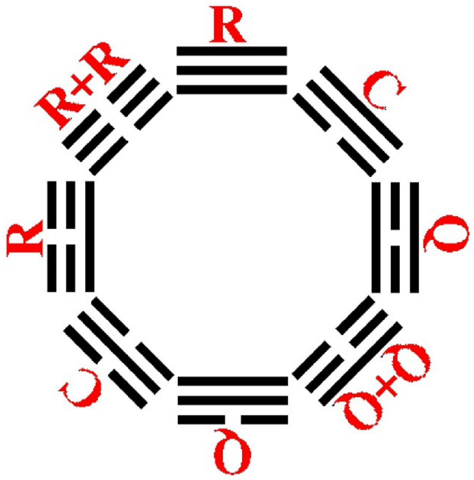
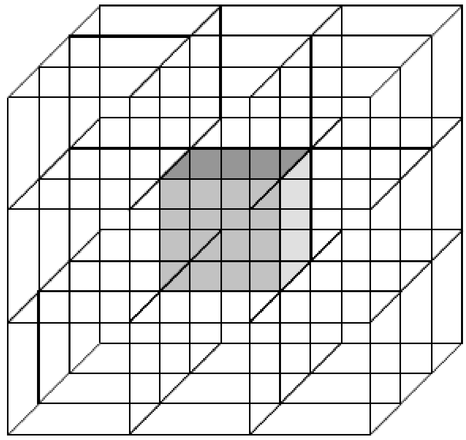
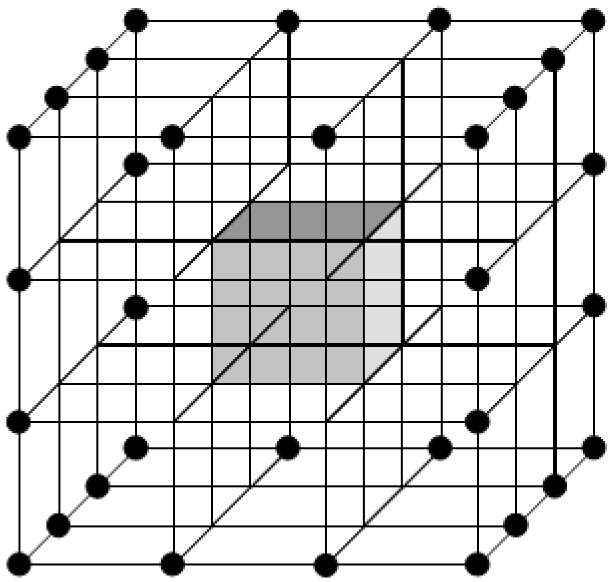
Clifford Clock and the Moolakaprithi Cube







By John Frederick Sweeney

Abstract: Matter begins with the Clifford Clock at the border of the Substratum state; then can be traced through to the Moolaprakrithi state. Along the way, this paper traces the Trigrams of Chinese metaphysics to the Clifford Clock and the Clock of Complex Spaces, the binary aspects of the trigrams and the binary aspects of the Octonions and the Fano Plane, then the construction of the 3 x 3 x 3 Cube from the Trigrams; then isomorphic relations between the Cube and the Klein Quartic and the Sierspinski Triangle; Clifford Algebras and their organization via Pascal's Triangle (Zhang Hui's Triangle or Mount Meru); connections to the Magic Triangle of Exceptional Lie Algebras which terminates with E8.

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Mind Map of Symmetries in Physics

*These groups and algebras are technically infinite dimensional

Introduction

The author began to write a series of papers on Vixra in order to provide the foundation in mathematical physics for Chinese divination, specifically for Qi Men Dun Jia, among the most advanced forms of divination known to the Chinese for two thousand years or longer. In order to achieve this purpose, it soon became necessary to provide the foundation for Vedic Physics in terms of contemporary science, since Vedic Physics provides the proper scientific paradigm necessary to prove that divination is not magic, but rather advanced science that contemporary science fails to explain.

Vedic Physics originates with the Vedas, or the most ancient books known to humanity, which in turn were committed to written language after millennia as an exclusive oral tradition, during the pre – Ice Age era at least 14,000 years ago. The science is highly advanced, representing a development of five to ten thousand years beyond our present level of science. Some books published in India explain this science but they are poorly written and lack editing, which makes it all the more difficult to establish credibility in the west, where people are resistant to doctrines from India and China.

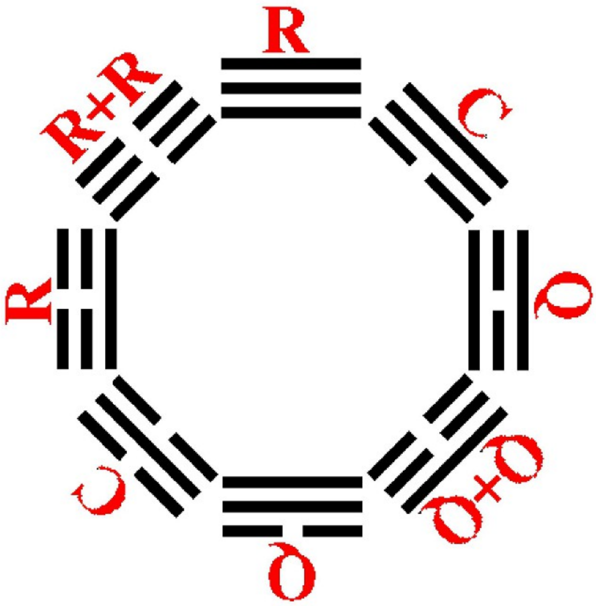
For this reason, the author of this paper took to editing and re – presenting some of those works in the hope of their gaining credibility in Europe and the Americas. Here is an excerpt from one of the books:

Vedic Physics demonstrates through algebraic mathematics that the infinitesimal displacement identifying the geodesic, itself has a coherent potential that can be expressed in a self-similar and perpetually coherent Moolaprakriti power series by a dimensionless variable, up to infinite levels, which seems to be new to physics.

This paper links the Octonions to the eight trigrams of the I Ching and then to an Octonion Power Series, the opposite of which is a series of logarithms moving back to the Substratum. In this sense, this current paper builds on two earlier papers which explained the Clifford Clock, Bott Periodicity and the Clock of Complex Spaces, which form isomorphic relationships with the basic Qi Men Dun Jia Cosmic Board.

The key to these connections lies in a 3 x 3 x 3 cube, which in Vedic Physics, forms the Moolaprakriti, a key component of the Substratum, the invisible black hole form of matter. At the same time, the 3 x 3 x 3 cube forms isomorphic relations with the eight trigrams of the I Ching, as well as the 64 amino acids of DNA, then with the Octonions, the Klein Quartic, the 24 Hurwitz Quarternions (Hurwitz Integers), and the tetrahedron.

Clifford Clock



John Baez on the Clifford Clock and Clock of Complex Spaces

Now for some math. It's always great when two subjects you're interested in turn out to be bits of the same big picture. That's why I've been really excited lately about Bott periodicity and the "super-Brauer group".

I wrote about Bott periodicity in "week105", and about the Brauer group in "week209", but I should remind you about them before putting them together.

Bott periodicity is all about how math and physics in n+8-dimensional space resemble math and physics in n-dimensional space. It's a weird and wonderful pattern that you'd never guess without doing some calculations. It shows up in many guises, which turn out to all be related. The simplest one to verify is the pattern of Clifford algebras.

You're probably used to the complex numbers, where you throw in just *one* square root of -1, called i. And maybe you've heard of the Quaternions, where you throw in *two* square roots of -1, called i and j, and demand that they anti - commute:

$$ij = -ji$$

This implies that k = ij is another square root of -1. Try it and see!

In the late 1800s, Clifford realized there's no need to stop here. He invented what we now call the "Clifford algebras" by starting with the real numbers and throwing in n square roots of -1, all of which anti - commute with each other. The result is closely related to rotations in n+1 dimensions, as I explained in "week82".

I'm not sure who first worked out all the Clifford algebras - perhaps it was Cartan - but the interesting fact is that they follow a periodic pattern. If we use

C_n to stand for the Clifford algebra generated by n anti - commuting square roots of -1, they go like this:

	Number		
C_0	R		
C_1	C		
C_2	H		
C_3	H + H		
C_4	H(2)		
C_5	C(4)		
C_6	R(8)		
C_7	R(8) + R(8)		

where:

- R(n) means n x n real matrices,
- C(n) means n x n complex matrices, and
- H(n) means n x n quaternionic matrices.

All these become algebras with the usual addition and multiplication of matrices. Finally, if A is an algebra, A + A consists of pairs of guys in A, with pairwise addition and multiplication.

What happens next? Well, from then on things sort of "repeat" with period 8:

C_{n+8} consists of 16 x 16 matrices whose entries lie in C_n!

So, you can remember all the Clifford algebras with the help of this eight-hour clock:

0

R

7 **1**

R+R **C**

6 R **H 2**

C **H+H**

5 **3**

H

4

To use this clock, you have to remember to use matrices of the right size to get C_n to have dimension 2^n. So, when I write "R + R" next to the "7" on the clock, I don't mean C_7 is really R + R.

To get C_7, you have to take R + R and beef it up until it becomes an algebra of dimension $2^7 = 128$. You do this by taking R(8) + R(8), since this has dimension 8 x 8 + 8 x 8 = 128.

Similarly, to get C_{10}, you note that 10 is 2 modulo 8, so you look at "2" on the clock and see "H" next to it, meaning the Quaternions.

But to get C_{10}, you have to take H and beef it up until it becomes an algebra of dimension $2^{10} = 1024$.

You do this by taking H(16), since this has dimension 4 x 16 x 16 = 1024.

This "beefing up" process is actually quite interesting. For any associative algebra A, the algebra A(n) consisting of n x n matrices with entries in A is a lot like A itself. The reason is that they have equivalent categories of representations!

To see what I mean by this, remember that a "representation" of an algebra is a way for its elements to act as linear transformations of some vector space.

For example, R(n) acts as linear transformations of R^n by matrix multiplication, so we say R(n) has a representation on R^n. More generally, for any algebra A, the algebra A(n) has a representation on A^n.

More generally still, if we have any representation of A on a vector space V, we get a representation of A(n) on V^n. It's less obvious, but true, that *every* representation of A(n) comes from a representation of A this way.

In short, just as n x n matrices with entries in A form an algebra A(n) that's a beefed-up version of A itself, every representation of A(n) is a beefed-up version of some representation of A.

Even better, the same sort of thing is true for maps between representations of A(n). This is what we mean by saying that A(n) and A have equivalent categories of representations.

If you just look at the categories of representations of these two algebras as abstract categories, there's no way to tell them apart! We say two algebras are "Morita equivalent" when this happens.

It's fun to study Morita equivalence classes of algebras - say algebras over the real numbers, for example. The tensor product of algebras gives us a way to multiply these classes. If we just consider the invertible classes, we get a *group*. This is called the "Brauer group" of the real numbers.

The Brauer group of the real numbers is just Z/2, consisting of the classes [R] and [H]. These correspond to the top and bottom of the Clifford clock! Part of the reason is that

H tensor H = R(4)

so when we take Morita equivalence classes we get

[H] x [H] = [R]
But, you may wonder where the complex numbers went! Alas, the Morita equivalence class [C] isn't invertible, so it doesn't live in the Brauer group. In fact, we have this little multiplication table for tensor product of algebras:

tensor	**R**	**C**	**H**
R	**R**	**C**	**H**
C	**C**	**C+C**	**C(2)**
H	**H**	**C(2)**	**R(4)**

Anyone with an algebraic bone in their body should spend an afternoon figuring out how this works! But I won't explain it now.
Instead, I'll just note that the complex numbers are very aggressive and infectious - tensor anything with a C in it and you get more C's. That's because they're a field in their own right - and that's why they don't live in the Brauer group of the real numbers.

