLUTION** these Authors say this,

> "*r_S has the interpretation of being $C/2\pi$, where C is the physically measured circumference of a surface of constant r_S, t and φ, or* $\sqrt{A/4\pi}$, *where A is the physically measured area of a surface of constant r_S and t. It is often called the Schwarzschild radial coordinate.*"

Note that they now 'interpret' r_S even though Hilbert's metric (2.3) above and the Authors' general metric numbered (1) in § **II. THE "TEXTBOOK" SOLUTION** of their paper actually contains the correct geometric meaning of *r* by virtue of its metric ground-form. These Authors do not know what *r* is in these metrics. They give it three different 'interpretations' in two sentences; (a) *C/2π*, which is just the "*reduced circumference"*; (b) $\sqrt{A/4\pi}$, which is just the "*areal radius*"; (3) "*the Schwarzschild radial coordinate*." Interpretation is nonsense. The metric determines the geometry of the space it describes.

To correctly identify *r* in Hilbert's metric (2.3) above, let us consider first the spatial section thereof. It is given by,

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{2.4}$$

Now consider the surface in the spatial section (2.4); it is given by

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) = r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2 \tag{2.5}$$

This is a simple case of the First Fundamental Form of a surface. The intrinsic geometry of a surface is entirely independent of any embedding space and so when it is embedded into a higher dimensional space the geometry of the surface is not altered in any way. So the intrinsic geometry of the surface (2.5) is the same in (2.3).

> "*And in any case, if the metric form of a surface is known for a certain system of intrinsic coordinates, then all the results concerning the intrinsic geometry of this surface can be obtained without appealing to the embedding space*."
>
> (Efimov, 1980)

A very important feature of the intrinsic geometry of a surface is its Gaussian curvature.

Gauss' Theorema Egregium: *The Gaussian curvature K at any point P of a surface depends only on the values at P of the coefficients in the First Fundamental Form and their first and second derivatives.*

The Gaussian curvature *K* of a surface can be calculated by means of the following relation (Crothers 2008a, 2008b),

$$K = \frac{R_{1212}}{g} \tag{2.6}$$

where R_{1212} is a component of the Riemann tensor of the first kind and *g* is the determinant of the metric tensor. Applying (2.6) to (2.5) gives,

$$K = \frac{1}{r^2} \tag{2.7}$$

From this it is plain that r is neither a radius nor a distance in (2.5) and that (2.5) does indeed describe a spherical surface (because the Gaussian curvature is of a constant positive value – a surface which has a constant positive Gaussian curvature is called a spherical surface; a surface which has a constant negative Gaussian curvature is called a pseudo-spherical surface). These Authors are clearly ignorant of the definition of a spherical surface. The spherically symmetric surface described by (2.5) does not have a radius because it is a surface, which is entirely independent of any embedding space. Since the intrinsic geometry of the surface is independent of any embedding space, it retains its character in Hilbert's metric (2.3) above. Thus, r in Hilbert's metric is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section thereof; it is neither the radius nor a distance in (2.4) or in (2.3).

The radius R_p of the spherically symmetric 3-space described by (2.4) is given by (Crothers 2005a, Crothers 2005b),

$$R_p = \int \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} dr = \sqrt{r\left(r - \frac{2GM}{c^2}\right)} + \frac{2GM}{c^2} \ln\left(\frac{\sqrt{r} + \sqrt{r - \frac{2GM}{c^2}}}{\sqrt{\frac{2GM}{c^2}}}\right) \tag{2.8}$$

This is the non-Euclidean radius in the spatial section of Hilbert's metric (2.3). The quantity r is not the radius and it is even not a distance in (2.3) and (2.4). The true geometric identity of r is given by (2.7), which also determines the nature of the surface (2.5) as a spherical surface. This is not an interpretation; it is a definite geometric quantity determined from the metric itself, according to Gauss' theory of surfaces. These Authors, as usual with all proponents of the black hole, do not know the true identity of r in Hilbert's metric and do not even know the definition of a spherical surface. That is why there are so many different 'interpretations' of Hilbert's r by proponents of the black hole.

Note that when $r = 2GM/c^2$ in expression (2.8), $R_p = 0$ and Hilbert's metric (2.3) is undefined. This fixes the range on r to $2GM/c^2 < r$ in Hilbert's metric. Since r is neither the radius nor a distance in Hilbert's metric, there is no *a priori* reason to even suppose that $0 \leq r$ therein.

