Higgs Field Theory of the Observed Fermions, and their Masses, Magnetic Moments and Renormalization

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Abstract: We demonstrate how the application of Higgs field theory to spin ½ fermions in a manner analogous to its application to spin 0 scalars enables fermion masses to be constructed entirely from a self-energy arising from gauge fields and "revealed" in the Dirac Lagrangian in a fully renormalizable manner with no "bare" aspect. When the observed fermions are taken to be Higgs fields based on expansion about the vacuum, it is found that the Dirac Lagrangian naturally produces an "anomalous" aspect for their magnetic moments. This enables us to deduce the gauge fields which underlie the self-energies for the three charged leptons on an entirely empirical basis, and then use this data to predict the impact of a time-dependent magnetic field on the lepton g-factors. We predict that a time-dependent magnetic field impacts the g-factor of the heaver mu and especially tau leptons much more substantially than it does the g-factor of the electron, and quantify how this should be detectable well within experimental ranges. We also show how this construction of fermion masses out of gauge fields permits these masses to remain invariant at all renormalization scales, wherein the variability of a fermion mass under renormalization is entirely equivalent to, and may be fully absorbed by, a gauge transformation of the vector potentials from which the fermion self-energies arise. Finally, the time and space dependencies of the electric and magnetic fields in Maxwell's equations are revealed to be embedded into Dirac's equation as a result of Heisenberg commutations. This develops multiple venues for further confirming Higgs Field Theory in the fermion sector, all of which appear to be new.

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Contents

1. Introduction, Novelty and Overview

The July 4, 2012 announcement from the Large Hadron Collider of experimental results consistent with the "long sought Higgs particle" [1] garnered an unusual degree of attention not only from the international physics community, but from the mainstream press and the general public as well. Yet, the full import of a validation of "Higgs mechanism" is, if anything, under-reported and underappreciated, even by knowledgeable physicists. This is signaled by the fact that the original work by Anderson [2], Englert and Brout [3], Higgs [4], Guralnik, Hagen and Kibble [5] is still often referred to as a "mechanism" rather than as a fundamental breakthrough in understanding the nature of the particles and fields that we actually observe in our laboratories and in our daily experience.

Of course, it is critically important that the Higgs "mechanism" provides a way to introduce non-zero masses for the vector bosons of a gauge theory without having to do so by hand, thus preserving renormalizability. And it was a major step forward when those features of the Higgs mechanism enabled Weinberg [6], Salam [7] and Glashow to develop an electroweak theory which correctly predicted the observed masses of the W and Z bosons which mediate weak interactions and to place them into a quadruplet set with the massless photon following spontaneous symmetry breaking all while keeping a renormalizable theory. But more than anything, all of this theoretical work which bears the name of "Higgs mechanism" involves expanding elementary fields about a non-zero vacuum, and teaches that these vacuum-expanded fields, and *not the original fields* in our Lagrangian or Hamiltonian, are the particles and fields we actually observe. Let us be specific and concrete:

Postulate a scalar field $\phi(x)$. Write down a Klein Gordon Lagrangian density for that scalar field given by $\mathfrak{L}=(\partial_\sigma\phi)(\partial^\sigma\phi)-m^2\phi^2$. Or, postulate a fermion field $\psi(x)$ and write down its $\mathfrak{L}=\overline{\psi}(i\partial-m)\psi$. *We do not observe, cannot observe, and never will observe these postulated fields* ϕ *and* ψ. But a) when we require that these Lagrangians remain invariant under the local gauge transformation $\phi\to\phi'=e^{i\theta(x)}\phi$ (or non-Abelian Yang-Mills extensions with $\Theta=\lambda^i\theta^i$ where λ^i, $i=1,2,3...N^2-1$ are the generator matrices of a gauge group SU(N)) and thus naturally introduce gauge fields A^μ (or their non-Abelian extensions $G^\mu=\lambda^i G^{i\mu}$), and b) after we introduce a vacuum vev v, find the vacuum minimum, and expand about this minimum via $\phi(x)=v+h(x)$ for scalar fields where $h(x)$ is a scalar Higgs field, and as we shall show here, $\psi(x)=v_\psi+h_\psi(x)$ for fermions where v_ψ is a suitable vacuum for fermion expansion and where $h_\psi(x)$ is a fermion Higgs field, then c) the Lagrangian with which we started *does and will* describe the physics of the particles and fields we observe in nature. *But the observed fields are not the* $\phi(x)$ *and* $\psi(x)$ *with which we started, but rather, the* $h(x)$ *and* $h_\psi(x)$ *with which we ended after the expansion about the vacuum.*

So the lesson of Higgs et al., is that (at least) the scalar particles we observe in nature are not $\phi(x)=v+h(x)$, but rather are $h(x)=\phi(x)-v$. They are the fields over and above a non-trivial, non-zero vacuum. Metaphorically, they are the visible portion of a boat above the waterline, with the portion underwater remaining invisible. Along the way of course, and very significantly, we uncover masses for gauge bosons and also uncover a mass (albeit with an unknown coupling λ) for the Higgs field, all in a renormalizable manner. Insofar as all of this is achieved, the Higgs approach provides a very important "mechanism." But the upshot of Higgs theory is that a "seed" field ϕ which is ***not*** observed is expanded about the vacuum to yield a Higgs field h which *is* observed. Therefore, this work by Higgs et al. is not only a mechanism for generating masses and keeping renormalizability, but is *a field theory about the fundamental character of the particles and fields that we observe in nature*. The import of the 2012 work at CERN was its direct validation of this broader viewpoint, because what appears to have been detected at CERN was the Higgs field h and not the seed scalar ϕ.

To date, these teachings of Higgs et al. have been well-developed for the scalar fields of a Klein-Gordon $\mathfrak{L} = (\partial_\sigma \phi)(\partial^\sigma \phi) - m^2\phi^2$. But they have not yet been satisfactorily extended to the elementary fermions we observe in nature, namely leptons and quarks which have been thoroughly observed with hordes of detailed attendant data in contrast to the comparatively paltry experimental data we have about the elementary Higgs scalars, the latter of which was the subject of the July 2012 announcement from CERN. The purpose of this paper is to show in detail how these teachings may be extended to apply just as fully to fermions as they apply to scalar fields, i.e., that the fermions we observe in nature are not the $\psi(x) = v_\psi + h_\psi(x)$ in $\mathfrak{L} = \overline{\psi}(i\partial\!\!\!/ - m)\psi$, but are $h_\psi(x) = \psi(x) - v_\psi$ expanded about a suitable v_ψ vacuum, just as in scalar Higgs theory.

The development of "Higgs fermions" in this manner enables us to reveal fermion masses in renormalizable fashion, yields additional insights into the magnetic moments of the fermions, and to the degree that these results can be experimentally validated as will be reviewed in section 13, strengthens the view that what Higgs and his colleagues invented is not merely a "mechanism" for generating mass and keeping renormalizability, and is not just a finding about spinless scalar fields, but rather, is a fundamental new insight into the very nature of the particles and fields that we observe in the physical world. And, as we shall show, this also deepens our knowledge of Dirac's equation which Dirac himself often said was "was more intelligent than its author," and intriguingly enough, of the Dirac equation's relationship to Maxwell's equations and Heisenberg's canonical matrix mechanics. In these and other ways, we develop a number of new venues for further confirming Higgs theory via the thoroughly catalogued data for the elementary fermions.

What renders this work novel is: 1) the ability to reveal fermion rest masses in the same manner that the Higgs masses are revealed in scalar Higgs theory without putting those masses into the theory by hand (see (6.16), (6.17)), 2) the revelation of fermion rest masses which are entirely self-energies (no "bare" mass) constructed entirely out of the gauge potentials of the fermion (also see (6.17)), 3) uncovering a deep connection whereby the time and space dependencies of the electric and magnetic fields **E** and **B** as they appear in Maxwell's equations are revealed to be embedded into Dirac's equation as a result of commutations of these fields and the gauge fields with canonical momentum and with the Hamiltonian (see after (7.6) and (9.12)), 4) showing how the "anomalous" portion of the fermion magnetic moments is already built into

Dirac's equation, when the observable fields are taken to be fermion Higgs fields $h_\psi(x)$ rather than the ordinary seed fields $\psi(x)$ (see (12.5), (12.10) and (12.11)), 5) three *numeric predictions* quantifying how the g-factors of the three charged leptons are changed when the magnetic field in the magnetic moment term $\boldsymbol{\sigma}\cdot\mathbf{B}$ is a time-dependent magnetic field appearing as $\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$ (see (13.10)), 6) a related *qualitative prediction* that the time-dependent magnetic field $\partial\mathbf{B}/\partial t$ impacts the heavy lepton (especially the tau lepton) magnetic moments much more substantially than it impacts those for the lighter leptons (especially the electron) (also see (13.10)), and 7) showing via "Invariant Mass, Variable Gauge Renormalization" how the fermion rest masses may be maintained as invariant masses over *all* renormalization scales, by absorbing any mass variation that might otherwise occur into an ordinary gauge transformation of the gauge fields from which the fermion self-energies are constructed (see section 14).

We now provide a brief overview of this paper: In section 2, as a prelude to studying fermions using Higgs field theory, we shall carefully review, step by step, the manner in which Higgs field theory is used for scalar fields. That is, section 2 is a review of known Higgs field theory for the Klein-Gordon equation and scalar fields, intended to establish a "prior art" template for considering fermions. Thereafter, the balance of the paper studies Higgs field theory for the Dirac equation and fermion fields.

In the general context of a Dirac Lagrangian density $\mathfrak{L}=T-V$ with both kinetic terms *T* and potential terms *V*, PART I of this paper focuses on the potential terms *V*. Section 3 postulates a fermion mass parameter μ fully analogous to the mass parameter of the same symbol in scalar Higgs theory, and develops the vacuum associated with the potential *V* so as to identify a stable minimum for expansion about this vacuum. Section 4 focuses on Dirac spinors and how they are expanded about this vacuum, in both four- and two-component representations. The former is mainly illustrative; the latter forms the basis for many subsequent calculations. We see how Dirac's equation requires a positive energy vacuum for fermions and a negative energy vacuum for antifermions, and we begin to review as a foundation for later development how time and space dependencies arise only via commutations with canonical momentum and not through the spacetime derivatives that first appear in a Lagrangian. In sum, and as developed throughout, spacetime dependencies are "revealed" just as are masses and observable fields. Section 5 implements the Higgs vacuum expansion for the terms in the Lagrangian potential *V*. In section 6, in a very central result, equations (6.16) and (6.17) demonstrate how the rest mass of the fermion is revealed strictly as a consequence of this Higgs expansion of the potential, in exactly the same way that a scalar Higgs field mass (the one apparently found at CERN) is revealed in scalar Higgs theory through the scalar Lagrangian potential. This is central to renormalizability, as is further developed in section 14. Section 7 completes our exploration of the Dirac potential *V*, by showing how a second-order magnetic moment term arises out of an anticommutator term uncovered in section 6. More important than this particular term, is that this presages two important aspects of the development to follow: first, how certain anticommutator terms end up producing magnetic moments, and secondly, how canonical commutators produce certain time and space dependencies in the gauge potentials such that Maxwell's equations end up becoming embedded in rather striking way right into the heart of Dirac's equation. The first appearance of this intriguing development appears in (7.6), and it permeates the later development as is seen most clearly in (12.5) and (12.11).

