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NEW CONCEPTS OF NEUTROSOPHIC SETS

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ABSTRACT

In this paper we will introduce and study some types of neutrosophic sets. Finally, we extend the concept of intuitionistic fuzzy ideal [8] to the case of neutrosophic sets. We can use the new of neutrosophic notions in the following applications: compiler, networks robots, codes and database.

KEYWORDS: Fuzzy Set, Intuitionistic Fuzzy Set, Neutrosophic Set, Intuitionistic Fuzzy Ideal, Neutrosophic Ideal

1-INTRODUCTION

The neutrosophic set concept was introduced by Smarandache [11, 12]. In 2012 neutrosophic sets have been investigated by Hanafy and Salama at el [4, 5, 6, 7, 8, 9]. The fuzzy set was introduced by Zadeh [13] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama at el [8] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts. In this paper we will introduce the definitions of normal neutrosophic set, convex set, the concept of α-cut and neutrosophic ideals, which can be discussed as generalization of fuzzy and fuzzy intuitionistic studies.

2-TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [11, 12], and Salama at el [6, 7].

3-SOME TYPES OF NEUTROSOPHIC SETS

Definition.3.1

A neutrosophic set A with $\mu_A(x)=1$, $\sigma_A(x)=1$, $\gamma(x)=1$ is called normal neutrosophic set.

In other words A is called normal if and only if $\max\limits_{x\in X}\mu_A(x)=\max\limits_{x\in X}\sigma_A(x)=\max\limits_{x\in X}\gamma_A(x)=1$.

Definition.3.2

When the support set is a real number set and the following applies for all $x\in[a,b]$ over any interval $[a,b]$:

$$\mu_A(x)\ge\mu_A(a)\wedge\mu_A(b) \quad ; \quad \sigma_A(x)\ge\sigma_A(a)\wedge\sigma_A(b) \quad and \quad \gamma_A(x)\ge\gamma_A(a)\wedge\gamma_A(b)$$

A is said to be convex.

Definition 3.3

When $A \subset X$ and $B \subset Y$, the neutrosophic subset $A \times B$ of $X \times Y$ that can be arrived at the following way is the direct product of A and B.

$$A \times B \leftrightarrow \mu_{A\times B}(x, y) = \mu_A(x) \wedge \mu_B(x)$$

$$\sigma_{A\times B}(x, y) = \sigma_A(x) \wedge \sigma_B(x)$$

$$\gamma_{A\times B}(x, y) = \gamma_A(x) \wedge \gamma_B(x)$$

We must first introduce the concept of α-cut

Definition 3.4

For a neutrosophic set A

$$A_\alpha = \{x : \mu_A(x), \sigma_A(x), \gamma_A(x) > \alpha\}; \qquad \alpha \in \left]{}^{-}0, 1^{+}\right[$$

$$A_{\overline{\alpha}} = \{x : \mu_A(x), \sigma_A(x), \gamma_A(x) \geq \alpha\}; \qquad \alpha \in \left]{}^{-}0, 1^{+}\right[$$

are called the weak and strong α-cut respectively.

Making use α-cut, the following relational equation is called the resolution principle.

Theorem 3.1

$$\mu_A(x) = \sigma_A(x) = \gamma_A(x) = \underset{x \in \left]{}^{-}0,1^{+}\right[}{Sup} \left[\alpha \wedge \chi_{A_\alpha}(x)\right]$$

$$\mu_A(x) = \sigma_A(x) = \gamma_A(x) = Sup\left[\alpha \wedge \chi_{A_{\overline{\alpha}}}(x)\right]$$

Proof

$$\underset{x \in \left]{}^{-}0,1^{+}\right[}{Sup}\left[\alpha \wedge \chi_{A_{\overline{\alpha}}}(x)\right] = \underset{\substack{\alpha \in \left({}^{-}0, \mu_{A_{\overline{\alpha}}}(x)\right) \\ \alpha \in \left({}^{-}0, \sigma_{A_{\overline{\alpha}}}(x)\right) \\ \alpha \in \left({}^{-}0, \gamma_{A_{\overline{\alpha}}}(x)\right)}}{Sup}\left[\alpha \wedge \chi_{A_{\overline{\alpha}}}(x)\right] = \underset{\substack{\alpha \in \left(\mu_{A_{\overline{\alpha}}}(x), 1^{+}\right) \\ \alpha \in \left(\sigma_{A_{\overline{\alpha}}}(x), 1^{+}\right) \\ \alpha \in \left(\gamma_{A_{\overline{\alpha}}}(x), 1^{+}\right)}}{Sup}\left[\alpha \wedge \chi_{A_{\overline{\alpha}}}(x)\right]$$

$$= \underset{\alpha \in (0, \mu_A(x)}{Sup}\left[\alpha \wedge 1^{+}\right] \vee Sup\left[\alpha \wedge 0^{-})\right]$$

$$= \underset{\substack{\alpha \in \left({}^{-}0, \mu_A(x)\right) \\ \alpha \in \left({}^{-}0, \sigma_A(x)\right) \\ \alpha \in \left({}^{-}0, \gamma_A(x)\right)}}{Sup} \alpha = \mu_A(x) = \sigma_A(x) = \gamma_A(x)$$