When $r = 2GM/c^2$,

$$K = \frac{c^4}{4G^2M^2} \tag{2.9}$$

In the final paragraph of § **II. THE "TEXTBOOK" SOLUTION** these Authors say of Hilbert's metric,

> "*This solution is so iconic that few textbooks even mention the existence of other radial coordinate systems, such as the isotropic coordinate r_{iso}, related to the Schwarzschild coordinate r_S by $r_S = r_{iso}(1 + M/2r_{iso})^2$, in which the spatial part of the metric is proportional to the Euclidean metric, or the harmonic coordinate $r_H = r_S - M$ …*"

They label r_{iso} "*the isotropic coordinate*" and r_H "*the harmonic coordinate*". They call both of them "*radial coordinate systems*". These are merely names that do not provide any geometric meaning. Lumping both r_{iso} and r_H together as "*radial coordinate systems*" gives each quantity two different 'interpretations', one of which is 'radius'; but they are both neither radii nor distances in their respective equivalent metrics. Let us first investigate Hilbert's metric in the so-called 'isotropic coordinates'. We shall again use the signature (-, +, +, +) adopted by these Authors. If $G = c = 1$ again, it is given by,

$$ds^2 = -\left(\frac{1-\frac{M}{2r}}{1+\frac{M}{2r}}\right)^2 dt^2 + \left(1+\frac{M}{2r}\right)^4 \left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \qquad (2.11)$$

$$0 \le r$$

This is again very deceptive, so let us write (2.11) with G and c explicit,

$$ds^2 = -c^2\left(\frac{1-\frac{GM}{2c^2 r}}{1+\frac{GM}{2c^2 r}}\right)^2 dt^2 + \left(1+\frac{GM}{2c^2 r}\right)^4 \left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \qquad (2.12)$$

$$0 \le r$$

Consider now the spatial section of metric (2.12),

$$ds^2 = \left(1+\frac{GM}{2c^2 r}\right)^4 \left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \qquad (2.13)$$

However, the surface in the spatial section is given by,

$$ds^2 = \left(1+\frac{GM}{2c^2 r}\right)^4 r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (2.14)$$

This is again a simple First Fundamental Form for a surface. Applying expression (2.6) to expression (2.14) the Gaussian curvature K of the surface (2.14) is,

$$K = \frac{1}{\left(1+\frac{GM}{2c^2 r}\right)^4 r^2} \qquad (2.15)$$

The quantity r is related to the Gaussian curvature of expression (2.14) and hence retains that role in (2.12) and (2.13). It acts now as a parameter for expressions (2.11) to (2.15). Equation (2.13) tells us the nature of this parameter – it is both the radius and the inverse square root of the Gaussian curvature of the spherically symmetric surface in the spatial section of metric (2.11) and hence of metric (2.12). Note that the terms within the square brackets in expressions (2.11), (2.12) and (2.13) is the Euclidean metric for 3-space. What now is the actual range of r in metric (2.12)? This is determined by the radius for (2.13), thus (Crothers 2006),

$$R_p = \int \left(1 + \frac{GM}{2c^2 r}\right)^2 dr = r + \frac{GM}{c^2}\ln\frac{2c^2 r}{GM} - \frac{G^2 M^2}{2c^4 r} + \frac{GM}{2c^2} \tag{2.16}$$

Note that when $r = GM/2c^2$, $R_p = 0$ (a scalar invariant) and the Gaussian curvature is,

$$K = \frac{c^4}{4G^2 M^2} \tag{2.17}$$

This is the very same result (2.9) for Hilbert's metric (2.3). This value of K is a scalar invariant.

It is also noted that at the outset these Authors state that they consider "*a vacuum, static spherical spacetime*" (see their **ABSTRACT**). Spherical symmetry has then been incorporated into their metric (1) in § **II. THE "TEXTBOOK" SOLUTION.** Hilbert's metric (2.3) above is consequently spherically symmetric, as is Schwarzschild's actual solution (1916), which is quite different to Hilbert's.

In § **III. SCHWARZSCHILD'S SOLUTION OF 1916** of their paper these Authors more or less reproduce Schwarzschild's derivation of his actual solution. However they do not write Schwarzschild's metric explicitly and they make a false claim about it. Here is Schwarzschild's actual solution,

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{2.18}$$

$$R = \left(r^3 + \alpha^3\right)^{1/3}$$

Note that Schwarzschild (1916) used the signature (+, -, -, -) and $c = 1$. These Authors assign $\alpha = 2M$ to get a mass M to appear in their rendition of Schwarzschild's actual solution, and falsely attribute this assignment to Schwarzschild. Schwarzschild made no such assignment to α. These Authors also assert that r in (2.18) can take values down to $r = -2M$, in other words they assert that r can take the values $-\alpha \le r < 0$ in (2.18), in order to produce a black hole from Schwarzschild's actual solution. Examination of Schwarzschild's paper shows that this is not so because in Cartesian coordinates $r = \sqrt{x^2 + y^2 + z^2}$, which can never be less than 0 by mathematical construction.