In PART II we move over to the kinetic T terms in $\mathfrak{L} = T - V$. Section 8 reviews and carefully develops the Gordon decomposition of the current density for the "seed" fermion ψ. Sections 9, 10 and 11 develop the magnetic moments, respectively, of the seed fermions themselves, of the self-interactions of the vacuum, and of the interaction between the vacuum and the seed fermions. From these three pieces of the puzzle, we are able in section 12 to identify magnetic moment g-factors (and two additional g-type factors) for the Higgs fermions h_ψ. Because Higgs theory tells us that the observed fermions are the Higgs fields following expansion about a non-trivial, non-zero vacuum, we identify this with the g-factor of the observed fermions, and we find that the magnetic moment in *Dirac's equation with no modification*, naturally has an "anomalous" portion (one which differs upwardly from the simple Dirac form $g/2=1$) when it is taken in relation to the Higgs field h_ψ rather than the seed field ψ. In section 13, we use the experimental data for the charged lepton (electron, mu and tau) masses and g-factors together with the low-energy running coupling $\alpha = 1/137.0359990740$ and the Fermi vev $v = 246.219650794137\ \text{GeV}$ to uniquely determine the low probe energy gauge potentials for each of these charged leptons, and related numerical data. We show in (13.9) how the fermion masses naturally arise as the self-energy owing to a difference between two related gauge potentials and in (13.10) we use these gauge potentials to *predict* how the g-factor for each charged lepton is modified in the presence of a *time-dependent* magnetic field (which has earlier been introduced because of the embedding of the Maxwell / Ampere equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ in the Dirac equation following one of several applications of canonical commutation to reveal a time dependency which was not apparent *ab initio*). We find that a time-dependent magnetic field impacts the g-factor of the heaver mu and especially tau lepton much more substantially and detectably than it does the g-factor of the electron, precisely because of the larger masses of heavier leptons. The same facilities which study and establish g-factors for $\boldsymbol{\sigma} \cdot \mathbf{B}$, should be able to discern these effects for $\boldsymbol{\sigma} \cdot \partial \mathbf{B}/\partial t$, as they are well within experimentally-detectable ranges.

In section 14 we directly demonstrate the renormalization benefits of being able to reveal fermion masses via a Higgs-type expansion about the vacuum rather than introducing the masses by hand as "bare" masses. In particular, we show how it is possible to keep the rest masses invariant at all renormalization scales, by instead renormalizing the gauge fields from which the fermion masses / self-energies arise in a manner that is *nothing more or less than an ordinary gauge transformation*. In other words, we show how the variability of a fermion mass under renormalization is entirely formally equivalent to, and may be fully absorbed by, a gauge transformation of the vector potentials from which the fermion self-energies arise. We refer to this as "Invariant Mass, Variable Gauge Renormalization." Section 15 lays out how the orbital angular momentum of these Higgs fermions may be developed, and section 16 offers some concluding observations.

2. A Step-by-Step Review of how Scalar and Vector Boson Masses are Revealed in a U(1) Gauge Theory for a Complex Scalar Field

To start our development, we shall first carefully review the manner in which a local U(1) gauge symmetry is broken in the standard model for the Klein-Gordon Lagrangian of a complex scalar field $\phi(x)$, in order to reveal masses for both a gauge boson field $A_\mu(x)$ and a scalar Higgs field $h(x)$. All of the development in this section is well-known. In this review, we follow closely on sections 14.6 through 14.9 of Halzen and Martin's [8], but with the purpose of establishing a template for a similar development that will reveal self-energy masses and magnetic moments for a Dirac fermion field $\psi(x)$ and specifically for its related Higgs field $h_\psi(x)$, in a fashion that is not yet known.

We begin our review with the relativistic energy relation $p_\sigma p^\sigma - m^2 = 0$ where p_σ is a canonical momentum and *m* is a rest mass. We postulate a scalar field $\phi(x)$ such that $i\partial_\mu \phi = p_\mu \phi$, and use this to rewrite the energy relation as $\left(\partial_\sigma \partial^\sigma + m^2\right)\phi = 0$. This is of course the Klein-Gordon (relativistic Schrödinger) equation for the scalar field ϕ. As is well-known and easily-derived, its Lagrangian density is $\mathfrak{L} = \frac{1}{2}\left(\partial_\sigma \phi\right)\left(\partial^\sigma \phi\right) - \frac{1}{2} m^2 \phi^2$. The mass *m*, of course, is classically interpreted as the mass of the scalar field ϕ. But this mass is introduced by hand, and we know that theories with hand-added masses are notoriously not renormalizable. Therefore, it is customary to rewrite the Klein-Gordon Lagrangian density as $\mathfrak{L} = \frac{1}{2}\partial_\sigma \phi \partial^\sigma \phi - \frac{1}{2}\mu^2 \phi^2$, where μ is a mass *parameter* which sits in the position of a mass in the Lagrangian density. That way, we are not introducing any masses at all. We are simply using μ as a "placeholder" for the mass term in the Klein-Gordon equation, and we leave it to the development of the theory to tell us how to understand this mass parameter, and specifically, how this mass parameter might be related to a *physically-observed* mass.

Next, we further postulate that ϕ is a complex scalar field $\phi \equiv \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$. By this definition the conjugate field $\phi^* \equiv \frac{1}{\sqrt{2}}\left(\phi_1 - i\phi_2\right)$, and so $\phi^*\phi = \frac{1}{2}\left(\phi_1^{\,2} + \phi_2^{\,2}\right)$. The $\frac{1}{2}$ coefficient in the Klein-Gordon $\mathfrak{L} = \frac{1}{2}\left(\partial_\sigma \phi\right)\left(\partial^\sigma \phi\right) - \frac{1}{2} m^2 \phi^2$ is then absorbed into this definition, and the Lagrangian density is rewritten as:

$$\mathfrak{L} = \left(\partial_\sigma \phi^*\right)\left(\partial^\sigma \phi\right) - \mu^2 \phi^* \phi . \tag{2.1}$$

At this juncture we are ready to begin. The first thing we do is introduce gauge theory, by requiring that (2.1) be invariant under the local gauge transformation $\phi \to \phi' = e^{i\theta(x)}\phi$ where $\theta(x)$ is a local gauge (really, phase) parameter. The prescription for doing this, which was first pioneered by Hermann Weyl [9], [10], [11] who modeled gauge theory on Einstein's use [12] of a spacetime-covariant derivative $\partial_\mu A_\nu \to \partial_{;\mu} A_\nu = \partial_\mu A_\nu - \Gamma^\sigma{}_{\mu\nu} A_\sigma$ to account for the curvature of Riemannian spacetime geometry, is to similarly replace ∂_σ with a gauge-covariant derivative $\partial_\sigma \to D_\sigma \equiv \partial_\sigma - ieA_\sigma$ to account for the "curvature" in a complex gauge /phase space. Consequently, using Weyl's gauge prescription and the help of $\phi^*\phi = \frac{1}{2}\left(\phi_1^{\,2} + \phi_2^{\,2}\right)$, (2.1) becomes:

$$\begin{aligned}\mathfrak{L} &= (D_\sigma\phi)^*(D^\sigma\phi) - \mu^2\phi^*\phi = (\partial_\sigma + ieA_\sigma)\phi^*(\partial^\sigma - ieA^\sigma)\phi - \mu^2\phi^*\phi \\ &= (\partial_\sigma\phi^*)(\partial^\sigma\phi) - e^2\phi^*\phi A_\sigma A^\sigma - \mu^2\phi^*\phi \\ &= (\partial_\sigma\phi^*)(\partial^\sigma\phi) - \tfrac{1}{2}(\phi_1{}^2 + \phi_2{}^2)e^2 A_\sigma A^\sigma - \mu^2\phi^*\phi\end{aligned} \quad . \tag{2.2}$$

We next note that $\mathfrak{L}_{\mathrm{KG}} = \frac{1}{2}(\partial_\sigma\phi)(\partial^\sigma\phi) - \frac{1}{2}m^2\phi^2$ (KG denotes Klein-Gordon) generalizes to the field equation for a gauge field A_μ by replacing $\phi \to A_\mu$ and properly adding spacetime indexes. Indeed, this yields the source-free Maxwell (M) Lagrangian density:

$$\mathfrak{L}_{\mathrm{KG}} = \tfrac{1}{2}(\partial_\sigma\phi)(\partial^\sigma\phi) - \tfrac{1}{2}m^2\phi^2 \Rightarrow \mathfrak{L}_{\mathrm{M}} = \tfrac{1}{2}(\partial_\sigma A_\tau)(\partial^{[\sigma}A^{\tau]}) - \tfrac{1}{2}m^2 A_\sigma A^\sigma = \tfrac{1}{4}F_{\sigma\tau}F^{\sigma\tau} - \tfrac{1}{2}m^2 A_\sigma A^\sigma , \tag{2.3}$$

where $F^{\sigma\tau} \equiv \partial^{[\sigma}A^{\tau]} \equiv \partial^\sigma A^\tau - \partial^\tau A^\sigma$ is the electromagnetic field strength tensor, and *m* is the Proca mass of the gauge boson A^σ. Contrasting $-\frac{1}{2}m^2 A_\sigma A^\sigma$ above with $-\frac{1}{2}(\phi_1{}^2 + \phi_2{}^2)e^2 A_\sigma A^\sigma$ in (2.2), for the first time we see a correspondence $m^2 \Leftrightarrow (\phi_1{}^2 + \phi_2{}^2)e^2$ which tells us that the simple application of gauge theory itself has revealed a boson mass $m^2 \equiv (\phi_1{}^2 + \phi_2{}^2)e^2$ (which we take to be >0), if we can now find some way to make sense of the term $(\phi_1{}^2 + \phi_2{}^2)$.

Now we turn to the term $-\mu^2\phi^*\phi$ in (2.2). This is a placeholder for the Klein-Gordon mass term. It is now customary to interpret this not as the mass of the scalar field ϕ, but as the leading term in a potential energy $-V(\phi)$. Specifically, working from the middle line of (2.2) we now write:

$$\mathfrak{L} = (\partial_\sigma\phi^*)(\partial^\sigma\phi) - e^2\phi^*\phi A_\sigma A^\sigma - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 = T(\phi) - V(\phi) . \tag{2.4}$$

where we define the kinetic (*T*) and potential (*V*) terms as:

$$T(\phi) \equiv (\partial_\sigma\phi^*)(\partial^\sigma\phi) - e^2\phi^*\phi A_\sigma A^\sigma , \tag{2.5}$$

$$V(\phi) \equiv \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 , \tag{2.6}$$

and where λ is a postulated but unknown coefficient for the next-order term $(\phi^*\phi)^2$. Given that $\mathfrak{L}$ and therefore *T* and *V* all have a mass-dimensionality of $D = +4$ in mass-length-time units of $\hbar = c = 1$, and that ϕ and A_σ each have $D = +1$, we see that λ is a dimensionless, $D = 0$ coefficient.