They do, however, live in the *super-Brauer* group of the real numbers, which is Z/8 - the Clifford clock itself!

But before I explain that, I want to show you what the categories of representations of the Clifford algebras look like:

1 **split Real vector spaces**	**0** **Real vector spaces**	**7** **Complex vector spaces**
6 **Real vector spaces**		**2** **Quaternionic vector**
5 **Complex vector spaces**	**4** **Quaternionic vector spaces**	**3** **split Quaternionic vector spaces**

0

Real vector spaces

7

1

split Real vector spaces Complex vector spaces

6 Real vector spaces Quaternionic vector spaces

2

Complex vector spaces split Quaternionic vector spaces

5 3

Quaternionic vector spaces

4

You can read this information off the 8-hour Clifford clock I showed you before, at least if you know some stuff:

- A real vector space is just something like R^n
- A complex vector space is just something like C^n
- A quaternionic vector space is just something like H^n

and a "split" vector space is a vector space that's been written as the direct sum of two subspaces.

Take C_4, for example -

The Clifford algebra generated by 4 anti - commuting square roots of -1. The Clifford clock tells us this is H + H. And if you think about it, a representation of this is just a pair of representations of H. So, it's two quaternionic vector spaces - or if you prefer, a "split" quaternionic vector space.

Or take C_7. The Clifford clock says this is R + R... or at least Morita equivalent to R + R: it's actually R(8) + R(8), but that's just a beefed-up version of R + R, with an equivalent category of representations. So, the category of representations of C_7 is *equivalent* to the category of split real vector spaces.

And so on. Note that when we loop all the way around the clock, our Clifford algebra becomes 16 x 16 matrices of what it was before, but this is Morita equivalent to what it was. So, we have a truly period-8 clock of categories!

But here's the really cool part: there are also arrows going clockwise and counterclockwise around this clock! Arrows between categories are called "functors".

Each Clifford algebra is contained in the next one, since they're built by throwing in more and more square roots of -1. So, if we have a representation of C_n, it gives us a representation of C_{n-1}. Ditto for maps between representations.

So, we get a functor from the category of representations of C_n to the category of representations of C_{n-1}. This is called a "forgetful functor", since it "forgets" that we have representations of C_n and just thinks of them as representations of C_{n-1}.

So, we have forgetful functors cycling around counterclockwise!

Even better, all these forgetful functors have "left adjoints" going back the other way. I talked about left adjoints in "week77", so I won't say much about them now. I'll just give an example.

Here's a forgetful functor:

```
                  forget complex structure
complex vector spaces --------------------------> real vector spaces
```

which is one of the counterclockwise arrows on the Clifford clock. This functor takes a complex vector space and forgets your ability to multiply vectors by i, thus getting a real vector space.

When you do this to C^n, you get R^{2n}.
This functor has a left adjoint:

```
                        complexify
complex vector spaces     <--------------------------     real vector spaces
```

where you take a real vector space and "complexify" it by tensoring it with the complex numbers. When you do this to R^n, you get C^n.
So, we get a beautiful version of the Clifford clock with forgetful functors cycling around counterclockwise and their left adjoints cycling around clockwise! When I realized this, I drew a big picture of it in my math notebook - I always carry around a notebook for precisely this sort of thing. Unfortunately, it's a bit hard to draw this chart in ASCII, so I won't include it here.

Instead, I'll draw something easier. For this, note the following mystical fact. The Clifford clock is symmetrical under reflection around the 3-o'clock/7-o'clock axis:

0

real vector spaces

7

1

split real vector spaces complex vector spaces

\
\
\
\

6 real vector spaces \ quaternionic vector spaces 2

\
\
\
\

complex vector spaces split quaternionic vector spaces

5 3

quaternionic vector spaces

4

0

Real Vector Spaces 7

1

Split Real Vector Spaces Complex Vector Spaces

6 Real Vector Spaces Quaternionic Vector Spaces 2

Complex Vector Spaces Split Quaternionic Vector spaces 5

3

Quaternionic Vector Spaces

4

It seems bizarre at first that it's symmetrical along *this* axis instead of the more obvious 0-o'clock/4-o'clock axis. But there's a good reason, which I already mentioned: the Clifford algebra C_n is related to rotations in n+1 dimensions.
I would be very happy if you had enough patience to listen to a full explanation of this fact, along with everything else I want to say. But I bet you don't... so I'll hasten on to the really cool stuff.

First of all, using this symmetry we can fold the Clifford clock in half... and the forgetful functors on one side perfectly match their left adjoints on the other side!

So, we can save space by drawing this "folded" Clifford clock:

```
              split real vector spaces

                        | ^
        forget splitting | | double
                        v |

               real vector spaces

                        | ^
            complexify | | forget complex structure
                        v |

              complex vector spaces

                        | ^
          quaternionify | | forget quaternionic structure
                        v |

             quaternionic vector spaces

                        | ^
                double | | forget splitting
                        v |

           split quaternionic vector spaces
```

The forgetful functors march downwards on the right, and their left adjoints march back up on the left! The arrows going between 7 o'clock and 0 o'clock look a bit weird:

```
          split real vector spaces

                   | ^
forget splitting | | double
                   V |

           real vector spaces
```

Why is "forget splitting" on the left, where the left adjoints belong, when it's obviously an example of a forgetful functor?

One answer is that this is just how it works. Another answer is that it happens when we wrap all the way around the clock - it's like how going from midnight to 1 am counts as going forwards in time even though the number is getting smaller.

A third answer is that the whole situation is so symmetrical that the functors I've been calling "left adjoints" are also "right adjoints" of their partners! So, we can change our mind about which one is "forgetful", without getting in trouble.

But enough of that: I really want to explain how this stuff is related to the super-Brauer group, and then tie it all in to the *topology* of Bott periodicity. We'll see how far I get before giving up in exhaustion....

What's a super-Brauer group? It's just like a Brauer group, but where we use Super Algebras instead of algebras! A "superalgebra" is just physics jargon for a Z/2-graded algebra - that is, an algebra A that's a direct sum of an "even" or "bosonic" part A_0 and an "odd" or "fermionic" part A_1:

$A = A_0 + A_1$

such that multiplying a guy in A_i and a guy in A_j gives a guy in A_{i+j}, where we add the subscripts mod 2.

The tensor product of Super Algebras is defined differently than for algebras. If A and B are ordinary algebras, when we form their tensor product, we decree that everybody in A commutes with everyone in B. For Super Algebras we decree that everybody in A "super - commutes" with everyone in B - meaning that

ab = ba

if either a or b are even (bosonic) while

ab = -ba

if a and b are both odd (fermionic).

Apart from these modifications, the super-Brauer group works almost like the Brauer group. We start with Super Algebras over our favorite field - here let's use the real numbers.

We say two Super Algebras are "Morita equivalent" if they have equivalent categories of representations. We can multiply these Morita equivalence classes by taking tensor products, and if we just keep the invertible classes we get a group: the super-Brauer group.

As I've hinted already, the super-Brauer group of the real numbers is Z/8 - just the Clifford algebra clock in disguise!

Here's why:

The Clifford algebras all become Super Algebras if we decree that all the square roots of -1 that we throw in are "odd" elements. And if we do this, we get something great:

C_n tensor C_m = C_{n+m}

The point is that all the square roots of -1 we threw in to get C_n *anticommute* with those we threw in to get C_m.

Taking Morita equivalence classes, this mean

$[C_n]$ $[C_m]$ = $[C_{n+m}]$

but we already know that

$[C_{n+8}]$ = $[C_n]$

so we get the group Z/8. It's not obvious that this is *all* the super-Brauer group, but it actually is - that's the hard part.

Now let's think about what we've got. We've got the super-Brauer group, Z/8, which looks like an 8-hour clock. But before that, we had the categories of representations of Clifford algebras, which formed an 8-hour clock with functors cycling around in both directions.

In fact these are two sides of the same coin - or clock, actually. The super-Brauer group consists of Morita equivalence classes of Clifford algebras, where Morita equivalence means "having equivalent categories of representations". But, our previous clock just shows their categories of representations!

This suggests that the functors cycling around in both directions are secretly an aspect of the super-Brauer group. And indeed they are! The functors going clockwise are just "tensoring with C_1", since you can tensor a representation of C_n with C_1 and get a representation of C_{n+1}. And the functors going counterclockwise are "tensoring with C_{-1}"... or C_7 if you insist, since C_{-1} doesn't strictly make sense, but 7 equals -1 mod 8, so it does the same job.

Hurwitz Quaternions (or Hurwitz integer)

In mathematics, a **Hurwitz quaternion** (or **Hurwitz integer**) is a quaternion whose components are *either* all integers *or* all half-integers (halves of an odd integer; a mixture of integers and half-integers is not allowed). The set of all Hurwitz Quaternions is

It can be confirmed that H is closed under quaternion multiplication and addition, which makes it a subring of the ring of all Quaternions **H**.

A **Lipschitz quaternion** (or **Lipschitz integer**) is a quaternion whose components are all integers. The set of all Lipschitz Quaternions

forms a subring of the Hurwitz Quaternions H.

As a group, H is free abelian with generators $\{½(1+i+j+k), i, j, k\}$. It therefore forms a lattice in $\mathbf{R}^4$. This lattice is known as the F_4 lattice since it is the root lattice of the semisimple Lie algebra F_4. The Lipschitz Quaternions L form an index 2 sublattice of H.

The group of units in L is the order 8 quaternion group $Q = \{±1, ±i, ±j, ±k\}$. The group of units in H is a nonabelian group of order 24 known as thebinary tetrahedral group. T

he elements of this group include the 8 elements of Q along with the 16 Quaternions $\{½(±1±i±j±k)\}$ where signs may be taken in any combination. The quaternion group is a normal subgroup of the binary tetrahedral group $U(H)$. The elements of $U(H)$, which all have norm 1, form the vertices of the 24-cell inscribed in the 3-sphere.

The Hurwitz Quaternions form an order (in the sense of ring theory) in the division ring of Quaternions with rational components. It is in fact a maximal order; this accounts for its importance. The Lipschitz Quaternions, which are the more obvious candidate for the idea of an *integral quaternion*, also form an order.

However, this latter order is not a maximal one, and therefore (as it turns out) less suitable for developing a theory of left ideals comparable to that of algebraic number theory. What Adolf Hurwitz realised, therefore, was that this definition of Hurwitz integral quaternion is the better one to operate with. This was one major step in the theory of maximal orders, the other being the remark that they will not, for a non-commutative ring such as **H**, be unique. One therefore needs to fix a maximal order to work with, in carrying over the concept of an algebraic integer.

The norm of a Hurwitz quaternion, given by (see below), is always the square root of an integer. By a theorem of Lagrange every nonnegative integer can be written as a sum of at most four squares. Thus, every nonnegative integer is the squared norm of some Lipschitz (or Hurwitz) quaternion. A Hurwitz integer is a prime element if and only if its norm is a prime number.