The Gaussian curvature K of the surface in the spatial section of Schwarzschild's actual solution is,

$$K = \frac{1}{R^2} = \frac{1}{\left(r^3 + \alpha^3\right)^{2/3}} \tag{2.19}$$

and the radius is given by,

$$R_p = \int \left(1 - \frac{\alpha}{R}\right)^{-\frac{1}{2}} dR = \sqrt{R(R-\alpha)} + \alpha \ln\left(\frac{\sqrt{R} + \sqrt{R-\alpha}}{\sqrt{\alpha}}\right) \tag{2.20}$$

$$R = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}$$

Thus, when $r = 0$, $R = \alpha$, $R_p = 0$ and $K = 1/\alpha^2$, and Schwarzschild's metric is undefined. Note also that by (2.20) R can never take values $0 \leq R < \alpha$, and so r in (2.18) can never take values $-\alpha \leq r < 0$.

In § **IV. SOLUTION USING THE LANDAU-LIFSHITZ FORMULATION** these Authors obtain, on page 10, the case **A. C = 0**, their "*harmonic radial coordinate*" form of Hilbert's solution, by means of a complicated mathematical method that involves the Landau-Lifshitz pseudotensor. The resulting solution is merely a single equivalent form of Hilbert's solution.

$$ds^2 = -\left(\frac{1 - M/r}{1 + M/r}\right)dt^2 + \left(\frac{1 + M/r}{1 - M/r}\right)dr^2 + (r + M)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{2.21}$$

$$-M \leq r$$

Their method does not generate what they seek – a means by which many different looking but equivalent forms of Hilbert's metric can be generated. Their result, which has no physical meaning, is mathematically trivial. Furthermore, contracting the Landau-Lifshitz pseudotensor produces a 1st-order intrinsic differential invariant, i.e. an invariant that depends solely upon the components of the metric tenor and their first derivatives. However, the pure mathematicians G. Ricci-Curbastro and T. Levi-Civita (1900) point out that 1st-order intrinsic differential invariants do not exist! Thus, by *reductio ad absurdum*, the Landau-Lifshitz pseudotensor is a meaningless collection of mathematical symbols. Einstein's pseudotensor suffers from the very same fatal defect.

It is rather easy to generalise Schwarzschild's actual solution to generate all possible equivalent forms (Crothers 2005a, 2005b). Using the signature (-, +, +, +) and $c = 1$ the metric takes the form,

$$ds^2 = -\left(1 - \frac{\alpha}{R}\right)dt^2 + \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 + R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{2.22}$$

$$R = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}}$$

$$r, r_o \in \mathbf{R},\ n \in \mathbf{R}^+,\ r \neq r_o$$

where r_o and n are entirely arbitrary. For instance, setting $r_o = 0$, $n = 3$, $r > r_o$, yields Schwarzschild's actual solution. Setting $r_o = \alpha$, $n = 1$, $r > r_o$, yields Droste's (1917) solution (which is the correct from of Hilbert's 'solution'). Setting $r_o = 0$, $n = 1$, $r > r_o$, $\alpha = 2M$, yields Brillouin's (1923) solution. Setting $r_o = M$, $n = 1$, $r > r_o$, $\alpha = 2M$ yields the correct form of the solution in what these Authors' call the "*harmonic radial coordinate*". One can also generate from (2.22) equivalent solutions in which r is always less than zero. These Authors cannot generate all these equivalent solutions by their method, which produces only one, their "*harmonic radial coordinate*" form. None of the equivalent solutions generated by equations (2.22) permit $g_{oo} = (1 - \alpha/R)$ to change signature. To do so would change the metric into a non-static solution for static problem and thereby bear no relation to the required static form, stipulated at the outset as a requirement by these Authors, and by Einstein and all proponents of the black hole.

It is also rather easy to generalise the isotropic form for the alleged solution to $R_{\mu\nu} = 0$. Using the signature (-, +, +, +) and $c = 1$ the generalised from is (Crothers 2006),

$$ds^2 = -\frac{\left(1-\frac{\alpha}{4R}\right)^2}{\left(1+\frac{\alpha}{4R}\right)^2}dt^2 + \left(1+\frac{\alpha}{4R}\right)^4 dR^2 + R^2\left(1+\frac{\alpha}{4R}\right)^4\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (2.23)$$

$$R = \left[\left|r - r_o\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{1/n} \qquad r, r_o \in \mathbf{R} \quad n \in \mathbf{R}^+ \quad r \neq r_o$$

If $\alpha = 2M$, $n = 1$, $r_o = \alpha/4$, then the metric (2.11) is obtained, except that by (2.16) the range on r is $M/2 < r$, not $0 \leq r$. Note that no matter what values are chosen for n and r_o the Gaussian curvature of the spherically symmetric geodesic surface at $r = r_o$ is always the same for (2.22) and for (2.23), although all the equivalent metrics are actually undefined then. Expressions (2.22) and (2.23) are equivalent.