Now that we are interpreting $\mu^2\phi^*\phi$ as the leading term in $V(\phi)$ and not as the mass of the scalar field, we may use (2.6) to find out a few more important things. First, we may take $\partial V/\partial\phi$ and set this to zero to find the min/max points in the potential, that is, we now deduce:

$$\frac{\partial V}{\partial\phi}=\mu^2\phi^*+2\lambda(\phi^*\phi)\phi^*=0\,. \tag{2.7}$$

This means that:

$$(\phi^*\phi)_{\text{min/max}}=-\frac{\mu^2}{2\lambda}\,. \tag{2.8}$$

But we do not yet know if (2.8) is a minimum or a maximum until we take another derivative $\partial/\partial\phi^*$ from (2.7) and then evaluate ($|(\phi^*\phi)_{\text{min/max}}$) that derivative at the min/max point (2.8). This yields:

$$\frac{\partial^2 V}{\partial\phi^*\partial\phi}|(\phi^*\phi)_{\text{min/max}}=\mu^2+4\lambda(\phi^*\phi)_{\text{min/max}}=\mu^2-2\mu^2=-\mu^2\,. \tag{2.9}$$

So, if we want $V(\phi)$ in (2.6) to be a real potential with a stable minimum, then the second derivative (2.9) evaluated at the min/max point (2.8) *must* be *positive*. Therefore, it is (2.9) which tells us that we must set $\mu^2<0$. We now do so. Then (2.8) will define a non-zero, local minimum of a real potential so long as we also have $\lambda>0$. We then return to (2.8) to define the vacuum expectation value (vev) v according to:

$$(\phi^*\phi)_{\text{min}}=\frac{1}{2}(\phi_1{}^2+\phi_2{}^2)_{\text{min}}=-\frac{\mu^2}{2\lambda}\equiv\frac{1}{2}v^2>0\,. \tag{2.10}$$

With $\mu^2<0$ and $\lambda>0$, the vev establishes a real, positive number that defines a local minimum in the vacuum. Thus, the scalar fields may be expanded around this ground state. The vacuum itself is the square root of the above, and so with the mathematically permissible $\pm$ values, is:

$$\pm v=\sqrt{(\phi_1{}^2+\phi_2{}^2)_{\text{min}}}=\sqrt{2(\phi^*\phi)_{\text{min}}}=i\frac{\mu}{\sqrt{\lambda}}\,. \tag{2.11}$$

The next step is to define a scalar Higgs field $h(x)$ and its imaginary counterpart $i\xi(x)$, and expand the scalar field about the vev. Normally, one chooses to expand around $+v$. But it will be important for illustration to make this choice, not right now, but at the very end. Thus, for the moment, preserve both options (2.11) by setting:

$$\phi(x)\equiv\tfrac{1}{\sqrt{2}}\left(\pm v+h(x)+i\xi(x)\right). \tag{2.12}$$

From this, the conjugate $\phi^*(x) = \frac{1}{\sqrt{2}}(\pm v + h(x) - i\xi(x))$ and so:

$$\phi^*\phi = \tfrac{1}{2}\left(v^2 \pm 2vh + h^2 + \xi^2\right). \tag{2.13}$$

$$(\phi^*\phi)^2 = \tfrac{1}{4}\left(v^4 \pm 4v^3h + 6v^2h^2 \pm 4vh^3 + h^4 + 2v^2\xi^2 \pm 4v\xi^2h + 2\xi^2h^2 + \xi^4\right). \tag{2.14}$$

We then substitute (2.12) and its conjugate and (2.13), (2.14) into the Lagrangian density (2.4) via its kinetic and potential terms (2.5) and (2.6). For what is now $T(h,\xi)$, we find that:

$$T(h,\xi) = \tfrac{1}{2}(\partial_\sigma h)(\partial^\sigma h) + \tfrac{1}{2}(\partial_\sigma \xi)(\partial^\sigma \xi) - \tfrac{1}{2}e^2\left(\pm 2vh + h^2 + \xi^2\right)A_\sigma A^\sigma - \tfrac{1}{2}\left(e^2v^2\right)A_\sigma A^\sigma. \tag{2.15}$$

and for $V(h,\xi)$:

$$V(h,\xi) = \tfrac{1}{2}\mu^2\left(v^2 \pm 2vh + h^2 + \xi^2\right) + \tfrac{1}{4}\lambda\left(v^4 \pm 4v^3h + 6v^2h^2 \pm 4vh^3 + h^4 + 2v^2\xi^2 \pm 4v\xi^2h + 2\xi^2h^2 + \xi^4\right). \tag{2.16}$$

With the help of $\mu^2 \equiv -\lambda v^2$ from (2.10) we may consolidate the two main terms in (2.16) into:

$$V(h,\xi) = -\tfrac{1}{4}\lambda v^4 + \lambda v^2h^2 \pm \lambda vh^3 + \tfrac{1}{4}\lambda h^4 \pm \lambda v\xi^2h + \tfrac{1}{2}\lambda\xi^2h^2 + \tfrac{1}{4}\lambda\xi^4. \tag{2.17}$$

Finally placing (2.15) and (2.17) into (2.4) we obtain:

$$\begin{aligned}\mathfrak{L} &= T(h,\xi) - V(h,\xi)\\ &= \tfrac{1}{2}(\partial_\sigma h)(\partial^\sigma h) + \tfrac{1}{2}(\partial_\sigma \xi)(\partial^\sigma \xi) - \tfrac{1}{2}e^2\left(\pm 2vh + h^2 + \xi^2\right)A_\sigma A^\sigma - \tfrac{1}{2}\left(e^2v^2\right)A_\sigma A^\sigma.\\ &+ \tfrac{1}{4}\lambda v^4 \mp \lambda vh^3 - \tfrac{1}{4}\lambda h^4 \mp \lambda v\xi^2h - \tfrac{1}{2}\lambda\xi^2h^2 - \tfrac{1}{4}\lambda\xi^4 - \tfrac{1}{2}\left(2\lambda v^2\right)h^2\end{aligned} \tag{2.18}$$

At the end of the middle line, we have a term $-\tfrac{1}{2}\left(e^2v^2\right)A_\sigma A^\sigma$. Comparing that to the term $-\tfrac{1}{2}m^2A_\sigma A^\sigma$ in the Maxwell Lagrangian density (2.3) and also using (2.10), we see that:

$$m_A = ev = ie\mu/\sqrt{\lambda}. \tag{2.19}$$

This is the *revealed mass* of the gauge boson, and is the mass which we expect to observe for the physically-observable gauge field. Our willingness to simply use μ as a place holder for the Klein-Gordon mass term and not regard this as the actual physical mass of the scalar particle pays off by comparing $-\tfrac{1}{2}\left(2\lambda v^2\right)h^2$ at the very end of (2.18) with the original Klein-Gordon $\mathfrak{L} = \tfrac{1}{2}(\partial_\sigma\phi)(\partial^\sigma\phi) - \tfrac{1}{2}m^2\phi^2$. Here, we see that:

$$m_h = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} = i\sqrt{2}\mu. \tag{2.20}$$

So the original scalar field ϕ has disappeared from the Lagrangian entirely, and been replaced by the Higgs scalar *h*. The mass of this new scalar – which we take to be what is physically observed – is related to the original mass parameter μ via $m_h = i\sqrt{2}\mu$. But because we found along the way that $\mu^2 < 0$, this is still a real mass value.

The final step we take is to actually break the symmetry of the new Lagrangian (2.18), which we have not done yet. While often conflated together with an early choice of $\pm v \to +v$, this breaking of symmetry really entails two steps. First, we go back to the original definition before (2.1) of the scalar field as a complex field $\phi \equiv \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$ and relate this to the later redefinition (2.12) of this field as an expansion about the vev, thus finding that:

$$\phi \equiv \tfrac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right) \equiv \tfrac{1}{\sqrt{2}}\left(\pm v + h + i\xi\right). \tag{2.21}$$

We know of course that $\phi^*\phi = \frac{1}{2}\left(\phi_1{}^2 + \phi_2{}^2\right)$. So now we break symmetry by *choosing* to make this field entirely real, so that $\phi_2 = 0$ and $\xi = 0$, thus $\phi^*\phi = \frac{1}{2}\phi_1{}^2$. So in the new Lagrangian (2.18), we set $\xi = 0$ throughout, and also use (2.19) and (2.20) to write:

$$\begin{aligned}\mathfrak{L} &= \tfrac{1}{2}\left(\partial_\sigma h\right)\left(\partial^\sigma h\right) - \tfrac{1}{2} m_h{}^2 h^2 - \tfrac{1}{2} m_A{}^2 A_\sigma A^\sigma \\ &\quad - \tfrac{1}{2} e^2 \left(\pm 2vh + h^2\right) A_\sigma A^\sigma \mp \lambda v h^3 - \tfrac{1}{4}\lambda h^4 + \tfrac{1}{4}\lambda v^4\end{aligned}. \tag{2.22}$$

We then restructure this slightly and use (2.19) and (2.20) to show the masses m_A, m_h throughout. We now arrive at:

$$\begin{aligned}\mathfrak{L} &= \tfrac{1}{2}\left(\partial_\sigma h\right)\left(\partial^\sigma h\right) - \tfrac{1}{2} m_h{}^2 h^2 - \tfrac{1}{2} m_A{}^2 A_\sigma A^\sigma \\ &\quad - \left(\pm 1 + \frac{h}{2v}\right)\frac{h}{v} m_A{}^2 A_\sigma A^\sigma - \frac{1}{2}\left(\pm 1 + \frac{h}{4v}\right)\frac{h}{v} m_h{}^2 h^2 + \frac{1}{8}\frac{m_A{}^2 m_h{}^2}{e^2}\end{aligned}. \tag{2.23}$$

We also note that $4\pi\alpha = e^2/\hbar c$ where in electromagnetic theory $\alpha = 1/137.035999074$ is the electromagnetic coupling at low impact / probe / renormalization, so that the final term may be put into the alternative form $m_h{}^2 m_A{}^2 / 32\pi\alpha$.

Now with (2.23) we have broken symmetry so as to be in the real plane with $\phi_2 = 0$ and and $\xi = 0$, defined by ϕ_1 along the horizontal axis and potential $V\left(\phi_1\right)$ along the vertical axis. The second symmetry breaking choice is to set the minimum of $V\left(\phi_1\right)$ at either $\phi_{1\min} = +v$ or $\phi_{1\min} = -v$, see (2.10). This yields two differently-appearing Lagrangians based on the choice we make. For the choice $\phi_{1\min} = +v$ with some further term consolidation which will be a helpful benchmark when we turn to fermions, the above becomes:

$$\mathfrak{L}(+v)=\frac{1}{2}\left[(\partial_\sigma h)(\partial^\sigma h)-\left(1+\frac{h}{v}+\frac{h^2}{4v^2}\right)m_h{}^2h^2-\left(1+2\frac{h}{v}+\frac{h^2}{v^2}\right)m_A{}^2A_\sigma A^\sigma+\frac{m_A{}^2m_h{}^2}{4e^2}\right]. \quad (2.24)$$

For the other choice $\phi_{1\min}=-v$, (2.23) becomes:

$$\mathfrak{L}(-v)=\frac{1}{2}\left[(\partial_\sigma h)(\partial^\sigma h)-\left(1-\frac{h}{v}+\frac{h^2}{4v^2}\right)m_h{}^2h^2-\left(1-2\frac{h}{v}+\frac{h^2}{v^2}\right)m_A{}^2A_\sigma A^\sigma+\frac{m_A{}^2m_h{}^2}{4e^2}\right]. \quad (2.25)$$

While waiting until the very end to make the final choice as between $\phi_{1\min}=\pm v$ may seem to be a minor point, it reappears much more starkly when it comes to understanding the vacuum for fermion interactions, and actually appears to involve the breaking of a particle and antiparticle symmetry.

Comparing (2.23) to the original Lagrangian densities (2.1) and (2.2), we see each top line is identical in form to $(\partial_\sigma\phi^*)(\partial^\sigma\phi)-\mu^2\phi^*\phi-\frac{1}{2}(\phi_1{}^2+\phi_2{}^2)e^2A_\sigma A^\sigma$ from the bottom line of (2.2). But through the entire process of introducing gauge symmetry to get from (2.1) to (2.2), then using $-\mu^2\phi^*\phi$ as the leading term of $-V(\phi)$, then establishing a stable ground state for the vacuum, then expanding about this vacuum, then breaking symmetry such that the scalar field becomes real, and finally choosing between $\phi_{1\min}=\pm v$, the original complex scalar field ϕ is replaced by the real Higgs scalar field *h*, this new scalar field *h* has ended up with a revealed mass m_h which is not introduced by hand but emerges naturally, and a gauge field A_σ which did not exist *at all* in (2.1) not only exists, *but also has its own revealed mass* m_A. And because none of these masses were introduced by hand, the theory based on (2.22) is fully renormalizable.