Definition 4.1. (Hurwitz Quaternion). *Hurwitz Quaternion* is a Quaternion where the coefficients are either all integers or half-integers. The set of all Hurwitz Quaternion are denoted as

$$H = \{a+bi+cj+dk \in \mathbb{H} | a,b,c,d \in \mathbb{Z} \text{ or } a,b,c,d \in \mathbb{Z}+\frac{1}{2}\}$$

or

$$H = \{A\frac{1+i+j+k}{2}+Bi+Cj+Dk \in \mathbb{H} | A,B,C,D \in \mathbb{Z}\}$$

Theorem 1 [5] *Let $\pi \in H(\mathcal{Z})$. Then $H(\mathcal{Z})_\pi$ has $N(\pi)^2$ elements.*

Definition 4 [7] *The set of all Hurwitz integers is*

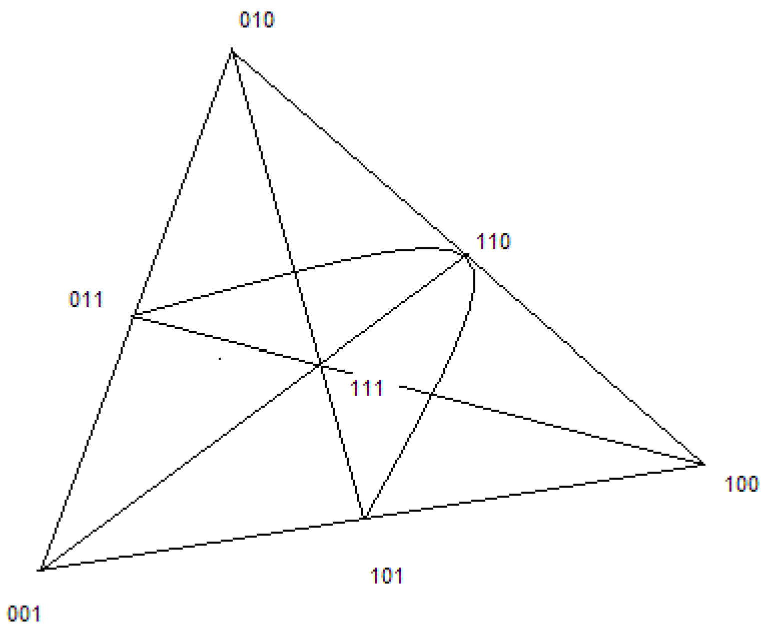
$$\begin{aligned}\mathcal{H} = \left\{a_0 + a_1\widehat{e}_1 + a_2\widehat{e}_2 + a_3\widehat{e}_3 \in H(R) : a_0, a_1, a_2, a_3 \in \mathcal{Z} \text{ or } a_0, a_1, a_2, a_3 \in \mathcal{Z} + \tfrac{1}{2}\right\} \\ = H(\mathcal{Z}) \cup H\left(\mathcal{Z} + \tfrac{1}{2}\right).\end{aligned}$$

Binary Trigrams and Octonions

0	1	10	11	100	101	110	111

0	1	2	3	4	5	6	7

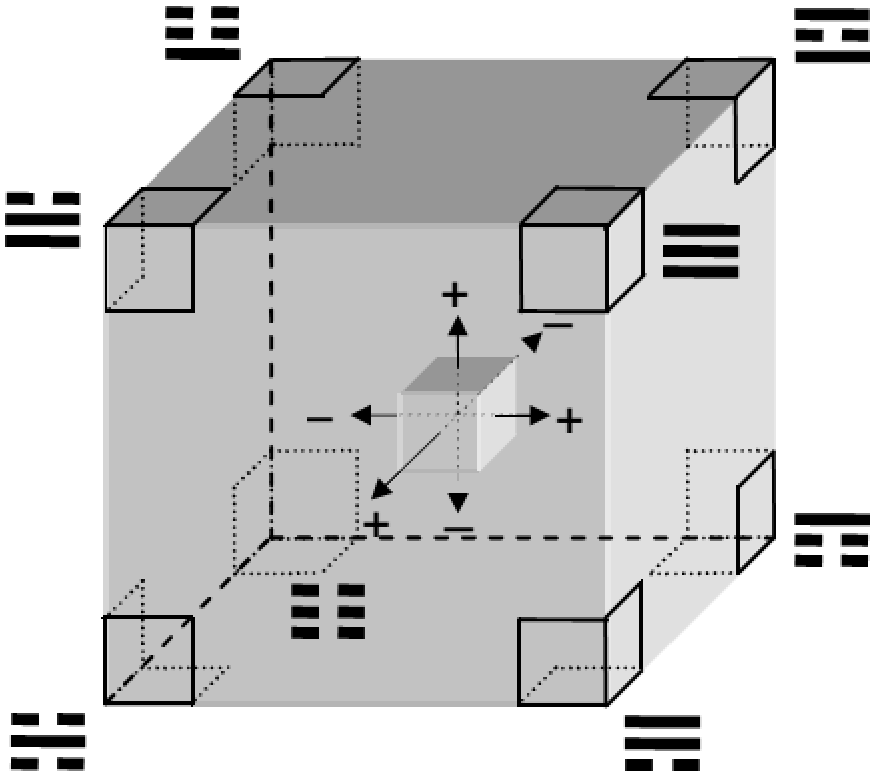
Each of the eight trigrams of Chinese metaphysics can be assigned a binary value as above. The same can be done for each of the Octonions.



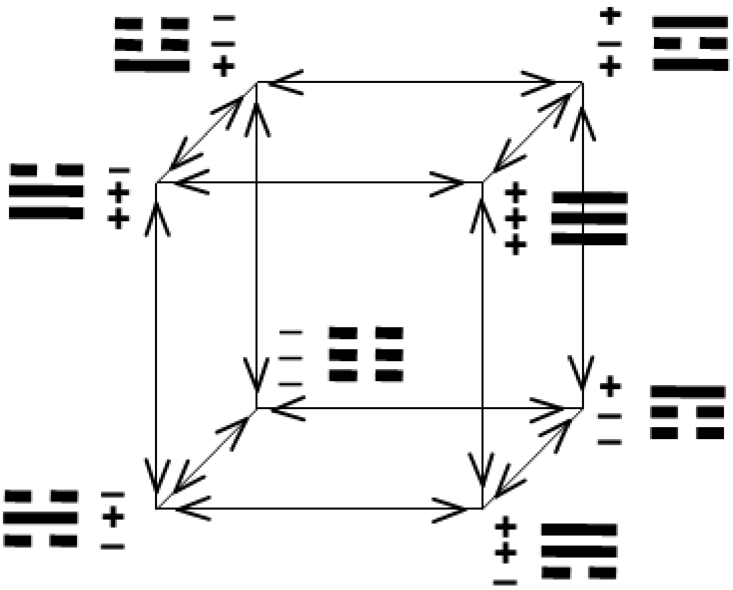
With both systems translated into binary form, we can now begin to explore the relationships between the 8 x 8 = 64 hexagrams and the 168 aspects of the Octonions.

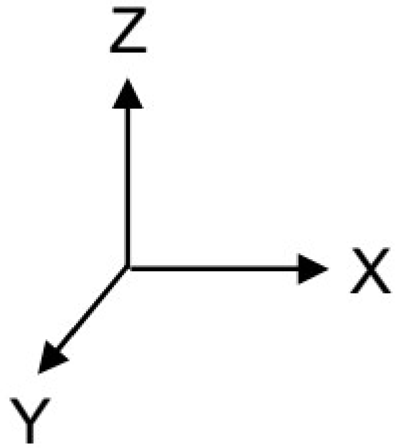
In the advanced Chinese divination method known as Qi Men Dun Jia, one of the eigth trigram positions (palaces) is always vacant, which means in real terms that there are only seven functioning trigrams at any time. The missing eighth trigram represents a vacancy, which means the unknown or not yet manifest Event - Body which will come into realization at some distant point in the future, from later the same day to years later.

3 x 3 x 3 Cube of Trigrams



With binary assignments made, it is possible to assign corner values per each trigram. (This section courtesy SM Philipps).





Clifford Algebras consist of tri - vectors.

The eight trigrams define the eight corners of a cube because their three yin/yang lines correspond to the three faces (positive or negative) that intersect at these corners.

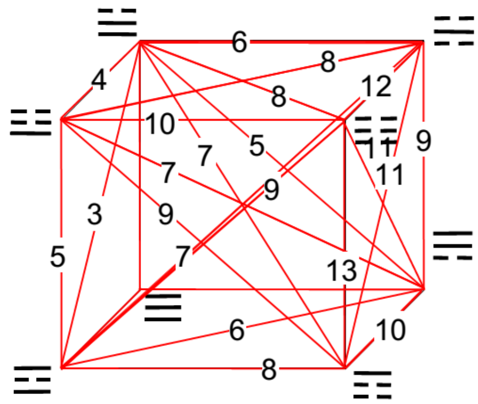
The left, back and bottom faces are (–) faces. Their intersection defines the Earth (K'un) trigram with three yin (negative) lines. Triple combinations of (+) and (–) faces define the remaining six corners of the cube.

The arrows along the edges of the cube indicate the positivity or negativity of a face. It is a (+) face if the arrow points along the positive X- , Y- or Z-axes and a (–) face if it points along the negative axes. The table below interprets the trigrams in terms of the faces of the cube:

Chien		**Sun**	
Right	—	Left	—
Front	—	Front	—
Top	—	Top	– –
Tui		**K'an**	
Left	– –	Left	– –
Front	—	Front	—
Bottom	—	Bottom	– –
Li		**Kên**	
Right	—	Left	—
Rear	– –	Rear	– –
Top	—	Top	– –
Chên		**K'un**	
Right	– –	Left	– –
Top	– –	Rear	– –
Rear	—	Bottom	– –

As in the case of their binary number representations, in which the integers 0 and 1 in each binary number signify the values of the Cartesian coordinates of the corners of a cube, the first four trigrams define corners of one face and the last four trigrams define corners of the face parallel to it (Fig. 7). The difference here is that the two sets of four trigrams correspond to the top and bottom faces instead of to the left and right faces.

0	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10
4	5	6	7	8	9	10	11
5	6	7	8	9	10	11	12
6	7	8	9	10	11	12	13
7	8	9	10	11	12	13	14



Note the even - numbers forming the diagonal.

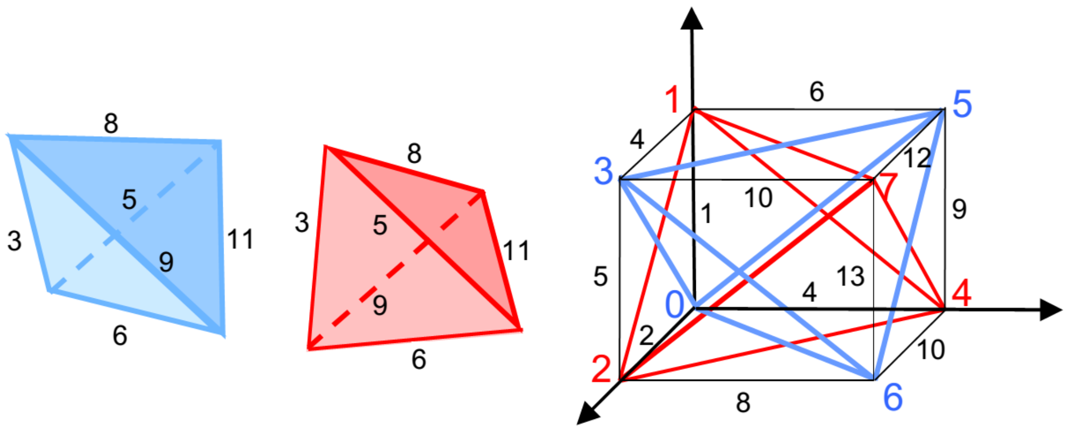


Figure 12. The sum of the integers associated with the 12 sides of the two tetrahedra is 84. This is also the sum of the integers associated with the 12 sides of the cube. The sum of the 24 integers is 168.