If we defined the neutrosophic set αA_α here as

$$\alpha A_\alpha \leftrightarrow \mu_{\alpha A_\alpha} = \alpha \wedge \chi_{A_{\overline{\alpha}}}(x) = \sigma_{\alpha A_{\overline{\alpha}}}(x) = \gamma_{\alpha A_{\overline{\alpha}}}(x)$$

The resolution principle is expressed in the form

$$A = \bigcup_{\alpha \in \left[\overset{-}{0}, \overset{+}{1}\right]} \alpha A_\alpha$$

In other words, a neutrosophic set can be expressed in terms of the concept of α-cuts without resorting to grade functions μ, *б* and γ. This is what wakes up the representation theorem, and we will leave it at that α-cuts are very convenient for the calculation of the operations and relations equations of neutrosophic sets.

Next let us discuss what is called the extension principle; we will use the functions from *X* to *Y*.

Definition 3.5

Extending the function $f: X \to Y$, the neutrosophic subset A of X is made to correspond to neutrosophic subset $f(A) = \left(\mu_{f(A)}, \sigma_{f(A)}, \gamma_{f(A)}\right)$ of $\boldsymbol{Y}$ may be the following ways (type1, 2)

- $$\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\}, & if \quad f^{-1}(\text{y}) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

  $$\sigma_{f(A)}(y) = \begin{cases} \wedge\{\sigma_A(x) : x \in f^{-1}(y)\}, & if \quad f^{-1}(\text{y}) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

  $$\gamma_{f(A)}(y) = \begin{cases} \wedge\{\gamma_A(x) : x \in f^{-1}(y)\}, & if \quad f^{-1}(\text{y}) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

- $$\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\}, & if \quad f^{-1}(\text{y}) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

  $$\sigma_{f(A)}(y) = \begin{cases} \vee\{\sigma_A(x) : x \in f^{-1}(y)\}, & if \quad f^{-1}(\text{y}) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

  $$\gamma_{f(A)}(y) = \begin{cases} \wedge\{\gamma_A(x) : x \in f^{-1}(y)\}, & if \quad f^{-1}(\text{y}) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

Let B neutrosophic set in Y. Then the preimage of B, under f, denoted by $f^{-1}(B) = \left(\mu_{f^{-1}(B)}, \sigma_{f^{-1}(B)}, \gamma_{f^{-1}(B)}\right)$ defined by $\mu_{f^{-1}(B)} = \mu\left(f(B)\right), \sigma_{f^{-1}(B)} = \sigma\left(f(B)\right), \gamma_{f^{-1}(B)} = \gamma\left(f(B)\right)$.

Theorem.3.2

Let A, A_i in X, B *and* B_j, $i \in I$, $j \in J$ in Y are neutrosophic subsets and $f: X \to Y$ be a function. Then

- $A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2)$.

- $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$,
- $A \subset f(f^{-1}(A))$, the equality holds if f is injective,
- $f(f^{-1}(B)) \subset B$, the equality holds if f is surjective,
- $f^{-1}(\cup_j B_j) = \cup_j f^{-1}(B_j)$,
- $f^{-1}(\cap_j B_j) = \cap_j f^{-1}(B_j)$,
- $f(\cup_i A_i) = \cup_i f(A_i)$,

Proof

Clear.

4- NEUTROSOPHIC IDEALS

Definition.4.1

Let X is non-empty set and L a non–empty family of NSs. We will call L is a neutrosophic ideal (NL for short) on X if

- $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],
- $A \in L$ and $B \in \mathrm{L} \Rightarrow A \vee B \in \mathrm{L}$ [Finite additivity].

A neutrosophic ideal L is called a σ-neutrosophic ideal if $\{A_j\}_{j \in N} \leq L$, implies $\underset{j \in J}{\vee} A_j \in L$ (countable additivity).

The smallest and largest neutrosophic ideals on a non-empty set X are $\{0_N\}$ and NSs on X. Also, $N.\mathrm{L_f}$, $\mathrm{N.L_c}$ are denoting the neutrosophic ideals (NL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NS : B \subseteq A\}$ is an NL on X. This is called the principal NL of all NSs of denoted by NL $\langle A \rangle$.

Remark 4.1

- If $1_N \notin L$, then L is called neutrosophic proper ideal.
- If $1_N \in L$, then L is called neutrosophic improper ideal.
- $O_N \in L$.

Example.4.1

Any Initiutionistic fuzzy ideal ℓ on X in the sense of Salama is obviously and NL in the form $L = \{A : A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in \ell\}$.

Example.4.2

Let $X = \{a,b,c\}$ $A = \langle x, 0.2, 0.5, 0.6 \rangle$, $B = \langle x, 0.5, 0.7, 0.8 \rangle$, and $D = \langle x, 0.5, 0.6, 0.8 \rangle$, then the family

$L = \{O_N, A, B, D\}$ of NSs is an NL on X.