The main point to be made from this review is that all the way back in (2.1), before we even applied gauge theory much less did anything else, we started out by using a mass parameter μ^2 in mass position in the Klein Gordon Lagrangian, but made no pre-supposition whatsoever about the this mass parameter. We left it to the development of the theory to inform us as to the nature of μ. We later found out in (2.8) that to give rise to a stable minimum in the vacuum, $\mu^2<0$, which means that μ itself is imaginary and that as written, (2.1) therefore contains the wrong sign were we to regard μ as an actual particle mass. But in the end, we ended up in (2.19) uncovering a revealed mass $m_A=ev=ie\mu/\sqrt{\lambda}$ for the gauge boson as well as in (2.20) uncovering $m_h=i\sqrt{2}\mu$ for the Higgs scalar. So μ itself did not turn out to be the mass of anything observable. It turned out to be an imaginary number which is related by (2.19) and (2.20) to the real, observable masses of a gauge boson and a Higgs scalar.

At the same time, we started out by positing a scalar field ϕ which we may think of as a "seed" field. This field is a mathematical device but not a physical observable. Through the course of development after expanding about the minimum of vacuum, this seed field ϕ turned into a Higgs scalar field *h* which *is regarded as an observable, complete with its own revealed mass*. The heart of this Higgs process, therefore, is to start with a seed field (above, ϕ) and a mass parameter (above, μ) which are both not observables, and to have these get turned into an observable field (above, *h*) with an observable mass (above, m_h). In the process, as a bonus, we also revealed a gauge field A_σ and its observable mass m_A, which is to say, we also revealed a second observable field and also its mass.

One final point to be made is this: As can be seen most directly in (2.18) through (2.20), the mass m_h of the Higgs field *h*, which is the field descended from the seed scalar field ϕ, arose from the *potential* term $-V(h,\xi)$ of the Lagrangian density. On the other hand, the "bonus field" A_σ, as well as its mass m_A , arose from the process of introducing gauge symmetry, and emanated from the *kinetic* portion $T(h,\xi)$.

All of this will be very critical to keep in mind as we now embark on a parallel approach for spin ½ fermions. For fermions, the fermion mass will arise in a fashion analogous to how the Higgs scalar mass was revealed in (2.20) above, see (6.17) infra. For gauge bosons, we will also obtain a "bonus" result. Above, for scalar theory, the bonus was the mass of the gauge boson revealed in (2.19). But for fermion fields, as we shall establish throughout the course of the subsequent development, the "bonus" is not the mass of the gauge field A^μ, but the magnetic moment of the fermion which is observed when that fermion is placed into a magnetic field $\mathbf{B} = B^i = -\frac{1}{2}\varepsilon^{ijk}F^{jk}$ which is related to the gauge field in the usual manner according to the field strength relation $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, including "anomalous" portions of the fermion g-factor. In other words, the bonus which in scalar Higgs field theory is the revelation of a gauge field mass, in Higgs field theory for fermions becomes the revelation of a complete magnetic moment for the fermion / magnetic field interaction, including anomalous portions. What is further revealed in (13.10) infra, is the effect which a *time-dependent* magnetic field $\partial\mathbf{B}/\partial t$ has on the g-factors of the three charged leptons, and this apparently-new result may well provide a basis for experimental validation.

PART I: REVEALING FERMION MASSES – LAGRANGIAN POTENTIAL

3. The Fermion Mass Parameter and the Minimum of Its Related Lagrangian Potential

In order to reveal a fermion mass out of gauge symmetry and spontaneous symmetry breaking in a similar manner to what was just reviewed, we once again start with the relativistic energy relation $p_\sigma p^\sigma - m^2 = 0$. If we write this in flat spacetime as $\eta^{\sigma\tau} p_\sigma p_\tau = m^2$ and then apply $\eta^{\sigma\tau} = \frac{1}{2}\left(\gamma^\sigma \gamma^\tau + \gamma^\tau \gamma^\sigma\right) = \frac{1}{2}\left\{\gamma^\sigma, \gamma^\tau\right\}$ where $\eta^{\sigma\tau}$ is the contravariant Minkowski metric tensor with $\mathrm{diag}\left(\eta^{\sigma\tau}\right) = (1,-1,-1,-1)$, one obtains $\frac{1}{2}\left(\gamma^\sigma \gamma^\tau + \gamma^\tau \gamma^\sigma\right) p_\sigma p_\tau - m^2 = 0$. Then using the Feynman slash notation $\not{p} \equiv \gamma^\sigma p_\sigma$ this becomes $\not{p}\not{p} = m^2$. Separating the two square roots and using the resulting expression to operate from the left on a Dirac spinor *u* yields $\left(\not{p} - m\right) u = 0$ in which the mass *m* represents eigenvalues of the slashed momentum matrix $\not{p}$. Upon promoting the spinor to a wavefunction $u \to \psi$ simultaneously with substituting $\not{p} \to i\not{\partial}$, the new wavefunction equation becomes $\left(i\not{\partial} - m\right)\psi = 0$, which is Dirac's equation. In essence, this is the path Dirac followed to derive his equation in [13], [14], which included uncovering a specific (Dirac) representation for the γ^σ matrices. Just as we posited a scalar "seed" field ϕ in section 2 with an unobservable mass parameter μ and turned it into an observable Higgs field *h* with a revealed mass m_h and obtained a gauge boson A^μ and its mass m_A as a bonus, here we shall posit a "seed" fermion field ψ with an unobservable mass parameter also denoted μ and try to turn it into an observable fermion Higgs field h_f with its own observable fermion mass *m*, and in the process see what other "bonuses" also emerge.

As already noted following (2.1), when moving from a "flat" gauge space (no particle interaction with gauge fields) to a "curved" gauge space (where the particles to interact via gauge fields), one makes the substitution $\partial_\sigma \to D_\sigma \equiv \partial_\sigma - ieA_\sigma$, or multiplying through by *i*, $i\partial_\sigma \to iD_\sigma \equiv i\partial_\sigma + eA_\sigma$. When we work with gauge theory in momentum space where $i\partial_\sigma \varphi \to p_\sigma \varphi$ for whatever generalized field $\varphi = \phi, \psi$, etc. one is considering at a given moment, then the canonical momentum p_σ goes over into a kinetic momentum $p_\sigma \to \pi_\sigma \equiv p_\sigma + eA_\sigma$. At the same time, the energy relation becomes.

$$m^2 = p_\sigma p^\sigma \to m^2 = \pi_\sigma \pi^\sigma = \left(p_\sigma + eA_\sigma\right)\left(p^\sigma + eA^\sigma\right). \tag{3.1}$$

In component form $p^\mu \equiv \left(E, p_x, p_y, p_z\right) = \left(E, \mathbf{p}\right)$ and $A^\mu = \left(\phi, A_x, A_y, A_z\right) = \left(\phi, \mathbf{A}\right)$, so the kinetic momentum $\pi^\mu = p^\mu + eA^\mu = \left(E + e\phi, p_x + eA_x, p_y + eA_y, p_z + eA_z\right) = \left(E + e\phi, \mathbf{p} + e\mathbf{A}\right)$. (<u>Important Note</u>: from here on, we use $\phi \equiv A^0$ to denote the time component of the vector potential A^μ

which physically is a voltage, and not the scalar field ϕ of the Klein-Gordon equation which was reviewed in section 2. Occasional references back to the scalar field ϕ should be easily distinguishable by context.) If we separate the space from the time components and raise indexes with $\eta^{\mu\nu}$, then (3.1) becomes:

$$m^2 = \pi_\sigma \pi^\sigma = \left(E + e\phi, \mathbf{p} + e\mathbf{A}\right)\left(E + e\phi, -\mathbf{p} - e\mathbf{A}\right) = \left(E + e\phi\right)^2 - \left(\mathbf{p} + e\mathbf{A}\right)^2 . \tag{3.2}$$

Because π^μ is proportional to the four-velocity of the mass *m*, in the rest frame of the mass, we have $\mathbf{p} + \mathbf{A} = 0$ and $m^2 = \left(E + e\phi\right)^2$.

Now let us turn to Dirac's equation $\left(i\partial - m\right)\psi = 0$ and its associated $\mathfrak{L} = \overline{\psi}\left(i\partial - m\right)\psi$. Although normally written with a mass *m*, let us instead write this with a mass parameter μ in the form:

$$\mathfrak{L} = \overline{\psi}\left(i\partial - \mu\right)\psi . \tag{3.3}$$

This is just like what was done in (2.1). As with the breaking of symmetry reviewed in section 2, we make no presuppositions about how this parameter μ relates to the actual observed mass *m* of a fermion such as an electron. We shall let the development of the theory advise us about that. However, to set the stage, we shall *define by postulate* the mass parameter μ by forming a four-vector *defined* as $\rho^\mu \equiv \left(\mu, 0, 0, 0\right) - e\left(\phi, A_x, A_y, A_z\right)$, such that in flat spacetime with $\operatorname{diag}\left(\eta_{\mu\nu}\right) = \left(1, -1, -1, -1\right)$ we have:

$$\mu^2 \equiv \rho_\mu \rho^\mu = \left(\mu - e\phi, -e\mathbf{A}\right)\left(\mu - e\phi, +e\mathbf{A}\right) = \left(\mu - e\phi\right)^2 - e\mathbf{A}^2 = \mu^2 - 2\mu e\phi + e^2\phi^2 - e^2\mathbf{A}^2 . \tag{3.4}$$

Contrasting this to the relativistic energy relationship $m^2 = p_\sigma p^\sigma = E^2 - \mathbf{p}^2$, we see the parallel structural correspondences $m \Leftrightarrow \mu$, $E = \left(m + W\right) \Leftrightarrow \left(\mu - e\phi\right)$ and $\mathbf{p} \Leftrightarrow -e\mathbf{A}$.

There are many reasons for defining μ as in (3.4) which will become apparent as we proceed through the development. But one of the many reasons for making the definition in (3.4) is that after subtracting μ^2 from each side, (3.4) embeds a quadratic

$$0 \equiv e^2\phi^2 - 2\mu e\phi - e^2\mathbf{A}^2 . \tag{3.5}$$

It is (3.5) which is really the definition of μ. Equation (3.4) shows the definition (3.5) in a form that allows ready comparison to equation (3.2) for the observed mass *m*, and that allows μ to enter into the Dirac Lagrangian (3.3) as a Lorentz-invariant scalar. In (3.2), *m* is understood to *be entirely independent* of ϕ, **A**, *e*. But in (3.5), the mass parameter μ is defined entirely in terms of ϕ, **A**, *e*, and nothing else. So, if we can convert μ into a revealed fermion mass in the same way that its cousin μ is converted into a revealed scalar mass in (2.20) for scalar Higgs theory, this reveled mass (which we find in (6.17) infra) will have been constructed solely out of the gauge field $A^{\mu}=\left(\phi, A_x, A_y, A_z\right)=\left(\phi, \mathbf{A}\right)$ and the charge strength e and nothing else, and so can be interpreted simply as a fermion self-energy arising from the gauge fields and the charge strength of the fermion, with no "bare" aspect to the mass. This has substantial benefits in many ways, including renormalization as will be reviewed in section 14.