One might as well assign the 24 Hurwitz Algebras to these sides, which brings a relationship with the Octonions, for which 168 is a key number:

There are **168** permutations of "twisted **Octonions**" for each of the 30 sets of triads (**168***30=5040=7!).

The 7 points of the Fano plane represent the 7 imaginary octonions and the **168** symmetries are permutation symmetries that preserve an **octonion** multiplicatio

The musical counterpart of the **168** permutations of the seven unit imaginary **octonions**and their conjugates are the **168** repetitions of all rising and falling

Remarkable correspondence exists between the **168** permutations of the seven 3-tuples of **octonions**, the **168** automorphisms of the Klein Quartic, the **168**

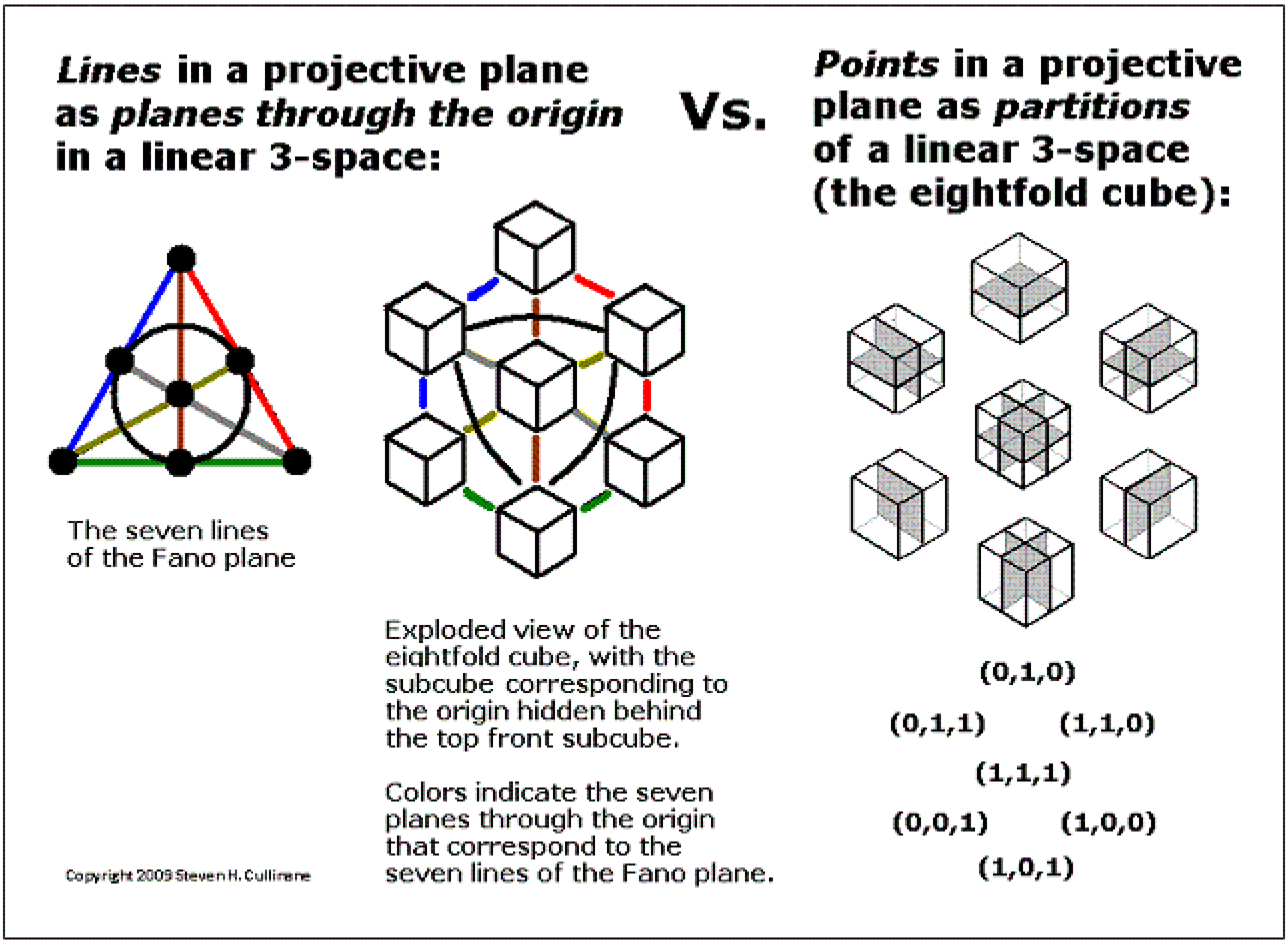
Combinations of one, two and three **octonions** from the two sets of seven 3-tuples with (84+84=**168**) possible orderings of the pairs and triplets in them.

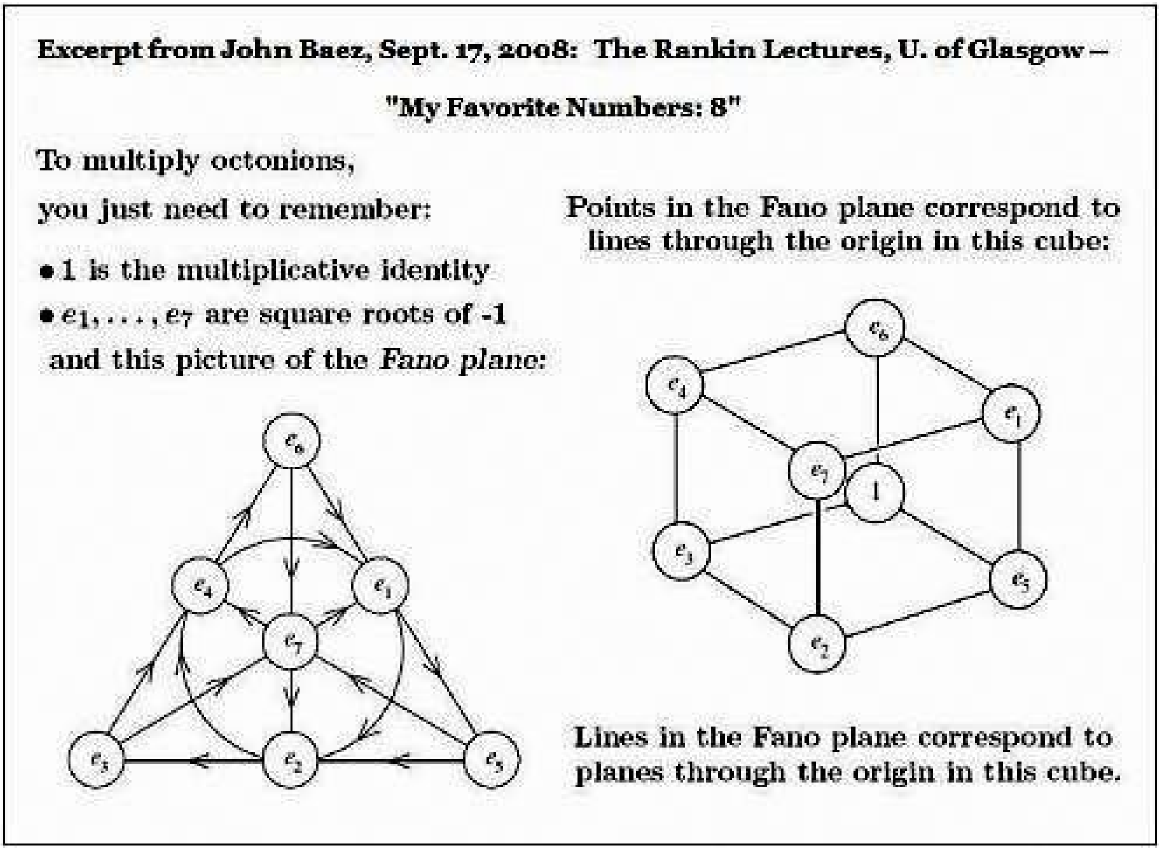
Double cover of the finite subgroup of SU(3) of order **168**,

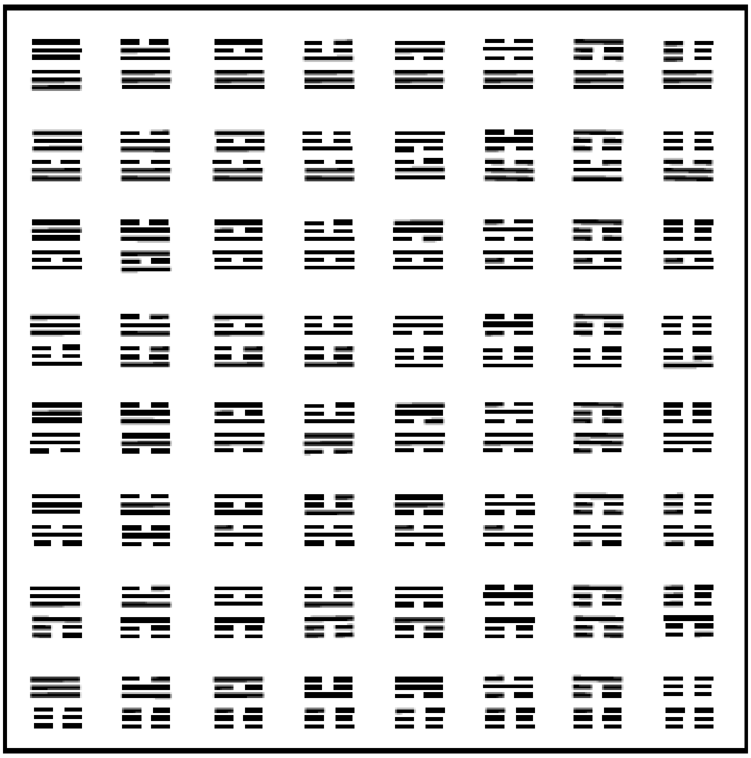
Octonions constituting the root system of E8 form a closed algebra where the root **...**subgroup is of order **168** with 8 conjugacy classes possessing the structure

The heptagonal hyperbolic plane is linked to the number **168**, and by extension to the Klein Quadric, PSL (2,7), the Fano Plane and **Octonions**.

The Fano plane is the simple group of group order**168** (Klein 1870)







Hexagrams along the diagonal of the table symbolise the three cube faces whose intersection defines a corner of a cube. The pairing of each trigram with a different one symbolises an association between the three faces defining a corner of the central cube

The three faces of one of the cubes surrounding it that is contiguous with this corner. Each corner of the cube at the centre of the 3×3×3 array of cubes is the point of intersection of the corners of seven other cubes.

