The Continuum Hypothesis

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In this paper we give a proof of the Continuum Hypothesis

Theorem "Continuum Hypothesis": There exist a bijection between the set of real numbers numbers $\mathbb{R}$ and the power set of $\mathbb{N}$.

Proof: Let $\mathbb{R}^+ \subset \mathbb{R}$ be the complete free $\mathbb{N}$-semialgebra of nonnegative numbers and let the multiplicative cyclic subgroup $\langle 2 \rangle$ be a basis of $\mathbb{R}^+$, let $\mathbb{R}_2^+$ be the $\mathbb{N}$-semialgebra of dyadic nonnegative numbers, which, by the existence of the identity bijection on the basis $\langle 2 \rangle$, it is isomorphic to $\mathbb{R}^+$, then, for $\mathbb{R}_2^+$ is complete free with $\langle 2 \rangle$ as basis, it is also isomorphic to the cartesian product $\prod_{i \in \mathbb{Z}} B_i$ where $B_i = \{0, 1\}$ for every i, then, since the sets $\mathbb{R}$ and $\mathbb{R}^+$ are equipotent, that is, the cardinal number $|\mathbb{R}|$ of $\mathbb{R}$ and that of $\mathbb{R}^+$, $|\mathbb{R}^+|$, are equal,

$$|\mathbb{R}| = |\mathbb{R}^+| = \left|\mathbb{R}_2^+\right| = |\prod_{i \in \mathbb{Z}} B_i| = 2^{\aleph_0},$$

therefore, by the Cantor-Bernstein-Schröder theorem, there exist a bijection between $\mathbb{R}$ and the power set of $\mathbb{N}$.

Thereby, by the ZFC axioms and definition of cardinal number, this theorem proves that there is no cardinal number between the initial transfinite cardinal number $\aleph_0$ and $2^{\aleph_0}$. In general, we have

Corollary "Generalized Continuum Hypothesis": For every cardinal number α, there is no cardinal number between α and 2^α.

Thus this theorem proves not only that the class of cardinal numbers forms a complete strict monoidal category, with arrows, the polynomial maps and the exponential maps, which is an e-algebra under the covariant exponential functor e, but also, by definition of ordinal number, that the category of limit ordinal numbers is isomorphic to the countable complete strict monoidal category of transfinite cardinals.