Working from the definition (3.5), we may solve the quadratic for $e\phi$ to obtain:

$$e\phi=\frac{2\mu\pm\sqrt{4\mu^2+4e^2\mathbf{A}^2}}{2}=\mu\pm\mu\sqrt{1+e^2\mathbf{A}^2/\mu^2}=\mu\left(1\pm\sqrt{1+\frac{e^2\mathbf{A}^2}{\mu^2}}\right) \tag{3.6}$$

We may also solve (3.5) directly for μ, to obtain:

$$\mu=\frac{1}{2}\left(e\phi-\frac{e^2\mathbf{A}^2}{e\phi}\right)=\frac{1}{2}e\left(\phi-\frac{\mathbf{A}^2}{\phi}\right) \tag{3.7}$$

We shall for the moment remain agnostic as to which is the appropriate quadratic choice in (3.6). Later, at equation (13.6) infra, we shall finally be in a position to choose the correct sign in (3.6), but this will be based on empirical data rather than human predisposition. Finally, at the end of section 14, we will have developed enough new knowledge based on (3.5) that it will become possible to appreciate the many reasons underlying the definition (3.5), and how this connects together a whole range of issues including renormalization and the fact that even in a fermion's own rest frame, there is a kinetic aspect that is never removed from a fermion because the circulating energy flow of the fermion's spin never ceases [15].

Moving on, we now introduce a gauge field into (3.3) via Weyl's gauge prescription, also using (3.7) and so write:

$$\mathfrak{L}=\overline{\psi}\left(i\partial\!\!\!/-\mu\right)\psi\Rightarrow\mathfrak{L}=\overline{\psi}\left(i\partial\!\!\!/+eA\!\!\!/-\mu\right)\psi=\overline{\psi}\left(i\partial\!\!\!/+eA\!\!\!/-\frac{1}{2}\left(e\phi-\frac{e^2\mathbf{A}^2}{e\phi}\right)\right)\psi\,. \tag{3.8}$$

Separating mass terms in a more familiar form flagged by $\overline{\psi}\psi$, the above becomes:

$$\mathfrak{L}=\overline{\psi}\left(i\partial\!\!\!/+eA\!\!\!/\right)\psi-\frac{1}{2}e\left(\phi-\frac{\mathbf{A}^2}{\phi}\right)\overline{\psi}\psi-\lambda_f\left(\overline{\psi}\psi\right)^2=T\left(\psi\right)-V\left(\psi\right) \tag{3.9}$$

where just as in (2.4) to (2.6), we now *define* kinetic (T) and potential (V) Lagrangian portions which include a newly-introduced first higher-order term $\lambda_f\left(\overline{\psi}\psi\right)^2$ and associated fermion coupling λ_f analogous to the λ first postulated in (2.6) for scalar Higgs theory, thus:

$$T(\psi) \equiv \overline{\psi}(i\partial + eA)\psi \tag{3.10}$$

$$V(\psi) = \mu\overline{\psi}\psi + \lambda_f\left(\overline{\psi}\psi\right)^2 = \frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right)\overline{\psi}\psi + \lambda_f\left(\overline{\psi}\psi\right)^2. \tag{3.11}$$

Lo and behold, contrasting with the usual Dirac Lagrangian density $\mathfrak{L} = \overline{\psi} i\partial\psi - m\overline{\psi}\psi$, we now have a term which takes the appearance of a fermion mass. Because the mass dimensionality $D = +6$ for $\left(\overline{\psi}\psi\right)^2$, this means that $D = -2$ for this new λ_f, so it is analogous to, but not the same as, the dimensionless λ from (2.6) of the scalar theory.

Now, we must develop the vacuum based on the potential (3.11). First, let us find the min/max point of this potential, which is calculated to be:

$$\frac{\partial V}{\partial\psi} = \frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right)\overline{\psi} + 2\lambda_f\left(\overline{\psi}\psi\right)\overline{\psi} = 0 \tag{3.12}$$

Analogously to (2.8), this tells us, also using (3.7), that:

$$\left(\overline{\psi}\psi\right)_{\text{min/max}} = -\frac{1}{4\lambda_f}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right) = -\frac{\mu}{2\lambda_f} \tag{3.13}$$

Comparing, we see exactly the same expression as in (2.8), but with μ rather than μ^2.

To distinguish minimum from maximum, we take the next derivative $\partial/\partial\overline{\psi}$ of (3.12) and evaluate ($|\left(\phi^*\phi\right)_{\text{min/max}}$) at (3.13). Now we find, contrast (2.9), that:

$$\begin{aligned}\frac{\partial^2 V}{\partial\overline{\psi}\partial\psi}\,|\left(\overline{\psi}\psi\right)_{\text{min/max}} &= \frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right) + 4\lambda_f\left(\overline{\psi}\psi\right) = \frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right) - e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right) \\ &= -\frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right) = -\mu\end{aligned} \tag{3.14}$$

This too is just like (2.9), but with μ rather than μ^2.

For this to be a local minimum (not maximum) of the vacuum and therefore energetically stable, the above expression must be positive, which now imposes the requirements:

$$\frac{\partial^2 V}{\partial\overline{\psi}\partial\psi}\,|\left(\overline{\psi}\psi\right)_{\text{min/max}} = -\frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right) = -\mu > 0, \tag{3.15}$$

or more directly:

$$\mu < 0; \quad \frac{1}{2} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right) > 0 \tag{3.16}$$

So if we also define $\lambda_f > 0$, then from (3.13) we may define the vacuum vev v via:

$$\left(\overline{\psi}\psi\right)_{\min} = \frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right) = -\frac{\mu}{2\lambda_f} \equiv v^3 > 0 \tag{3.17}$$

This is analogous to (2.10) with which we uncovered the vev for a scalar field ϕ. Based on (3.16) and $\lambda_f > 0$ this $v^3 > 0$ which means that

$$v = \sqrt[3]{\frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right)} > 0 \tag{3.18}$$

But because (2.10) contained the scalar construct $\left(\phi^* \phi\right)_{\min}$ with mass-dimension $D = +2$ the vev uncovered there was $\propto v^2$, while in (3.17), because the fermion construct $\left(\overline{\psi}\psi\right)_{\min}$ has mass dimension $D = +3$, the vev number here must be $\propto v^3$. This is solely a function of the fact that now we are dealing with fermion sources ψ which have a mass dimension $D = +1.5$ whereas earlier we were dealing with scalar sources with a mass dimension $D = +1$. This is a first important point of departure from scalar Higgs theory.

But the next step, which is to expand the fermion around the vacuum using a Higgs field, reveals an even more pronounced difference. Specifically, in (2.11), we took the square root $\pm v$ of the $\left(\phi^* \phi\right)_{\min} = \frac{1}{2} v^2$ found in (2.10) and expanded about this vacuum in the form $\phi(x) \equiv \frac{1}{\sqrt{2}}\left(\pm v + h(x) + i\xi(x)\right)$. Here, the different finding of a *cubed* v^3 in (3.17) raises the question of how we expand the fermion about this vacuum. This question actually has three parts: First, do we expand around $\sqrt{v^3} = v^{1.5}$ or around $\sqrt[3]{v^3} = v$? Second, if we expand around $\sqrt{v^3} = v^{1.5}$ (which does appear to be the right answer for a fermion with $D = +1.5$), then what is the nature of the $v^{1.5}$? Is it too a scalar number like v^3? Or, does this vacuum, like a fermion, have a four-component Dirac structure? Third, because $v^3 > 0$ from (3.17) versus $v^2 > 0$ in (2.10), this means that $v > 0$ per (3.18) because this is now a *cubed root* not a square root of a positive number as in section 2. So the $\pm v$ freedom that we found in (2.11) and finally broke in (2.24) and (2.25) does not appear for fermions, at least on the surface, and we appear to be *forced* into choosing a positive energy vacuum. So this raises the third question: does this $\pm v$ freedom *reappear* for fermions in some other guise? The answer to this third question will emerge in the next section as we develop the answers to the first and second questions, and it reveals a breaking of symmetry between particles and antiparticles.

As to the first and second questions, it appears that structurally, the only sensible answer is that we expand around $v^{1.5}$, and that although v^3 is itself the scalar number $e\left(\mathbf{A}^2/\phi-\phi\right)/4\lambda_f$ per (3.17), it needs to be constructed out of a $v^{1.5}$ *defined* according to:

$$v^3 \equiv \overline{v^{1.5}}v^{1.5} = v^{1.5\dagger}\gamma^0 v^{1.5} \equiv \frac{1}{4\lambda_f}e\left(\frac{\mathbf{A}^2}{\phi}-\phi\right) > 0 \tag{3.19}$$

That is, we need to find a $v^{1.5}$ which, like a Dirac spinor, is a four-component object which includes complex components, which has an *adjoint* vacuum $\overline{v^{1.5}} \equiv v^{1.5\dagger}\gamma^0$, and which reproduces (3.19). To be clear: (3.19) is the *definition* of this $v^{1.5}$ spinor which we shall seek to explicitly find in the next section. Then, given $v^{1.5}$, we will *define* a Higgs field $h_f(x)$ which is a *Higgs fermion field* which itself has a four-component Dirac structure, and which itself is complex and has an adjoint Higgs field $\overline{h_f} \equiv h_f^{1.5\dagger}\gamma^0$, which is itself defined by an expansion about the $v^{1.5}$ vacuum spinor according to:

$$\psi(x) \equiv v^{1.5} + h_f(x), \tag{3.20}$$

$$\overline{\psi}(x) \equiv \overline{v^{1.5}} + \overline{h_f}(x). \tag{3.21}$$

Analogously to scalar Higgs theory, we shall regard the Higgs fermion field $h_f(x)$ to be the *physically-observed fermion field*, which acquires a revealed observable mass equal to that of the observed fermions. In section 2 $\phi \to h$ where ϕ is the seed scalar and h is the observed scalar. Here, $\psi \to h_f$ where ψ is the seed fermion and h_f is the observed fermion.

So now the next question becomes simply mathematical question: How do we deduce a $v^{1.5}$ which has the structure of a fermion and satisfies the definition (3.19)? Then, we can make use of this $v^{1.5}$ to carry out the expansions (3.20) and (3.21) and ascertain the masses of the fermions via the terms of the form $-m\overline{h_f}h_f$ in whatever Lagrangian density emerges to correspond to (2.24).

4. Dirac Spinors and their Expansion Vacuum

The customary way to derive a Dirac spinor (e.g., section (5.3) of [8]) is to start with Dirac's equation in the form $\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0$, posit a free fermion $\psi = u\left(p^\mu\right)e^{-ip_\sigma x^\sigma}$ thus $i\partial_\mu\psi = p_\mu\psi$, and then cast Dirac's equation into the spinor form $\left(ip_\mu - m\right)u = 0$. With $\gamma^\mu = \left(\gamma^0, \gamma^0\alpha^1, \gamma^0\alpha^2, \gamma^0\alpha^3\right)$ this is then recast into $Hu \equiv \left(\alpha^i p^i + \mu\gamma^0\right)u = Eu$, where $E = p^0$ represents the eigenvalues of Hamiltonian H. The spinors are then derived as four independent solutions to this Dirac equation in Hamiltonian form. These spinors are well known, see, e.g., [16]. Importantly, each component of the spinor is now time and space *independent*, that is, the spinors are $u\left(p^{\mu}\right)$, *not* $u\left(p^{\mu},x^{\mu}\right)$. Any space dependence is segregated into the plane wavefunction $e^{-ip_{\sigma}x^{\sigma}}$ in $\psi=u\left(p^{\mu}\right)e^{-ip_{\sigma}x^{\sigma}}$, which is simply a "canonical Fourier kernel" containing the canonical momentum p_{σ}.