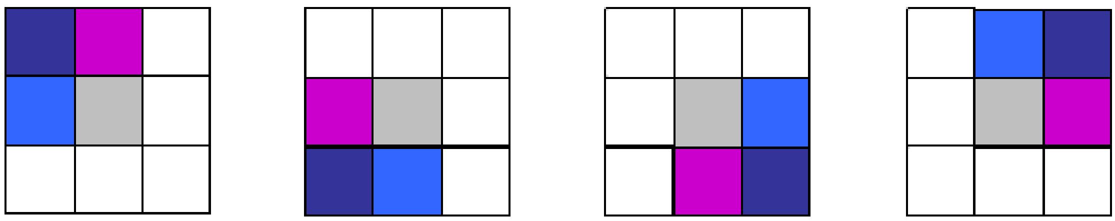
Each corner of the grey cube (a) at the centre of a 3×3×3 array of cubes touches three cubes (coloured blue, indigo & violet in (b)) at the same level as itself and four cubes (red, orange, yellow & green) that are either above or below it.

These (the violet, indigo and blue cubes) are at the same height as the cube and the remaining four (the green, yellow, orange and red ones) are above or below it.

Let us express the pattern of eight cubes centred on any corner of the central cube as

$$8 = (1 + 3) + 4, \quad (1)$$

where '1' always denotes the central cube, '3' denotes the three cubes contiguous with it at the same height and '4' denotes the four contiguous cubes above or below it. As we jump from corner to corner in the top face of the central cube, the L-shaped pattern



of three coloured cubes in the same plane rotates (Fig. 3), as do the four cubes above them. Likewise, as we go from corner to corner in the bottom face of the central cube, the three cubes in the same plane change, as do the four cubes below them. Movement Figure 3. Three cubes on the same level are contiguous with each corner of the central (grey) cube.

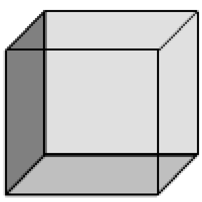


Figure 4. Each corner of a cube is the intersection of three faces of each of the seven cubes that surround it.

through all eight corners of the central cube involves every one of the 26 ^1 cubes in the 3×3×3 array because they are all contiguous with its corners.

Each corner of the central cube is the point of intersection of the three orthogonal faces of each of the seven cubes that are contiguous with it. The number of such faces generating the corners of the central cube is 8×3×7 = 168.

Counting these corners in the same way as the pattern of cubes, that is, differentiating any corner from the three corners at the same height and the four corners below them, then

$$\mathbf{168} = [(1+3) + 4]\times\underline{3}\times 7$$
$$= 24\times 7,$$

where

$$24 = [(1+3) + 4]\times\underline{3}.$$

The three orthogonal faces consist of the face perpendicular to one coordinate axis (for convenience, let us choose the X -axis) and the two faces perpendicular to the two other axes. So

$$\underline{3} = 1_X + 1_Y + 1_Z.$$

$$24 = [(1+3) + 4]\times(1_X + 1_Y + 1_Z)$$

Courtesy SM Philipps

$$= (1_X + 1_Y + 1_Z) + (3_X + 4_X) + (3_Y + 4_Y) + (3_Z + 4_Z),$$

where 3_X ($\equiv 3\times 1_X$)

signifies the three faces of cubes that are orthogonal to the X-axis and associated with the three corners at the same height as the corner labeled ‘1’ (and similarly for the Y - and Z-axes), and 4X (≡4×1X) denotes the four faces orthogonal to the X-axis associated with the four corners below corner ‘1’ (and similarly for 4Y and 4Z).

Therefore, substituting Equation 6 in Equation 3,

$$\mathbf{168} = [(1_X + 1_Y + 1_Z) + (3_X + 4_X) + (3_Y + 4_Y) + (3_Z + 4_Z)]\times 7,$$

The 168 faces of the 26 cubes surrounding the central cube that touch its eight corners consist of seven sets of 24 faces.

Each set consists of the three orthogonal faces of the cube corresponding to corner ‘1’, the seven faces perpendicular to the X-axis at the seven other corners, the seven faces perpendicular to the Y-axis at these corners and the seven faces perpendicular to the Z -axis at these corners.

Each group of seven faces comprises a face for each of the three corners of the central cube at the same height as corner ‘1’ and a face for each of the four corners below it. The 24 faces naturally divide into two groups of twelve:

Courtesy SM Philipps

$$24 = (1_X + 1_Y + 1_Z) + (3_X + 4_X) + (3_Y + 4_Y) + (3_Z + 4_Z),$$
$$= (4_X + 4_Y + 4_Z) + [(1_X + 1_Y + 1_Z) + 3_X + 3_Y + 3_Z].$$
$$= 12 + 12',$$

where

$$12 \equiv 4_X + 4_Y + 4_Z$$

and

$$12' \equiv (1_X + 1_Y + 1_Z) + 3_X + 3_Y + 3_Z.$$

Hence,

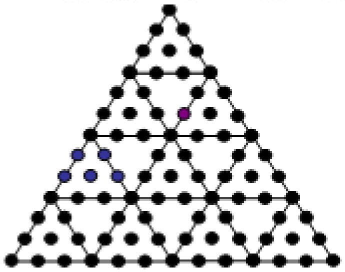
$$\mathbf{168} = 24\times7 = (12 + 12')\times7 = 84 + 84',$$

where

$$84 \equiv (4_X + 4_Y + 4_Z)\times7$$

and

$$84' \equiv [(1_X + 1_Y + 1_Z) + 3_X + 3_Y + 3_Z]\times7.$$

$85 = 4^0 + 4^1 + 4^2 + 4^3 =$ 

Courtesy SM Philipps

The figure above is the Sierspinski Triangle.

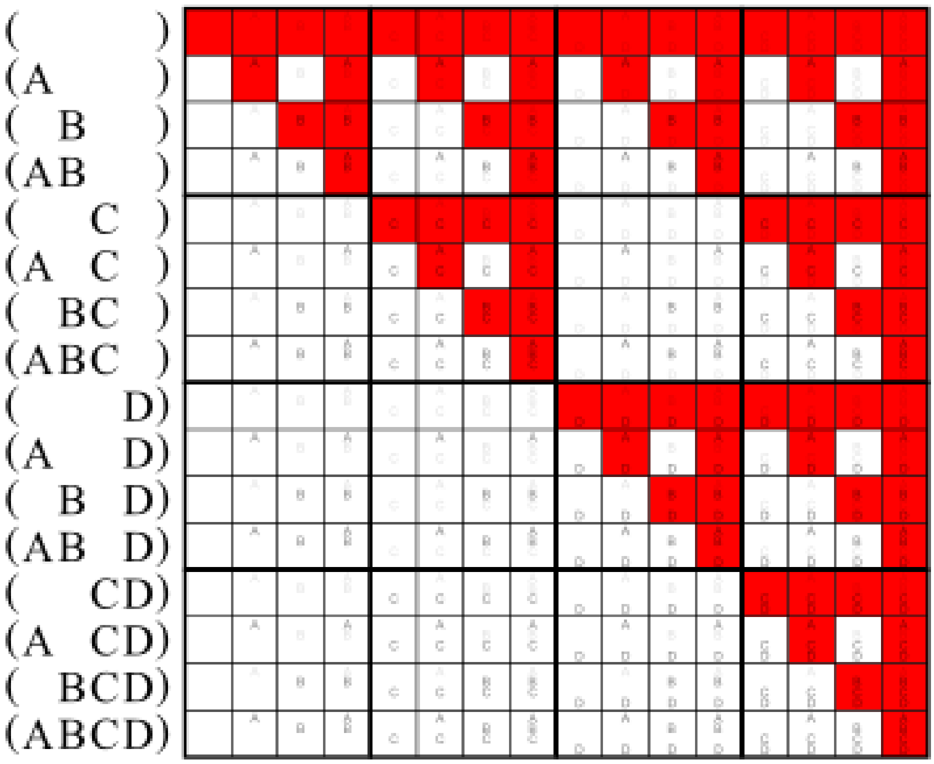
Wikipedia on the Sierspinski Triangle

The **Sierpinski triangle** (also with the original orthography *Sierpiński*), also called the **Sierpinski gasket** or the**Sierpinski Sieve**, is a fractal and attractive fixed set named after the Polish mathematician Wacław Sierpiński who described it in 1915.

However, similar patterns appear already in the 13th-century Cosmati mosaics in the cathedral ofAnagni, Italy,[1] and other places, such as in the nave of the Roman Basilica of Santa Maria in Cosmedin.[2]

Originally constructed as a curve, this is one of the basic examples of self-similar sets, i.e. it is a mathematically generated pattern that can be reproducible at any magnification or reduction.

Comparing the Sierpinski triangle or the Sierpinski carpet to equivalent repetitive tiling arrangements, it is evident that similar structures can be built into any rep-tile arrangements.



Sierpinski triangle in logic: The first 16 conjunctions of lexicographically ordered arguments The columns interpreted as binary numbers give 1, 3, 5, 15, 17, 51... (sequence A001317 in OEIS)

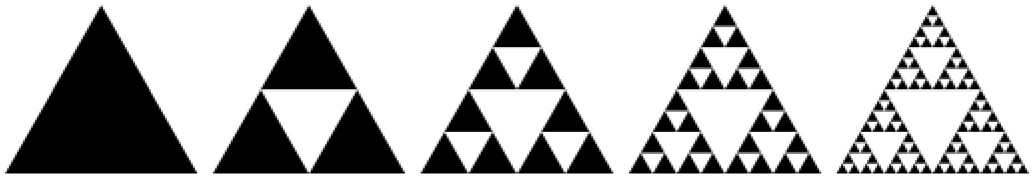
Magic Triangle

This shape formed by the Exceptional Lie Algebras fits neatly into the white spaces in the logic diagram above. A point of further research for the author is to explore the relationships between the Exceptional Lie Algebras, Sierspinski Triangle and Clifford Algebras (see below). The author of this diagram felt that the triangle should be further extended, but this may show the natural limit.

						0 0	3 A_1 2
					0 0	1 $U(1)$ 1	8 A_2 3
				0 0	0 1	3 A_1 3	14 G_2 7
			0 0	0 1	2 $2U(1)$ 2	9 $3A_1$ 4	28 D_4 8
		0 0	0 2	3 A_1 5	8 A_2 8	21 C_3 14	52 F_4 26
	0 0	0 1	2 $2U(1)$ 3	8 A_2 6	16 $2A_2$ 9	35 A_5 15	78 E_6 27
0 0	1 $U(1)$ 2	3 A_1 4	9 $3A_1$ 8	21 C_3 14	35 A_5 20	66 D_6 32	133 E_7 56
3 A_1 3	8 A_2 8	14 G_2 14	28 D_4 28	52 F_4 52	78 E_6 78	133 E_7 133	248 E_8 248

An algorithm for obtaining arbitrarily close approximations to the Sierpinski triangle is as follows:

Note: each removed triangle (a *trema*) is topologically an open set.[3]



1. Start with any triangle in a plane (any closed, bounded region in the plane will actually work). The canonical Sierpinski triangle

uses an equilateral triangle with a base parallel to the horizontal axis (first image).

2. Shrink the triangle to ½ height and ½ width, make three copies, and position the three shrunken triangles so that each triangle touches the two other triangles at a corner (image 2). Note the emergence of the central hole - because the three shrunken triangles can between them cover only 3/4 of the area of the original. (Holes are an important feature of Sierpinski's triangle.)
3. Repeat step 2 with each of the smaller triangles (image 3 and so on).

This process of recursively removing triangles is an example of a finite subdivision rule.