Since (2.23) can be obtained by a transformation on (2.22) either set of expressions can be used to generate all admissible equivalent metrics for Schwarzschild spacetime. Thus, the generator of all equivalent metrics that these Authors seek but have not attained has already been obtained elsewhere (Crothers 2005, 2006).

In (2.22) and (2.23) α is an entirely arbitrary constant from the strictly mathematical standpoint. It is however alleged to be associated with a material source, as assigned by the proponents of the black hole in Hilbert's metric (2.2) and (2.3) above. However, since $R_{\mu\nu} = 0$ contains no matter by mathematical construction, if the constant α is to be associated with material sources then it must be that $\alpha = 0$ (no matter). In this case all metrics (2.22) and (2.23) reduce trivially to that for empty Minkowski spacetime, expression (1.5), taking $c = 1$.

(3) Newton's Expression for Escape Velocity

Let's now examine Hilbert's metric in the revealing form (2.3). The proponents of the black hole assert that when $r = 2GM/c^2$ this is the radius of the event horizon on the associated black hole, their so-called 'Schwarzschild radius',

$$r = \frac{2GM}{c^2} \tag{2.24}$$

However, not only is r neither a radius nor a distance in Hilbert's metric, at this value of r there is division by zero in the metric, viz,

$$g_{11} = \frac{1}{(1-1)} = \frac{1}{0} \tag{2.25}$$

Furthermore, the proponents of the black hole maintain that

$$\frac{1}{0} = \infty \tag{2.26}$$

Division by zero is however undefined. Division by zero therefore does not produce ∞. Nonetheless the proponents of the black hole not only divide by zero they also assign a physical property to it. Rearranging expression (2.24) for c gives,

$$c = \sqrt{\frac{2Gm}{r}} \tag{2.27}$$

This is immediately recognised as Newton's expression for escape velocity. From this it is asserted that the escape velocity of the associated black hole is the speed of light c. But Newton's expression for escape velocity is an implicit two-body relation: one body escapes from another body. It cannot therefore appear in what is alleged to be a solution for a one-body problem, but which in fact is a no-body problem. Furthermore, if the black hole event horizon has an escape velocity c, according to equation (2.27), then, by definition, light can escape, contrary to the frequent claim that it cannot even leave.

The mass appearing in Hilbert's solution is obtained by arbitrarily and inadmissibly inserting Newton's expression for escape velocity in order to satisfy the false claim that a material source is present in $R_{\mu\nu} = 0$. McMahon (2006) says in relation to Hilbert's solution,

> "*… is known as the Schwarzschild radius. In terms of the mass of the object that is the source of the gravitational field, it is given by*
>
> $$r_s = \frac{2GM}{c^2}$$ "

On the one hand it is claimed that the black hole has an escape velocity:

"***black hole** A region of spacetime from which the escape velocity exceeds the velocity of light*"

(Dictionary of Geophysics, Astrophysics and Astronomy 2001)

"***black hole** A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light.*"

(Collins Encyclopædia of the Universe 2001)

Yet on the other hand it is also claimed that nothing can even leave a black hole.

"*I had already discussed with Roger Penrose the idea of defining a black hole as a set of events from which it is not possible to escape to a large distance. It means that the boundary of the black hole, the event horizon, is formed by rays of light that just fail to get away from the black hole. Instead, they stay forever hovering on the edge of the black hole.*"

(Hawking 2002)

"*The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge.*"

(Chandrasekhar 1972)

Thus, the black hole is alleged to have an escape velocity and not to have an escape velocity simultaneously, which is impossible. The very concept of black hole escape velocity is nothing but a play on the words "*escape velocity*" (McVittie, 1978).

Here is how Einstein surreptitiously introduces multiple masses into an alleged one-mass universe which is in fact a no-mass universe, by applying the Principle of Superposition where the Principle of Superposition is invalid. In relation to Hilbert's solution Einstein (1967) says,

"*M denotes the sun's mass, centrally symmetrically placed about the origin of co-ordinates; the solution (109a) is valid only outside of this mass, where all the $T_{\mu\nu}$ vanish. If the motion of the planet takes place in the $x_1 - x_2$ plane then we must replace (109a) by*

$$ds^2 = \left(1 - \frac{A}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{A}{r}\right)} - r^2 d\phi^2 \text{ "}$$

wherein $A = \kappa M/4\pi$. Notice that Einstein has not only introduced a material source M he has also introduced a planet outside this alleged source.