One simple way to deduce $v^{1.5}$ (the "vacua$^{1.5}$") is to contrast the relativistic energy relationship $m^2=p_{\sigma}p^{\sigma}=E^2-\mathbf{p}^2$ with the Lorentz-scalar definition $\mu^2\equiv\left(\mu-e\phi\right)^2-e\mathbf{A}^2$ of the μ parameter in (3.4). From the correspondence $m\Leftrightarrow\mu$, $E=\left(m+W\right)\Leftrightarrow\left(\mu-e\phi\right)$ and $\mathbf{p}\Leftrightarrow-\mathbf{A}$, we simply start with the usual canonical Dirac spinors derived from $\left(\alpha^i p^i+\mu\gamma^0\right)u=Eu$ in the Dirac representation (again, e.g., [16]), and make the corresponding substitutions to find the four distinct Dirac spinors which can be made to yield (3.19). These correspond to particle and antiparticle, each with spin up and spin down. In this way we find two particle vacua$^{1.5}$:

$$v_1^{1.5}=\frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\begin{bmatrix}1\\0\\\frac{\phi A^3}{\mathbf{A}^2}\\\frac{\phi\left(A^1+iA^2\right)}{\mathbf{A}^2}\end{bmatrix};\quad v_2^{1.5}=\frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\begin{bmatrix}0\\1\\\frac{\phi\left(A^1-iA^2\right)}{\mathbf{A}^2}\\\frac{-\phi A^3}{\mathbf{A}^2}\end{bmatrix}. \tag{4.1}$$

and two antiparticle vacua$^{1.5}$:

$$v_3^{1.5}=\frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\begin{bmatrix}\frac{\phi A^3}{\mathbf{A}^2}\\\frac{\phi\left(A^1+iA^2\right)}{\mathbf{A}^2}\\1\\0\end{bmatrix};\quad v_4^{1.5}=\frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\begin{bmatrix}\frac{\phi\left(A^1-iA^2\right)}{\mathbf{A}^2}\\\frac{-\phi A^3}{\mathbf{A}^2}\\0\\1\end{bmatrix} \tag{4.2}$$

which are both designed *such that they reproduce (3.19) by definition*, as we now show:

Starting with the particle vacua, using $v_1^{1.5}$ as an example, we form $\overline{v^{1.5}}\equiv v^{1.5\dagger}\gamma^0$ with $\mathrm{diag}\left(\gamma^0\right)=\left(1,1,-1,-1\right)$ in the Dirac representation to find that:

$$v_1^{\ 3} = \overline{v_1^{1.5}} v_1^{1.5} = v_1^{1.5\dagger}\gamma^0 v_1^{1.5} = \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi}\begin{bmatrix} 1 & 0 & -\frac{\phi A^3}{\mathbf{A}^2} & -\frac{\phi\left(A^1 - iA^2\right)}{\mathbf{A}^2}\end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ \frac{\phi A^3}{\mathbf{A}^2} \\ \frac{\phi\left(A^1 + iA^2\right)}{\mathbf{A}^2}\end{bmatrix} \tag{4.3}$$

$$= \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi}\left(1 - \frac{\phi^2 \mathbf{A}^2}{\mathbf{A}^2\mathbf{A}^2}\right) = \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi}\left(\frac{\mathbf{A}^2\mathbf{A}^2 - \phi^2\mathbf{A}^2}{\mathbf{A}^2\mathbf{A}^2}\right) = \frac{1}{4}\frac{e}{\lambda_f}\left(\frac{\mathbf{A}^2 - \phi^2}{\phi}\right) = \frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right) > 0$$

As we can see, this faithfully reproduces (3.19) with $v_1^{\ 3} > 0$ as in (3.17), and therefore, $v_1 > 0$ as in (3.18). A like-calculation using the spin-down $v_2^{1.5}$ in (4.1) will do the same.

Now let's look at the antiparticle spinors, for which we use $v_3^{1.5}$ of (4.2) as an example with $v_4^{1.5}$ yielding a like result. The some calculation now leads us to:

$$v_3^{\ 3} = \overline{v_1^{1.5}} v_3^{1.5} = v_3^{1.5\dagger}\gamma^0 v_3^{1.5} = \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi}\begin{bmatrix} \frac{\phi A^3}{\mathbf{A}^2} & \frac{\phi\left(A^1 - iA^2\right)}{\mathbf{A}^2} & -1 & 0\end{bmatrix}\begin{bmatrix} \frac{\phi A^3}{\mathbf{A}^2} \\ \frac{\phi\left(A^1 + iA^2\right)}{\mathbf{A}^2} \\ 1 \\ 0\end{bmatrix} \tag{4.4}$$

$$= \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi}\left(\frac{\phi^2 \mathbf{A}^2}{\mathbf{A}^2\mathbf{A}^2} - 1\right) = \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi}\left(\frac{\phi^2\mathbf{A}^2 - \mathbf{A}^2\mathbf{A}^2}{\mathbf{A}^2\mathbf{A}^2}\right) = \frac{1}{4}\frac{e}{\lambda_f}\left(\frac{\phi^2 - \mathbf{A}^2}{\phi}\right) = -\frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right) < 0$$

Contrasting with (3.17), we see that here $v_3^{\ 3} < 0$, which implies that the *antiparticle* vacuum $v_3 < 0$. This now answers the third question as to whether the $\pm v$ freedom which we reviewed in section 2 for scalars, appears for fermions in some other guise. The answer: this twofold $\pm v$ does reappear for fermions, *but it is not a freedom that we can freely choose*. We are not free to choose to expand about $\pm v$ for fermions. Rather, positive energy fermions are *forced* to expand about the $+v$ vacuum and negative energy fermions (antiparticles when Feynman- Stückelberg is applied) are *forced* to expand about the $-v$ vacuum. In both (4.3) and (4.4) we maintain the requirement $\frac{1}{2} e\left(\mathbf{A}^2 / \phi - \phi\right) > 0$ of (3.15), (3.16) so we are always expanding – whether for particles or antiparticles – about a *minimum* of the potential and thus have a stable vacuum for expansion. But the choice of a $\pm v$ expansion that we have for scalars is no longer a choice for fermions. Particle fermions always expand about $+v$ and antiparticle fermions always expand about $-v$.

This is also retrospectively suggestive as will be expanded upon momentarily that (2.24) is the symmetry breaking choice that leads to a Higgs scalar *particle*, while (2.25) is the one that leads to a Higgs scalar *antiparticle*. This is why we took some pains to show the impact of the $\phi_{1\min} = \pm v$ choices in the two differently appearing Lagrangian densities (2.24) and (2.25): what we see in (4.3) and (4.4) is the first sign of a fermion analogue to the $\phi_{1\min} = \pm v$ choice in the scalar Higgs theory reviewed in section 2.

Specifically, and fundamentally, in (4.3) and (4.4) we see that this sign flips of the negative energy vacuum entirely originate in $\mathrm{diag}\left(\gamma^0\right) = (1,1,-1,-1)$ which is at the core of particle and antiparticle eigenstates in Dirac theory. So here we see Dirac theory at work once again creating a symmetry between particles and antiparticles – this time, between the particle and antiparticle vacua! This means that we must go back to (3.18) and amend this as follows:

$$v = \sqrt[3]{\frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right)} > 0 \text{ for fermions;} \quad v = -\sqrt[3]{\frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right)} < 0 \text{ for antifermions} \tag{4.5}$$

To highlight how the Dirac γ^0 is at the root of this, which is seen by contrasting (4.3) and (4.4), if we write $\mathrm{diag}\left(\gamma^0\right) = (1,-1)$ in two-component form, the sign difference in (4.5) is directly traceable to $\mathrm{diag}\left(\gamma^0\right) = (1,-1)$. This is the fermion counterpart to the $\pm v$ vacua that we reviewed in section 2 for the scalar fields. So this correspondence between the scalar and the fermion Higgs theories as regards the vacuum may be summarized by $\sqrt{v^2} = \pm v \Leftrightarrow \mathrm{diag}\left(\gamma^0\right) = (1,-1)$, and is laid right at the doorstep of Dirac's γ^0.

Mathematically, we can attribute this in part to the fact that the scalars ϕ reviewed in section 2 have a mass dimension $D = +1$ and so yielded a v^2 in (2.10), while the fermions presently under review have a mass dimension $D = +1.5$ and so yielded a v^3 in (3.16). Taking a square root of a positive number $\sqrt{+v^2} = \pm v$ always presents a choice of sign and this carried over to the very final Lagrangians (2.24), (2.25). But for fermions with $D = +1.5$ we are endemically forced to the cubic level, not the square level. Here, taking the cubed root of a positive number $\sqrt{+v^3} = +v$ *always* yields a positive number and while taking $\sqrt{-v^3} = -v$ *always* yields a negative number. So the *choice* of $\pm v$ that we earlier had in (2.11) is absent when dealing with fermions, and instead it reemerges in different guise via (4.3) and (4.3). Now, we find that $v_1, v_2 > 0$ and $v_3, v_4 < 0$, which replaces the $\pm v$ choice in (2.11). *Dirac fields remove this choice and assign a $v > 0$ to fermions and a $v < 0$ to antifermions without giving a choice.* This all originates in (3.14) and (3.16) where we ensured that the vacuum has an energetically-stable minimum point for expansion. If we require a stable minimum for both fermions and antifermions, then fermions and antifermions must have oppositely-signed vacua, with a positive minimum for fermions and a negative minimum for antifermions.

Physically, this also suggests that one way to view the choice of (2.24), (2.25) is to view (2.24) as the Lagrangian for an observable Higgs scalar particle, and (2.25) as that for an observable Higgs scalar *antiparticle*. The hidden, broken symmetry as between (2.24), (2.25) is then seen as a broken symmetry between particles and antiparticles. As we shall soon see in (5.9) and (5.10) infra, (4.3) and (4.3) similarly break the particle and antiparticle symmetry for fermions. Further, if we keep in mind Feynman-Stückelberg [17] whereby a past-oriented negative energy particle is reinterpreted as a future-oriented positive energy antiparticle, then (4.3) forces a positive energy vacuum for positive energy fermions travelling forward in time, and (4.4) forces a negative energy vacuum for negative energy fermions travelling backwards in time. In this light, (4.4) and (4.3) are yet another example of the matter / antimatter symmetries which are endemic to Dirac's equation.

Although less visible on the surface than these particle / antiparticle results, the other point which needs to be made about the vacuum spinors (4.1) and (4.2) is that these spinors – just like the usual spinors derived from $\left(\alpha^i p^i + \mu\gamma^0\right)u = Eu$ – must be functions $v^{1.5}\left(p^\mu\right)$ only of the energy-momentum, and *not* $v^{1.5}\left(p^\mu, x^\mu\right)$ of spacetime. Because (4.1) and (4.2) also contain the gauge fields A^μ, an interesting paradox appears to emerge. Gauge fields of course are functions $A^\mu\left(p^\mu, x^\mu\right)$ of space and time. But as they appear in (4.1) and (4.2) they are not. So the question now arises: how do these gauge fields $A^\mu = \left(\phi, A^1, A^2, A^3\right)$ acquire their spacetime dependency if they are not spacetime-dependent in (4.1) and (4.2)?