Note that this infinite process is not dependent upon the starting shape being a triangle—it is just clearer that way. The first few steps starting, for example, from a square also tend towards a Sierpinski triangle. Michael Barnsley used an image of a fish to illustrate this in his paper "V-variable fractals and superfractals."[4]

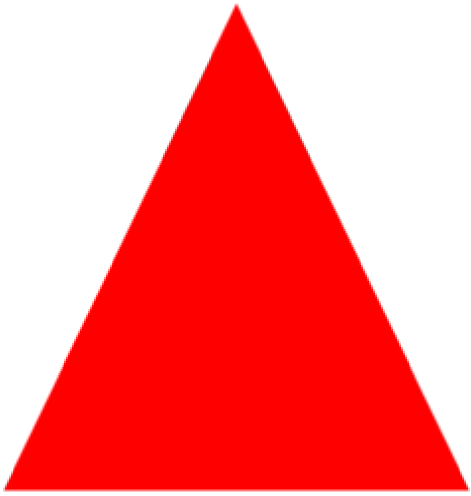
The actual fractal is what would be obtained after an infinite number of iterations. More formally, one describes it in terms of functions on closed sets of points. If we let d_a note the dilation by a factor of ½ about a point a, then the Sierpinski triangle with corners a, b, and c is the fixed set of the transformation $d_a \cup d_b \cup d_c$.

This is an attractive fixed set, so that when the operation is applied to any other set repeatedly, the images converge on the Sierpinski triangle. This is what is happening with the triangle above, but any other set would suffice.

If one takes a point and applies each of the transformations d_a, d_b, and d_c to it randomly, the resulting points will be dense in the Sierpinski triangle, so the following algorithm will again generate arbitrarily close approximations to it:

Start by labeling $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ as the corners of the Sierpinski triangle, and a random point $\mathbf{v}_1$. Set $\mathbf{v}_{n+1}$ = ½ ($\mathbf{v}_n$ + $\mathbf{p}_{r_n}$), where r_n is a random number 1, 2 or 3. Draw the points $\mathbf{v}_1$ to $\mathbf{v}_\infty$. If the first point $\mathbf{v}_1$ was a point on the Sierpiński triangle, then all the points $\mathbf{v}_n$ lie on the Sierpinski triangle.

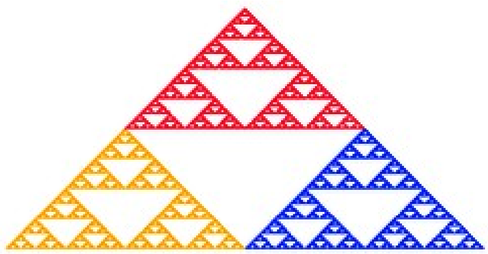
If the first point $\mathbf{v}_1$ to lie within the perimeter of the triangle is not a point on the Sierpinski triangle, none of the points $\mathbf{v}_n$ will lie on the Sierpinski triangle, however they will converge on the triangle. If $\mathbf{v}_1$ is outside the triangle, the only way $\mathbf{v}_n$ will land on the actual triangle, is if $\mathbf{v}_n$ is on what would be part of the triangle, if the triangle was infinitely large.



Or more simply:

1. Take 3 points in a plane to form a triangle, you need not draw it.
2. Randomly select any point inside the triangle and consider that your current position.
3. Randomly select any one of the 3 vertex points.
4. Move half the distance from your current position to the selected vertex.
5. Plot the current position.
6. Repeat from step 3.

Note: This method is also called the Chaos game. You can start from any point outside or inside the triangle, and it would eventually form the Sierpinski Gasket with a few leftover points. It is interesting to do this with pencil and paper. A brief outline is formed after placing approximately one hundred points, and detail begins to appear after a few hundred.



Sierpinski triangle using IFS

Or using an Iterated function system

An alternative way of computing the Sierpinski triangle uses an Iterated function system and starts by a point at the origin ($x_0 = 0$, $y_0 = 0$). The new points are iteratively computed by randomly applying (with equal probability) one of the following three coordinate transformations (using the so-called chaos game):

$x_{n+1} = 0.5\, x_n$

$y_{n+1} = 0.5\, y_n$; a half-size copy

This coordinate transformation is drawn in yellow in the figure.

$x_{n+1} = 0.5\, x_n + 0.25$

$y_{n+1} = 0.5\, y_n + 0.5\, \frac{\sqrt{3}}{2}$; a half-size copy shifted right and up
This coordinate transformation is drawn using red color in the figure.

$x_{n+1} = 0.5\, x_n + 0.5$
$y_{n+1} = 0.5\, y_n$; a half-size copy doubled shifted to the right
When this coordinate transformation is used, the triangle is drawn in blue.

Or using an L-system — The Sierpinski triangle drawn using an L-system.

bitwise AND - The 2D AND function, z=AND(x,y) can also produce a white on black right angled Sierpinski triangle if all pixels of which z=0 are white, and all other values of z are black.

bitwise XOR - The values of the discrete, 2D XOR function, z=XOR(x,y) also exhibit structures related to the Sierpinski triangle.

For example, one could generate the Sierpinski triangle by setting up a 2 dimensional matrix, [rows][columns] placing the uppermost point on [1][n/2], then cycling through the remaining cells row by row the value of the cell being XOR([i-1][j-1],[i-1][j+1])

Other means — The Sierpinski triangle also appears in certain cellular automata (such as Rule 90), including those relating to Conway's Game of Life. The automaton "12/1" when applied to a single cell will generate four approximations of the Sierpinski triangle.

If one takes Pascal's triangle with 2^n rows and colors the even numbers white, and the odd numbers black, the result is an approximation to the Sierpinski triangle. More precisely, the limit as n approaches infinity of this parity-colored 2^n-row Pascal triangle is the Sierpinski triangle.

Properties[edit source]

For integer number of dimensions d, when doubling a side of an object, 2^d copies of it are created, i.e. 2 copies for 1 dimensional object, 4 copies for 2 dimensional object and 8 copies for 3 dimensional object.

For Sierpinski triangle doubling its side creates 3 copies of itself. Thus Sierpinski triangle has Hausdorff dimension log(3)/log(2) ≈ 1.585, which follows from solving $2^d = 3$ for d. [5]

The area of a Sierpinski triangle is zero (in Lebesgue measure). The area remaining after each iteration is clearly 3/4 of the area from the previous iteration, and an infinite number of iterations results in zero.

Intuitively one can see this applies to any geometrical construction with an infinite number of iterations, each of which decreases the size by an amount proportional to a previous iteration.[citation needed]

Higher - Dimension Analogues

The tetrix is the three-dimensional analogue of the Sierpinski triangle, formed by repeatedly shrinking a regular tetrahedron to one half its original height, putting together four copies of this tetrahedron with corners touching, and then repeating the process.

This can also be done with a square pyramid and five copies instead. A tetrix constructed from an initial tetrahedron of side-length L has the property that the total surface area remains constant with each iteration.

The initial surface area of the (iteration-0) tetrahedron of side-length L is $L^2\sqrt{3}$. At the next iteration, the side-length is halved

$$L \rightarrow \frac{L}{2}$$

and there are 4 such smaller tetrahedra. Therefore, the total surface area after the first iteration is:

$$4\left(\left(\frac{L}{2}\right)^2\sqrt{3}\right) = 4\frac{L^2}{4}\sqrt{3} = L^2\sqrt{3}.$$

This remains the case after each iteration. Though the surface area of each subsequent tetrahedron is 1/4 that of the tetrahedron in the previous iteration, there are 4 times as many—thus maintaining a constant total surface area.

The total enclosed volume, however, is geometrically decreasing (factor of 0.5) with each iteration and asymptotically approaches 0 as the number of iterations increases. In fact, it can be shown that, while having fixed area, it has no 3-dimensional character.

The Hausdorff dimension of such a construction is $\frac{\ln 4}{\ln 2} = 2$ which agrees with the finite area of the figure. (A Hausdorff dimension strictly between 2 and 3 would indicate 0 volume and infinite area.)

Clifford Algebra Classification

All told there are three properties which determine the class of the algebra $C\ell_{p,q}(\mathbf{R})$:

- signature mod 2: n is even/odd: central simple or not
- signature mod 4: $\omega^2 = \pm 1$: if not central simple, center is $\mathbf{R} \oplus \mathbf{R}$ or $\mathbf{C}$
- signature mod 8: the Brauer class of the algebra (n even) or even subalgebra (n odd) is $\mathbf{R}$ or $\mathbf{H}$

Each of these properties depends only on the signature $p - q$ modulo 8. The complete classification table is given below. The size of the matrices is determined by the requirement that $C\ell_{p,q}(\mathbf{R})$ have dimension 2^{p+q}.

$p-q$ mod 8	ω^2	$C\ell_{p,q}(\mathbf{R})$ $(n = p+q)$	$p-q$ mod 8	ω^2	$C\ell_{p,q}(\mathbf{R})$ $(n = p+q)$
0	+	$\mathbf{R}(2^{n/2})$	1	+	$\mathbf{R}(2^{(n-1)/2}) \oplus \mathbf{R}(2^{(n-1)/2})$
2	−	$\mathbf{R}(2^{n/2})$	3	−	$\mathbf{C}(2^{(n-1)/2})$
4	+	$\mathbf{H}(2^{(n-2)/2})$	5	+	$\mathbf{H}(2^{(n-3)/2}) \oplus \mathbf{H}(2^{(n-3)/2})$
6	−	$\mathbf{H}(2^{(n-2)/2})$	7	−	$\mathbf{C}(2^{(n-1)/2})$

It may be seen that of all matrix ring types mentioned, there is only one type shared between both complex and real algebras: the type $\mathbf{C}(2^m)$.

For example, $C\ell_2(\mathbf{C})$ and $C\ell_{3,0}(\mathbf{R})$ are both determined to be $\mathbf{C}(2)$. It is important to note that there is a difference in the classifying isomorphisms used.

Since the $C\ell_2(\mathbf{C})$ is algebra isomorphic via a $\mathbf{C}$-linear map (which is necessarily $\mathbf{R}$-linear), and $C\ell_{3,0}(\mathbf{R})$ is algebra isomorphic via an $\mathbf{R}$-linear map, $C\ell_2(\mathbf{C})$ and $C\ell_{3,0}(\mathbf{R})$ are $\mathbf{R}$-algebra isomorphic.

A table of this classification for $p + q \leq 8$ follows. Here $p + q$ runs vertically and $p - q$ runs horizontally (e.g. the algebra $C\ell_{1,3}(\mathbf{R}) \cong \mathbf{H}(2)$ is found in row 4, column −2).