Example.3.3

Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ given by:

X	$\mu_A(x)$	$\sigma_A(x)$	$\nu_A(x)$
a	0.6	0.4	0.3
b	0.5	0.3	0.3
c	0.4	0.6	0.4
d	0.3	0.8	0.5
e	0.3	0.7	0.6

Then the family $L = \{O_N, A\}$ is an NL on X.

Definition.4.3

Let L_1 and L_2 be two NL on X. Then L_2 is said to be finer than L_1 or L_1 is coarser than L_2 if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1 or L_1 is strictly coarser than L_2.

Two NL said to be comparable, if one is finer than the other. The set of all NL on X is ordered by the relation L_1 is coarser than L_2 this relation is induced the inclusion in NSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.4.1

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are neutrosophic ideal on X,

In fact L is the smallest upper bound of the set of the L_j in the ordered set of all neutrosophic ideals on X.

Remark.4.2

The neutrosophic ideal by the single neutrosophic set O_N is the smallest element of the ordered set of all neutrosophic ideals on X.

Proposition.4.3

A neutrosophic set A in neutrosophic ideal L on X is a base of L iff every member of L contained in A.

Proof

(Necessity)Suppose A is a base of L. Then clearly every member of L contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic subset in X contained in A coincides with L by the Definition 4.3.

Proposition.4.4

For a neutrosophic ideal L_1 with base A, is finer than a fuzzy ideal L_2 with base B iff every member of B contained in A.

Proof

Immediate consequence of Definitions

Corollary.4.1

Two neutrosophic ideals bases A, B, on X are equivalent iff every member of A, contained in B and via versa.

Theorem.4.1

Let $\eta = \{\langle \mu_j, \sigma_j, \gamma_j \rangle : j \in J\}$ be a non empty collection of neutrosophic subsets of X. Then there exists a neutrosophic ideal L (η) = {A $\in$ NSs: A $\subseteq \vee$ A_j} on X for some finite collection {A_j: j = 1,2, ..., n $\subseteq \eta$}.

Proof

Clear.

Remark.4.3

ii) The neutrosophic ideal L (η) defined above is said to be generated by η and η is called sub base of L(η).

Corollary.4.2

Let L_1 be an neutrosophic ideal on X and A $\in$ NSs, then there is a neutrosophic ideal L_2 which is finer than L_1 and such that A $\in$ L_2 iff

A $\vee$ B $\in$ L_2 for each B$\in$ L_1.

Corollary.4.3

Let $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in L_1$ and $B = \langle x, \mu_B, \sigma_B, \nu_B \rangle \in L_2$, where L_1 and L_2 are neutrosophic ideals on the set X. then the neutrosophic set $A*B = \langle \mu_{A*B}(x), \sigma_{A*B}(x), \nu_{A*B}(x) \rangle \in L_1 \vee L_2$ on X where $\mu_{A*B}(x) = \vee\{\mu_A(x) \wedge \mu_B(x) : x \in X\}, \sigma_{A*B}(x)$ may be $= \vee\{\sigma_A(x) \wedge \sigma_B(x)\}$ or $\wedge\{\sigma_A(x) \vee \sigma_B(x)\}$ and $\nu_{A*B}(x) = \wedge\{\nu_A(x) \vee \nu_B(x) : x \in X\}$.

Theorem.4.2

If L is a neutrosophic ideal on X, then so is □ L= is a neutrosophic ideal on X.

Proof

Clear

Theorem.4.3

An NS $L = \{O_N, \langle \mu_A, \sigma_A, \nu_A \rangle\}$ is a neutrosophic ideal on X iff the fuzzy sets μ_A, σ_A and ν_A^c are intuitionistic fuzzy ideals on X.

Proof

Let $L = \{O_N, \langle \mu_A, \sigma_A, \nu_A \rangle\}$ be a NL of X, $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$, then clearly μ_A is a intuitionistic fuzzy ideal on

X. Then $\underset{A}{\overset{c}{\nu}}(x) = 1 - \nu_A(x) = \max\left\{\left(\underset{A}{\overset{c}{\nu}}(x), 0\right)\right\} = \min\left\{1, \nu_A^{c}(x)\right\}$ if $\underset{A}{\overset{c}{\nu}}(x) = O_N$ then is the smallest intuitionistic fuzzy ideal , or $\underset{A}{\overset{c}{\nu}}(x) = 1_N$ then is the largest intuitionistic fuzzy ideal on X.

Corollary.4.3

L is a neutrosophic ideal on X iff □L and ◊L are neutrosophic ideals on X.

Proof

Clear from the definition 1.3.

Example.4.4

Let L a non empty set and NL on X given by: $L = \{O_N, \langle 0.3, 0.6, 0.2\rangle, \langle 0.3, 0.5, 0.6\rangle \langle 0.2, 0.5, 0.5\rangle\}$.Then □L= $\{O_N, \langle 0.3, 0.7, 0.7\rangle, \langle 0.2, 0.8, 0.8\rangle\}$ and $\lozenge L = \{O_N, \langle 0.4, 0.6, 0.6\rangle, \langle 0.5, 0.5, 0.5\rangle\}$ and □L $\subseteq$ ◊L.

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