The answer, which we shall develop gradually in the subsequent discussion at the suitable junctures, is that once the gauge fields make their way into a spinor as in (4.1) and (4.2), they must be regarded as Hermitian linear operator observables in the "Heisenberg picture" sense, *but with no explicit, inherent space or time dependency*. Using X and P to represent the Heisenberg space and momentum matrices which are Hermitian and which follow the canonical commutation $\left[P, X\right] = -i\hbar = -i$, these gauge fields continue to be regarded as $A^\mu\left(P, X\right)$ in the sense of matrix mechanics, but not $A^\mu\left(p^\mu, x^\mu\right)$ with p^μ as a classical momentum and x^μ as classical spacetime coordinates. Then, in order to regain a classical spatial dependency, these $A^\mu\left(P, X\right)$ operators must undergo a commutation $\left[P^i, A^\mu\right] = -i\partial^i A^\mu$ with a three-momentum. And in order to regain a time-dependency, they must acquire a time evolution via a commutation $\left[H, A^\mu\right] = -i\partial^0 A^\mu$ with the Hamiltonian via the Heisenberg equation of motion. This is a subtle but very important point that will play a role throughout the subsequent development and which will end up revealing a very profound connection among Dirac's equation, the Heisenberg commutation relationships (both canonical with momentum and for motion with the Hamiltonian), and Maxwell's equations for which the time and space dependencies $\nabla \cdot \mathbf{E}$, $\nabla \times \mathbf{E}$, $\nabla \cdot \mathbf{B}$, $\nabla \times \mathbf{B}$, $\partial \mathbf{E} / \partial t$ and $\partial \mathbf{B} / \partial t$ are essential defining features. Again, we seek at the moment simply to draw attention to this. But its import and sweep is best developed not abstractly, but incrementally along the way in specific circumstances as we uncover various commutators which will be converted over into spacetime dependencies.

Now we turn to explicitly represent the Dirac spinor wavefunctions for each of the four Higgs fermion states $h_f\left(x\right)$ in the expansion (3.20). But before we can do so we will need to also specify ψ, because according to (3.20) the general form for $h_f\left(x\right)$ is the *difference*:

$$h_f = \psi - v^{1.5}. \tag{4.6}$$

These ψ should be the usual spinors of Dirac theory, but with two differences: First, because the Lagrangian (3.8) which we are developing is $\mathfrak{L} = \overline{\psi}\left(i\not{D} - \mu\right)\psi - \overline{\psi}\left(i\not{\partial} + e\not{A} - \mu\right)\psi$ with the mass parameter μ and not the usual $\mathfrak{L} = \overline{\psi}\left(i\not{\partial} + e\not{A} - m\right)\psi$ with a hand-added fermion mass *m* (a fundamental goal of this paper is to *reveal* such a mass rather than add it by hand), we may use the usual Dirac spinors but with *m* replaced throughout by μ. Second, because $\mathfrak{L} = \overline{\psi}\left(i\not{D} - \mu\right)\psi$ these spinors will contain the kinetic momentum $p_\sigma \to \pi_\sigma \equiv p_\sigma + eA_\sigma$ rather than the canonical momentum alone. Thus, in the Dirac representation (again, see e.g., [16]) and employing the kinetic momentum $p_\sigma \to \pi_\sigma \equiv p_\sigma + eA_\sigma$ to properly place the gauge fields into the spinors, for fermions with plane wavefunctions we may write:

$$\psi_1 = \mu\sqrt{E+\mu+e\phi}\begin{bmatrix} 1 \\ 0 \\ \dfrac{p^3+eA^3}{E+\mu+e\phi} \\ \dfrac{p^1+eA^1+ip^2+ieA^2}{E+\mu+e\phi}\end{bmatrix} e^{-i\pi_\sigma x^\sigma}; \quad \psi_2 = \mu\sqrt{E+\mu+e\phi}\begin{bmatrix} 0 \\ 1 \\ \dfrac{p^1+eA^1-ip^2-ieA^2}{E+\mu+e\phi} \\ \dfrac{-p^3-eA^3}{E+\mu+e\phi}\end{bmatrix} e^{-i\pi_\sigma x^\sigma}. \tag{4.7}$$

Likewise, for antifermions:

$$\psi_3 = \mu\sqrt{E+\mu+e\phi}\begin{bmatrix} \dfrac{p^3+eA^3}{E+\mu+e\phi} \\ \dfrac{p^1+eA^1+ip^2+ieA^2}{E+\mu+e\phi} \\ 1 \\ 0\end{bmatrix} e^{-i\pi_\sigma x^\sigma}; \quad \psi_4 = \mu\sqrt{E+\mu+e\phi}\begin{bmatrix} \dfrac{p^1+eA^1-ip^2-ieA^2}{E+\mu+e\phi} \\ \dfrac{-p^3-eA^3}{E+\mu+e\phi} \\ 0 \\ 1\end{bmatrix} e^{-i\pi_\sigma x^\sigma}. \tag{4.8}$$

Two other things should be noted above. First, we are employing the normalization $\mu\sqrt{E+\mu+e\phi}$ rather than the usual $\sqrt{E+m+e\phi}$. Aside from the $m \to \mu$ replacement just discussed, this is because we wish to ensure that the $D = +1.5$ dimensionality of these wavefunctions is explicitly represented, because we will be using these in a Lagrangian density $\mathfrak{L}$ which must have an overall mass dimension $D = 4$ in four-dimensional spacetime.

Second, rather than use the canonical Fourier kernel $e^{-ip_\sigma x^\sigma}$, we employ a *kinetic kernel* $e^{-i\pi_\sigma x^\sigma}$ containing $p_\sigma \to \pi_\sigma \equiv p_\sigma + eA_\sigma$. We do this mindful of the geometric view of gauge theory discussed following (2.1) wherein the gauge-covariant derivative $\partial_\sigma \to D_\sigma \equiv \partial_\sigma - ieA_\sigma$

represents curvature in gauge space just as $\partial_\mu A_\nu \to \partial_{;\mu} A_\nu = \partial_\mu A_\nu - \Gamma^\sigma{}_{\mu\nu} A_\sigma$ in represents curvature in spacetime. Free fermions are those which follow geodesic paths *in the gauge space*, so when the Dirac's equation is $(i\partial\!\!\!/ - \mu)\psi = 0$ the geodesically-free fermion spinors are solutions based on $\psi(x) = u(p)e^{-ip_\sigma x^\sigma}$ but when Dirac's equation is $(iD\!\!\!\!/ - \mu)\psi = 0$ the free fermion spinors are solutions based on $\psi(x) = u(\pi)e^{-i\pi_\sigma x^\sigma}$. However, because we are using a *kinetic* rather than *canonical* kernel, we have $i\partial_\mu \psi = \pi_\mu \psi$ not $i\partial_\mu \psi = p_\mu \psi$, and for these derivatives to work properly, the gauge field A_σ in $\pi_\sigma \equiv p_\sigma + eA_\sigma$, *must not have an explicit time or space dependence*, i.e., it must be a function of $A^\mu(P, X)$ where *P* and *X* are the Heisenberg matrix operators and not ordinary momenta or ordinary space coordinates. Just Higgs theory teaches that as the observed particles and fields in the physical universe are not those that appear as seed particles in a Lagrangian or Hamiltonian but rather are those Higgs fields which arise via expansion about the vacuum, so too we shall soon see that *the space and time dependencies which we observe in the physical universe are not those which appear in a Lagrangian such as* $\mathfrak{L} = \overline{\psi}(i\partial\!\!\!/ + eA\!\!\!/ - \mu)\psi$ *but rather are those space and time dependencies that arise out of quantum mechanical Heisenberg commutations*. That is, just like observed particle masses, *observed space and time dependencies are also hidden, revealed phenomena, not ab initio phenomena.* We will start to see specific examples of this in section 7 and through the later development here.

Now, via (4.1), (4.2), (4.7) and (4.8) we can explicitly represent each of the four Higgs fermions, by subtracting the vacuum spinor from the seed spinor with the kinetic kernel $e^{-i\pi_\sigma x^\sigma}$ used as the overall coefficient, as such:

$$h_{f1} = \psi_1 - v_1^{1.5} e^{i\pi_\sigma x^\sigma} = \left(\mu\sqrt{E+\mu+e\phi} \begin{bmatrix} 1 \\ 0 \\ \dfrac{p^3 + eA^3}{E+\mu+e\phi} \\ \dfrac{p^1 + eA^1 + ip^2 + ieA^2}{E+\mu+e\phi} \end{bmatrix} - \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} 1 \\ 0 \\ \dfrac{\phi A^3}{\mathbf{A}^2} \\ \dfrac{\phi\left(A^1 + iA^2\right)}{\mathbf{A}^2} \end{bmatrix} \right) e^{i\pi_\sigma x^\sigma} . \quad (4.9)$$

$$h_{f2} = \psi_2 - v_2^{1.5} e^{i\pi_\sigma x^\sigma} = \left(\mu\sqrt{E+\mu+e\phi} \begin{bmatrix} 0 \\ 1 \\ \dfrac{p^1 + eA^1 - ip^2 - ieA^2}{E+\mu+e\phi} \\ \dfrac{-p^3 - eA^3}{E+\mu+e\phi} \end{bmatrix} - \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} 0 \\ 1 \\ \dfrac{\phi\left(A^1 - iA^2\right)}{\mathbf{A}^2} \\ \dfrac{-\phi A^3}{\mathbf{A}^2} \end{bmatrix} \right) e^{-i\pi_\sigma x^\sigma} . \quad (4.10)$$

$$h_{f3}=\psi_3-v_3^{1.5}e^{i\pi_\sigma x^\sigma}=\left(\mu\sqrt{E+\mu+e\phi}\begin{bmatrix}\frac{p^3+eA^3}{E+\mu+e\phi}\\ \frac{p^1+eA^1+ip^2+ieA^2}{E+\mu+e\phi}\\ 1\\ 0\end{bmatrix}-\frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\begin{bmatrix}\frac{\phi A^3}{\mathbf{A}^2}\\ \frac{\phi\left(A^1+iA^2\right)}{\mathbf{A}^2}\\ 1\\ 0\end{bmatrix}\right)e^{-i\pi_\sigma x^\sigma}\ .\text{(4.11)}$$

$$h_{f4}=\psi_4-v_4^{1.5}e^{i\pi_\sigma x^\sigma}=\left(\mu\sqrt{E+\mu+e\phi}\begin{bmatrix}\frac{p^1+eA^1-ip^2-ieA^2}{E+\mu+e\phi}\\ \frac{-p^3-eA^3}{E+\mu+e\phi}\\ 0\\ 1\end{bmatrix}-\frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\begin{bmatrix}\frac{\phi\left(A^1-iA^2\right)}{\mathbf{A}^2}\\ \frac{-\phi A^3}{\mathbf{A}^2}\\ 0\\ 1\end{bmatrix}\right)e^{-i\pi_\sigma x^\sigma}\ .\text{(4.12)}$$

Note one other change in (4.9) through (4.12) which is motivated by needing to properly subtract the spinors in (4.1) and (4.2) from those in (4.7) and (4.8) to yield the spinors in (4.9) to (4.12): We have now modified the Higgs expansions for fermions from the presumed form (3.20) and (3.21), into a form which is motivated by the explicit spinors, namely:

$$\psi(x)=ue^{i\pi_\sigma x^\sigma}\equiv v^{1.5}e^{i\pi_\sigma x^\sigma}+h_f(x)=\left(v^{1.5}+u_h\right)e^{i\pi_\sigma x^\sigma}, \tag{4.13}$$

$$\overline{\psi}(x)=\overline{u}e^{-i\pi_\sigma x^\sigma}\equiv\overline{v^{1.5}}e^{-i\pi_\sigma x^\sigma}+\overline{h_f}(x)=\left(\overline{v^{1.5}}+\overline{u_h}\right)e^{-i\pi_\sigma x^\sigma}, \tag{4.14}$$

with $h_f(x)\equiv u_h(\pi)e^{i\pi_\sigma x^\sigma}$, $\overline{h_f}(x)\equiv\overline{u_h}(\pi)e^{-i\pi_\sigma x^\sigma}$ defining Higgs Dirac spinors $u_h(\pi)$, $\overline{u_h}(\pi)$ such that:

$$u(\pi)\equiv v^{1.5}+u_h(\pi), \tag{4.15}$$

$$\overline{u}(\pi)\equiv\overline{v^{1.5}}+\overline{u_h}(\pi). \tag{4.16}$$

This is the fully-developed fermion counterpart to the scalar expansion about the vacuum of (2.12), namely, $\phi(x)\equiv\frac{1}{\sqrt{2}}\left(\pm v+h(x)+i\xi(x)\right)$. We shall, however, prefer to work with the full wavefunctions $\psi(x)$ and $h_f(x)$ and not only the spinors as much as possible throughout the development. Noting that Fourier kernels generally cancel out from terms in a Lagrangian, e.g., $\overline{\psi}\psi=\overline{u}u$, as a notational convenience we define $v'^{1.5}(x)\equiv v^{1.5}e^{i\pi_\sigma x^\sigma}$ and $\overline{v'^{1.5}}(x)\equiv\overline{v^{1.5}}e^{-i\pi_\sigma x^\sigma}$ to be "vacua-prime" which absorb Fourier kernels into their definition. Then, $\overline{v'^{1.5}}v'^{1.5}=\overline{v^{1.5}}v^{1.5}=v^3$ give another example of how the kernels cancel in the Lagrangian. The only place these vacua-prime will appear in the Lagrangian will be in terms of the form $\overline{v'^{1.5}}h_f$ and $\overline{h_f}v'^{1.5}$, to maintain a proper balance and thus cancellation of the kernels in both $v'^{1.5}$ and h_f. This notation allows us to rewrite (4.13) and (4.14) as:

$$\psi(x) = v'^{1.5}(x) + h_f(x), \tag{4.17}$$

$$\overline{\psi}(x) = \overline{v'^{1.5}}(x) + \overline{h_f}(x). \tag{4.18}$$

This will be our preferred form to represent fermion Higgs field expansion about the vacuum.