	8	7	6	5	4	3	2	1	0	−1	−2	−3	−4	−5	−6	−7	−8
0									$\mathbf{R}$								
1								$\mathbf{R}^2$		$\mathbf{C}$							
2							$\mathbf{R}(2)$		$\mathbf{R}(2)$		$\mathbf{H}$						
3						$\mathbf{C}(2)$		$\mathbf{R}^2(2)$		$\mathbf{C}(2)$		$\mathbf{H}^2$					
4					$\mathbf{H}(2)$		$\mathbf{R}(4)$		$\mathbf{R}(4)$		$\mathbf{H}(2)$		$\mathbf{H}(2)$				
5				$\mathbf{H}^2(2)$		$\mathbf{C}(4)$		$\mathbf{R}^2(4)$		$\mathbf{C}(4)$		$\mathbf{H}^2(2)$		$\mathbf{C}(4)$			
6			$\mathbf{H}(4)$		$\mathbf{H}(4)$		$\mathbf{R}(8)$		$\mathbf{R}(8)$		$\mathbf{H}(4)$		$\mathbf{H}(4)$		$\mathbf{R}(8)$		
7		$\mathbf{C}(8)$		$\mathbf{H}^2(4)$		$\mathbf{C}(8)$		$\mathbf{R}^2(8)$		$\mathbf{C}(8)$		$\mathbf{H}^2(4)$		$\mathbf{C}(8)$		$\mathbf{R}^2(8)$	
8	$\mathbf{R}(16)$		$\mathbf{H}(8)$		$\mathbf{H}(8)$		$\mathbf{R}(16)$		$\mathbf{R}(16)$		$\mathbf{H}(8)$		$\mathbf{H}(8)$		$\mathbf{R}(16)$		$\mathbf{R}(16)$
ω^2	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+	+

Symmetries

There is a tangled web of symmetries and relationships in the above table. Write $A[n] := A \otimes M_n(\mathbf{R})$ for $n \times n$ matrices with coefficients in A, and$C\ell(p+1,q+1)$ for the real Clifford algebra.

$$C\ell(p+1, q+1) = C\ell(p, q)[2]$$
$$C\ell(p+4, q) = C\ell(p, q+4)$$

(Going over 4 spots in any row yields an identical algebra.)

From these Bott Periodicity follows:

$$C\ell(p+8, q) = C\ell(p+4, q+4) = C\ell(p, q)[2^4].$$

If the signature satisfies $p - q \equiv 1 \pmod 4$ then

$$C\ell(p+k, q) = C\ell(p, q+k).$$

(The table is symmetric about columns with signature 1, 5, −3, −7, and so forth.) Thus if the signature satisfies $p - q \equiv 1$ (mod 4),

$$C\ell(p+k, q) = C\ell(p, q+k) = C\ell(p-k+k, q+k) = C\ell(p-k, q)[2^k] = C\ell(p, q-k)[2^k].$$

Klein Quartic / Wikipedia

In hyperbolic geometry, the **Klein quartic**, named after Felix Klein, is a compact Riemann surface of genus 3 with the highest possible order automorphism group for this genus, namely order 168 orientation-preserving automorphisms, and 336 automorphisms if orientation may be reversed.

As such, the Klein quartic is the Hurwitz surface of lowest possible genus; see Hurwitz's automorphisms theorem. Its (orientation-preserving) automorphism group is isomorphic to PSL(2,7), the second-smallest non-abelian simple group. The quartic was first described in (Klein 1878b).

Klein's quartic occurs in many branches of mathematics, in contexts including representation theory, homology theory,octonion multiplication, Fermat's last theorem, and the Stark-Heegner theorem on imaginary quadratic number fields ofclass number one; see (Levy 1999) for a survey of properties.

Originally, the "Klein quartic" referred specifically to the subset of the complex projective plane $\mathbf{CP}^2$ defined by the equation given in the **As an algebraic curve** section. This has a specific Riemannian metric (that makes it a minimal surface in $\mathbf{CP}^2$), under which its Gaussian curvature is not constant.

But more commonly (as in this article) it is now thought of as any Riemann surface that is conformally equivalent to this algebraic curve, and especially the one that is a quotient of the hyperbolic plane $\mathbf{H}^2$ by a certain cocompact group **G** that acts freely on $\mathbf{H}^2$ by isometries.

This gives the Klein quartic a Riemannian metric of constant negative curvature = −1 that it inherits from $\mathbf{H}^2$. This set of conformally equivalent Riemannian surfaces is precisely the same as all compact Riemannian surfaces of genus 3 whose conformal automorphism group is isomorphic to the unique simple group of order 168.

This group is known as $\mathbf{PSL}(2,\mathbf{Z}/7\mathbf{Z})$, and also as the isomorphic group $\mathbf{PSL}(3, \mathbf{Z}/2\mathbf{Z})$. By covering space theory, the group **G** mentioned above is isomorphic to the fundamental group of the compact surface of genus 3.

The Klein quartic is related to various other surfaces.

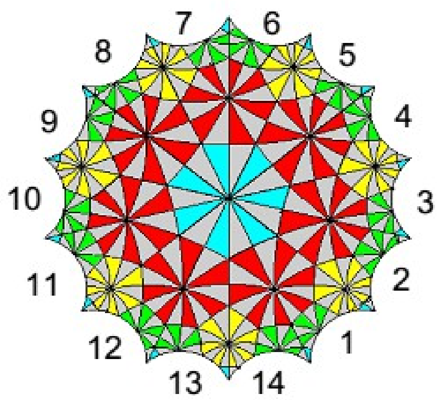
Geometrically, it is the smallest Hurwitz surface (lowest genus); the next is the Macbeath surface (genus 7), and the following is the First Hurwitz triplet (3 surfaces of genus 14). More generally, it is the most symmetric surface of a given genus (being a Hurwitz surface); in this class, the Bolza surface is the most symmetric genus 2 surface, while Bring's surface is a highly symmetric genus 4 surface – see isometries of Riemann surfaces for further discussion.

Algebraically, the (affine) Klein quartic is the modular curve X(7) and the projective Klein quartic is its compactification, just as the dodecahedron (with a cusp in the center of each face) is the modular curve X(5); this explains the relevance for number theory.

More subtly, the (projective) Klein quartic is a Shimura curve (as are the Hurwitz surface of genus 7 and 14), and as such parametrizes principally polarized abelian varieties of dimension 6.[11]

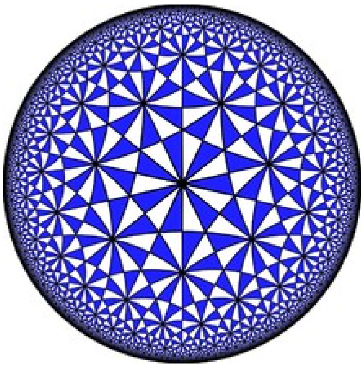
There are also other quartic surfaces of interest – see special quartic surfaces.

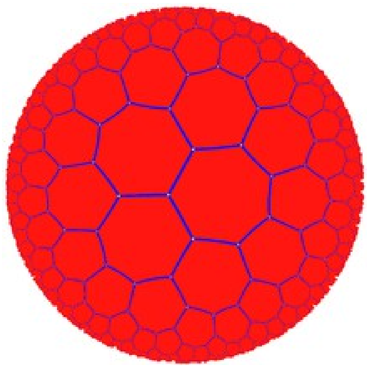
More exceptionally, the Klein quartic forms part of a "trinity" in the sense of Vladimir Arnold, which can also be described as a McKay correspondence. In this collection, the projective special linear groups PSL(2,5), PSL(2,7), and PSL(2,11) (orders 60, 168, 660) are analogous, corresponding to icosahedral symmetry (genus 0), the symmetries of the Klein quartic (genus 3), and the buckyball surface (genus 70).[12] These are further connected to many other exceptional phenomena, which is elaborated at "trinities".

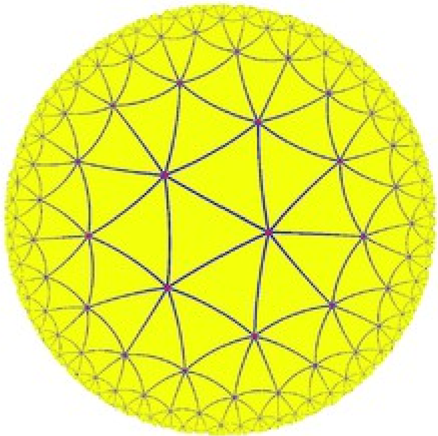


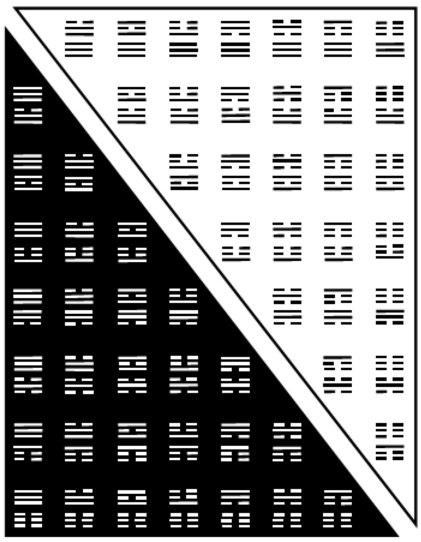
14 slices
12 coloured triangles per slice
14×12 = **168** coloured triangles in 24 heptagons

The Klein Configuration of the 168 elements of the group PSL(2,7) of transformations of the Klein Quartic.









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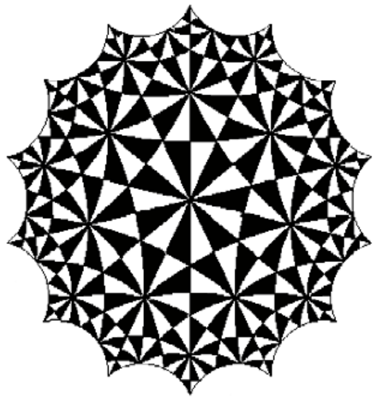


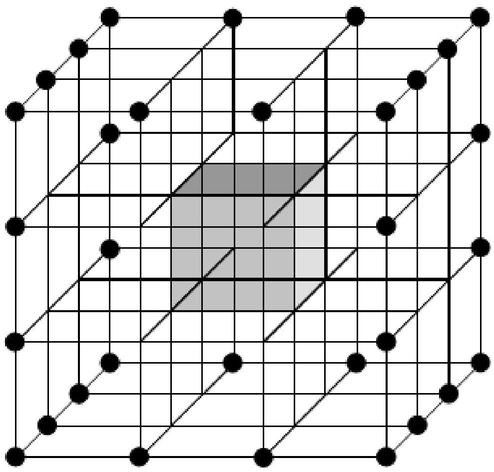


Figure 13. **168** faces of the **26** cubes in the 3×3×3 array surrounding the central (grey) cube define the 32 corners (●) along the 12 sides of the array.