(4) $R_{\mu\nu} = 0$ violates the physical principles of General Relativity

$R_{\mu\nu} = 0$ and the 'Schwarzschild solution' thereto violate the physical principles of General Relativity. Einstein asserted that his Principle of Equivalence and his laws of

Special Relativity must hold in sufficiently small finite regions of his gravitational field, and that these regions can be located anywhere in his gravitational field. According to Einstein (1967),

> "*Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K', uniformly accelerated with respect to K. Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were unaccelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K', we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For, according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to K) or under the combined action of inertia and gravitation (with respect to K').*
>
> "*Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of special relativity, which have been developed above, hold with remarkable accuracy*."

Clearly both the Principle of Equivalence and Special Relativity are defined in terms of the *a priori* presence of multiple arbitrarily large finite masses and photons. It is therefore impossible for the Principle of Equivalence and Special Relativity to manifest in a spacetime that by mathematical construction contains no matter. But $R_{\mu\nu} = 0$ is a spacetime that by mathematical construction contains no matter. Thus $R_{\mu\nu} = 0$ violates the physical principles of General Relativity and so it is inadmissible and therefore has no physical significance. Since Hilbert's solution is for $R_{\mu\nu} = 0$ it is also of no physical significance. But it is from Hilbert's solution that the black hole was first generated. It is also impossible for the Principle of Equivalence and Special Relativity to manifest in a universe that, allegedly, contains only one mass (the 'Schwarzschild solution'). Therefore, the black hole is not even consistent with General Relativity.

(5) Static and non-static 'solutions'

Recall that these Authors have stated that they deal with "*a vacuum, static spherical spacetime*" but also talk of black holes in binary systems. The black hole however requires a non-static solution. Consider Hilbert's metric (2.2) and (2.3).

"*The most obvious pathology at $r = 2M$ is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region $r > 2M$, the t direction, $\partial/\partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial/\partial r$, is spacelike ($g_{rr} > 0$); but in the region $r < 2M$, $\partial/\partial t$, is spacelike ($g_{tt} > 0$) and $\partial/\partial r$, is timelike ($g_{rr} < 0$).*

"*What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at r =2M always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside r =2M. Then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate r =2M to the later time coordinate r =0. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away 'the unforgiving minutes', with equal certainty, and with never one halt along the way, r drops from 2M to 0.*

"*At r =2M, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite.*"

(Misner, Thorne, and Wheeler, 1970)

"*There is no alternative to the matter collapsing to an infinite density at a singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all time-like trajectories must necessarily get to the singularity: 'all the King's horses and all the King's men' cannot prevent it.*"

(Chandrasekhar, 1972)

"*This is worth stressing; not only can you not escape back to region I, you cannot even stop yourself from moving in the direction of decreasing r, since this is simply the timelike direction. (This could have been seen in our original coordinate system; for r < 2GM, t becomes spacelike and r becomes timelike.) Thus you can no more stop moving toward the singularity than you can stop getting older.*"

(Carroll, S.)

"*For $r < 2GM/c^2$, however, the component g_{oo} becomes negative, and g_{rr}, positive, so that in this domain, the role of time-like coordinate is played by r, whereas that of space-like coordinate by t. Thus in this domain, the gravitational field depends significantly on time (r) and does not depend on the coordinate t.*"

(Vladmimirov, Mitskiévich, and Horský, 1984)

To amplify this, set $t = r^*$ and $r = t^*$, and so for $0 \le r < 2M$, Hilbert's solution (2.2) becomes,

$$ds^2 = -\left(1-\frac{2M}{t^*}\right)dr^{*2} + \left(1-\frac{2M}{t^*}\right)^{-1} dt^{*2} + t^{*2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \quad (2.28)$$

$$0 \le t^* < 2M$$

It now becomes quite clear that this is a time-dependent (non-static) metric since all the components of the metric tensor are now functions of the timelike t^*, and so this metric bears no relationship to the original time-independent (static) problem that was initially proposed (Droste 1917, Brillouin 1923). In other words, this metric is a non-static solution to a static problem: contra hype! Furthermore, the signature of the metric changes from (-, +, +, +) to (+, -, +, +).

DEDICATION

I dedicate this paper to my beloved late brother:

Paul Raymond Crothers

12TH MAY 1968 – 25TH DECEMBER 2008

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