Octonion Power Series

In other words, we obtain the exponential function. Therefore, we can write (11) as

$$e^{o} = e^{u_1}\left\{ \cos|\vec{u}| + \vec{u}\left(\frac{\operatorname{sen}|\vec{u}|}{|\vec{u}|}\right)\right\}. \tag{12}$$

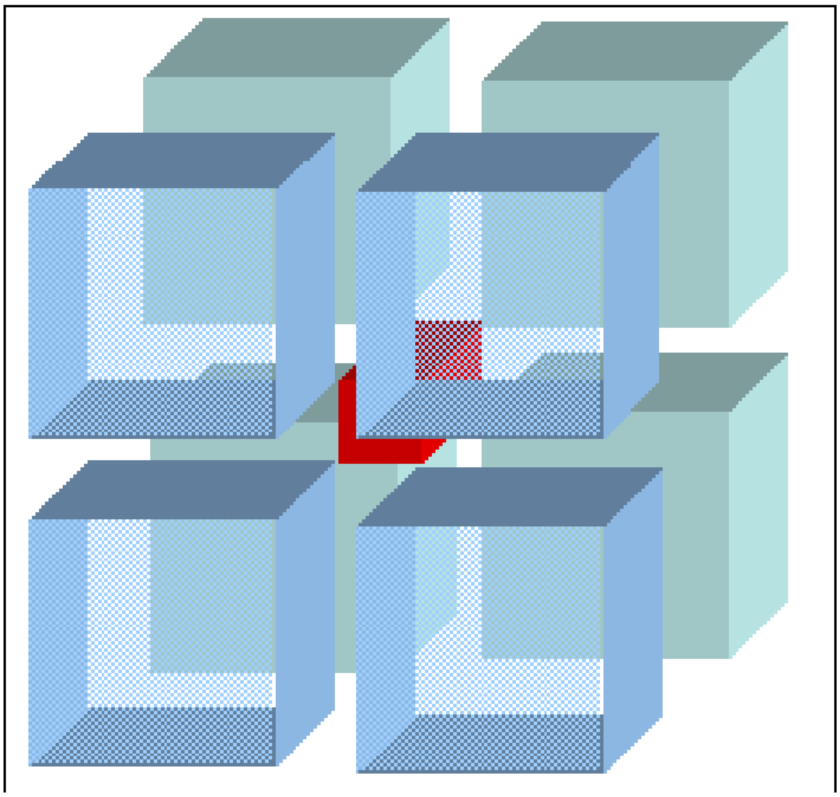
Considering the absolute value $|\vec{u}| = \sqrt{u_2^2+u_3^2+u_4^2+u_5^2+u_6^2+u_7^2+u_8^2}$, we write (12) as

$$\begin{aligned} e^{o} = e^{u_1}\Bigg\{ & \cos\left(\sqrt{u_2^2+u_3^2+u_4^2+u_5^2+u_6^2+u_7^2+u_8^2}\right) \\ & + \vec{u}\left(\frac{\operatorname{sen}(\sqrt{u_2^2+u_3^2+u_4^2+u_5^2+u_6^2+u_7^2+u_8^2})}{\sqrt{u_2^2+u_3^2+u_4^2+u_5^2+u_6^2+u_7^2+u_8^2}}\right)\Bigg\}. \end{aligned} \tag{13}$$

We call (13) an octonionic exponential function. We obtain thus the generalized De Moivre's relation for octonions.

The Octonion Power Series is essential in combinatorial Vedic Physics, and the exponents can be converted into logarithms.

MoolaKaprithi in Vedic Physics



The red cube is the Moolakaprithi .

It is shown as the red cube in the diagram and displays the cubic form maintained by the 8 vibrating larger blue cubes. That red cube can be described in terms of interactive counts of vibrations in a cyclic-period, assuming that all the eight blue cubes interact in the same location. The size of the red cube has a specific numerical relationship to the larger cube, which can be numerically related in powers of 2 or

$$2^N \text{ or } (2^3)^N = 8^N$$

The larger blue cube can be considered as just the combination of the smaller - sized red cubes. The collection of red cubes act as a blue cube by vibrating together as a coherent, synchronous, group displaying Thaamasic simultaneous activity.

If the coherent and synchronised state of interacting (vibrating together simultaneously) is disrupted, then that position produces the effect of the red cube. It then becomes a cube vibrating out of step or synchrony. It becomes a cube that does not remain in the same state or location as the rest and which displays a different condition.
This is the fundamental condition of commencement of activity called Moolaprakrithi. Moolaprakrarithi - the red cube is not a cube - but a state of a

cube in an active state or a state different from the rest.

It is a vibrating form that exists only because other vibrating cubes exist, and is a holographic form. The meeting point of three axes shares a single point, which in reality is cubic point. It is the fundamental concept of a unit of charge in Physics called a Purusha. An important change of condition takes place at this interactive interface. The impact between two cubes can result in 3 states of interactive reactions as defined in the Guna Theorems.

There follows instantaneous separation on impact, as an inelastic reaction of the Sathwa state. Or it may elastically vibrate and remain in a resonant Raja state.

Finally, the states may combine together to attain a uniform, singular, synchronous and coherent state of activity, as one larger cube in the Thaama state as shown in the diagram. The Sathwa state shows the radiation of a set of Moolaprakrithis as Vrithis (coherent particle states)) on sudden or inelastic collision.

The Raja state shows the resonant harmony of two sets of Moolaprakrithis interacting simultaneously at the same rate as a bound state. The Thaama state shows the absorption of two sets of Moolaprakrithis in a higher state of activity as superposing, compressing, or denser states.

If the red cube Moolaprakrithi is considered the elemental unitary state, then larger cubes can be created as multiples of the elemental unitary state, as vibrant but coherent and unitary states.

The Moolaprakrithi is a cube of space in a vibratory state and the non - vibratory state of this same unit of space cannot remain in that size because of the axiomatic nature of Guna interactions. Hence the elemental components in space combine, agglomerate, or join together as a larger, self-limiting unit of space, which can remain static, coherent, passive or non - manifest etc.

There is a single Guna law that acts in identical ways at every agglomerate level of phenomena. At each level there exist the same proportionate limits of maximum and minimum interactive counts, but the form and size may vary to attain balance at each level.

Saying it another way, perpetual self - similar oscillatory activity comes to a stop naturally only at the Purusha level. For this reason it is called the Andha - thaamshra or dark and dense state of Super - posed vibrations in space. The Guna principles explain why and how this has to be. When the oscillatory state becomes undetectable by super - positioning of counts, then communication with that state is cut off and the state enters into an isolated, black hole state.

The ability to discriminate the interval between interactive counts disappears, and it superposes on the previous count. This is the black hole state in

Physics. Therefore every unit of quiescent, apparently static, barely resonant and non - manifest unit of space is a Purusha - a massive black hole, a potential state of dormant, internal, stress and trans - migrational activity of elemental components in space. Conceptually, the black hole state behaves exactly like deep - sea components.

The natural drift of active states towards lower or reduced activity levels is purely due to the action and reaction counts not being cyclically equal. As an example: if a 20 interactions per cycle (ipc) unit inter - acta with a 10 ipc unit, the 20 ipc will move in towards the 10 ipc unit, because for every 2 counts, there is only one reacting count, to attain a balance of counts.

This is the fundamental cause of transmigration of counts between any two different count rate states. It is the only reason that all identified forces in Physics, like gravity, electromagnetic, weak and strong accelerate from a higher interactive count rate to a lower one.

At the basic elemental level, this drift of Moolaprakrithy counts towards the Purusha coherent states is observed as a gravitating phenomenon. At intermediate levels, this type of migration of counts display the Linga/Bhaava and Abiman/Ahankar changes in the Thaama-Raja-Sathwa Guna characteristics, which represent the strong, weak and electro - magnetic interactive spectrum.

The 1:3:4 pattern of cubes associated with the central cube corresponds in each sector of the Klein Configuration to the cyan hyperbolic triangle of the central heptagon, the

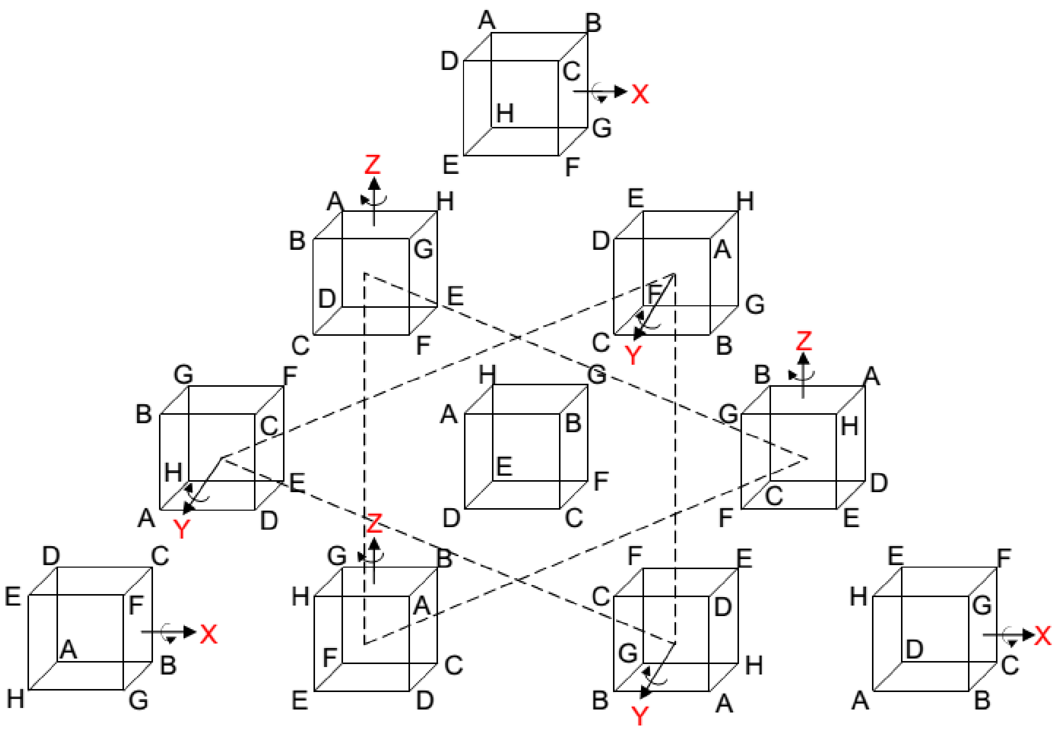


Figure 11. The three rotations of the central cube about the X-, Y- and Z-axes (labelled red) generate a tetractys array of ten orientations with the unrotated cube at its centre.

With each count, there may occur one rotation, giving a series of ten counts and ten rotations as shown above in the diagram by SM Philipps.

Conclusion

Philipps showed the relationship between the 3 x 3 x 3 Cube of the I Ching Trigrams, the Klein Quartic and the Sierspinski Triangle, which relates to Pascal's Triangle (Zhang Hui Triangle or Mount Meru in Vedic Physics) and combinatorial math.

Many additional connections prove possible here, especially with the Magic Triangle of Exceptional Lie Algebras, and the Pascal Triangle - like organization of Clifford Algebras. As John Baez points out, Clifford Algebras are based on the (n +1) equation.

Pascal's Triangle and the Octonions via the Fano Plane enjoy direct connections to the Fibonacci Numbers and to the Golden Ratio, and this is not accidental. The author hypothesizes this is presence as the border between two states of matter, either Thaamas, Raja or Satvic. The Golden Ratio has the function of modulating between states which vibrate at different ratios.

The binary qualities of the I Ching Trigrams immediately form an association with the binary qualities of the Octonions, the multiplication table for which is the Fano Plane, an equilateral triangle such as those that form the Sierspinski Triangle. Thus we begin to see a rich complex of inter - relationships between these different mathematical concepts, which focus around the 3 x 3 x 3 cube.

The author surmise that these relationships are not accidental by any means, and that the 3 x 3 x 3 cube, a product heretofore of a westerner's manipulation of an ancient Chinese concept, is in fact the Moolaprakrithy, the ancient Vedic concept and the very heart of Vedic Nuclear Physics. These relationships and identities indicate the importance of linking contemporary physics with sacred and religious constructs for the purpose of giving full articulation to Vedic Physics in terms of contemporary mathematical physics.

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Isomorphism Between the I Ching Table, the 3×3×3 Array of Cubes & the Klein Configuration

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DE MOIVRE EXTENDED EQUATION FOR OCTONIONS AND POWER SERIES

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Quaternion

Hoon Kwon

March 13, 2010

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Some men see things as they are and say *why?*
I dream things that never were and say *why not?*

Let's dedicate ourselves to what the Greeks wrote so many years ago: **to tame the savageness of man and make gentle the life of this world.**

Robert Francis Kennedy