At this point, we turn back to the development of the fermion Lagrangian density (3.9) and its $T(\psi)$ and $V(\psi)$ of (3.10) and (3.11) using the expansions (4.17), (4.18). Although these four-component $v^{1.5}$ of (4.1), (4.2), ψ of (4.7), (4.8) and observable Higgs fermions h_f of (4.9) through (4.12) are good for explicitly illustrating the development here, they are unwieldy for doing reasonably-tractable calculation. For this purpose, it will be preferable to consolidate (4.1), (4.2) and (4.7), (4.8) from four-component down to two-component Dirac form. We now denote by $v_+^{1.5} \equiv v_1^{1.5} = v_2^{1.5} > 0$ the positive energy vacuum used for expansion of positive energy (particle) fermions and by $v_-^{1.5} \equiv v_3^{1.5} = v_3^{1.5} < 0$ the negative energy vacuum with pairs up with negative energy (antiparticle) fermions. Therefore, (4.1) and (4.2) respectively consolidate in two-component form to:

$$v_+^{1.5} = \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\phi \sigma^i A^i}{\mathbf{A}^2} \chi^{(s)} \end{bmatrix}; \quad v_-^{1.5} = \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \dfrac{\phi \sigma^i A^i}{\mathbf{A}^2} \chi^{(s)} \\ \chi^{(s)} \end{bmatrix} \tag{4.19}$$

where, in the usual way, "up" and "down" spin states are respectively represented by:

$$\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4.20}$$

For the positive and negative energy "seed fermions" themselves, and also consolidating via the kinetic momentum $\pi_\sigma \equiv p_\sigma + eA_\sigma$, (4.7) and (4.8) consolidate to:

$$\psi = u e^{-i\pi_\sigma x^\sigma} = \mu\sqrt{\pi^0 + \mu} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\sigma^i \pi^i}{\pi^0 + \mu} \chi^{(s)} \end{bmatrix} e^{-i\pi_\sigma x^\sigma}; \quad \varphi = v e^{-i\pi_\sigma x^\sigma} = \mu\sqrt{\pi^0 + \mu} \begin{bmatrix} \dfrac{\sigma^i \pi^i}{\pi^0 + \mu} \chi^{(s)} \\ \chi^{(s)} \end{bmatrix} e^{-i\pi_\sigma x^\sigma} . \tag{4.21}$$

For notational distinction, we now use φ to denote the wavefunction for a seed antifermion and v for its associated spinor. The above are the two-component spinors solutions which emerge from Dirac's equation with gauge fields and a kinetic kernel $e^{-i\pi_\sigma x^\sigma}$. Upon application of Feynman-Stückelberg one further substitutes $\pi_\sigma \to -\pi_\sigma$ and $-E \to |E|$ as applicable. We shall not do this here, however, because the calculation is simpler if we retain this original form of negative energy fermions. One continues to keep in mind that the gauge fields A^μ such as they appear in the vacuum spinors (4.19) and via $\pi^\mu = p^\mu + eA^\mu$ in (4.21) are to be regarded as neither time-dependent nor space-dependent, but to the degree to which they are later found (as they will be) to commute with a Hamiltonian H to yield a time dependency, or with a three-momentum p^k to yield a space dependency. That is, these Dirac spinors are spinors in the sense of the Heisenberg picture, wherein the fields are not time or space-dependent *ab initio*, but only become so because of their commutation with the Hamiltonian and /or with a three- momentum.

Therefore, the observable Higgs fermions spinors corresponding to (4.9), (4.10) which we denote by $h_{f1,2} \to h_\psi$ consolidate in two-component form to:

$$h_\psi = \psi - v'^{1.5}_- = \left(\mu\sqrt{\pi^0+\mu} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\sigma^i \pi^i}{\pi^0+\mu}\chi^{(s)} \end{bmatrix} - \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\phi\sigma^i A^i}{\mathbf{A}^2}\chi^{(s)} \end{bmatrix} \right) e^{-i\pi_\sigma x^\sigma} \tag{4.22}$$

while the Higgs antifermions corresponding to (4.11), (4.12) which we denote by $h_{f3,4} \to h_\varphi$ consolidate to:

$$h_\varphi = \varphi - v'^{1.5}_+ = \left(\mu\sqrt{\pi^0+\mu} \begin{bmatrix} \dfrac{\sigma^i \pi^i}{\pi^0+\mu}\chi^{(s)} \\ \chi^{(s)} \end{bmatrix} - \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \dfrac{\phi\sigma^i A^i}{\mathbf{A}^2}\chi^{(s)} \\ \chi^{(s)} \end{bmatrix} \right) e^{-i\pi_\sigma x^\sigma} . \tag{4.23}$$

Before concluding this section, it is instructive to show the calculation (4.3) for the fermion vacuum using the two-component spinor $v_+^{\,1.5}$ of (4.19). Making use of the Hermitian conjugacy relation $\sigma^{i\dagger} = \sigma^i$ as well as $\chi^{(s)T}\chi^{(s)} = 1$, and $\sigma^i A^i \sigma^j A^j = \mathbf{A}^2$, the two-component form of the calculation (4.3) is:

$$\begin{aligned} v_+^{\,3} &= \overline{v_+^{\,1.5}} v_+^{\,1.5} = v_+^{\,1.5\dagger}\gamma^0 v_+^{\,1.5} = \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi} \begin{bmatrix} \chi^{(s)T} & -\chi^{(s)T}\dfrac{\phi\sigma^i A^i}{\mathbf{A}^2} \end{bmatrix} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \end{bmatrix} \\ &= \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\left(\chi^{(s)T}\chi^{(s)} - \chi^{(s)T}\frac{\phi\sigma^i A^i}{\mathbf{A}^2}\frac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \right) = \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\left(\chi^{(s)T}\chi^{(s)} - \chi^{(s)T}\frac{\phi^2\mathbf{A}^2}{\mathbf{A}^2\mathbf{A}^2}\chi^{(s)} \right) \\ &= \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\left(1 - \frac{\phi^2\mathbf{A}^2}{\mathbf{A}^2\mathbf{A}^2} \right) = \frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2}{\phi} - \phi \right) = \frac{1}{4\lambda_f} e\left(\frac{\mathbf{A}^2 - \phi^2}{\phi} \right) > 0 \end{aligned} \tag{4.24}$$

Later, we shall find it necessary to do a number of more complicated calculations built out of this basic form, which is why we show this now.

5. Fermion Field Expansion of the Lagrangian Potential

With all of this, we are now ready to use the expansions (4.17) and (4.18) about the vacua (4.1), (4.2) in the fermion Lagrangian density (3.9) via its kinetic (*T*) and potential (*V*) terms in (3.10) and (3.11). From (3.10), the kinetic portion $T(\psi)$ of the Lagrangian density with $i\partial \rightarrow \not{p}$ and $\not{\pi} = \not{p} + e\not{A}$ is:

$$\begin{aligned} T(\psi) &= \overline{\psi}(i\not{\partial} + e\not{A})\psi = \overline{\psi}(\not{p} + e\not{A})\psi = \overline{\psi}\not{\pi}\psi \\ &= \overline{h_f}\not{\pi}h_f + \overline{v'^{1.5}}\not{\pi}h_f + \overline{h_f}\not{\pi}v'^{1.5} + \overline{v^{1.5}}\not{\pi}v^{1.5} \end{aligned}. \tag{5.1}$$

For the potential portion $V(\psi)$ we must account for the findings in (4.3) and (4.4) as summarized in (4.5) that $v > 0$ for fermions and $v < 0$ for antifermions. For fermions with ${v_+}^3 = {v_1}^3 = {v_2}^3$, starting with (3.11), and using $-2\lambda_f {v_+}^3 = \frac{1}{2}e\left(\phi - \mathbf{A}^2/\phi\right)$ from (4.24), we have:

$$V(\psi) = \frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right)\overline{\psi}\psi + \lambda_f\left(\overline{\psi}\psi\right)^2 = -2\lambda_f {v_+}^3\overline{\psi}\psi + \lambda_f\left(\overline{\psi}\psi\right)^2 = \lambda_f\left(-2{v_+}^3\overline{\psi}\psi + \left(\overline{\psi}\psi\right)^2\right). \tag{5.2}$$

For antifermions with ${v_-}^3 = {v_3}^3 = {v_4}^3$ we also start with (3.11), but use $2\lambda_f {v_-}^3 = \frac{1}{2}e\left(\phi - \mathbf{A}^2/\phi\right)$ from (4.4) ((4.24) with reversed sign $-2{v_+}^3 \rightarrow +2{v_-}^3$ for antiparticles) to find a slightly different:

$$V(\psi) = \frac{1}{2}e\left(\phi - \frac{\mathbf{A}^2}{\phi}\right)\overline{\psi}\psi + \lambda_f\left(\overline{\psi}\psi\right)^2 = +2\lambda_f {v_-}^3\overline{\psi}\psi + \lambda_f\left(\overline{\psi}\psi\right)^2 = \lambda_f\left(+2{v_-}^3\overline{\psi}\psi + \left(\overline{\psi}\psi\right)^2\right). \tag{5.3}$$

Now we expand about the vacuum. First, via (4.17) and (4.18), it helps to form (again, $v'^{1.5}(x) \equiv v^{1.5}e^{i\pi_\sigma x^\sigma}$ and $\overline{v'^{1.5}}(x) \equiv \overline{v^{1.5}}e^{-i\pi_\sigma x^\sigma}$ to balance kernels):

$$\overline{\psi}\psi = \overline{h_f}h_f + \overline{v'^{1.5}}h_f + \overline{h_f}v'^{1.5} + \overline{v^{1.5}}v^{1.5}, \tag{5.4}$$

and then use (5.4) above to form:

$$\overline{\psi}\psi\overline{\psi}\psi = \begin{pmatrix} +\overline{h_f}h_f\overline{h_f}h_f + \overline{h_f}h_f\overline{v'^{1.5}}h_f + \overline{h_f}h_f\overline{h_f}v'^{1.5} + \overline{h_f}h_f\overline{v^{1.5}}v^{1.5} \\ +\overline{v'^{1.5}}h_f\overline{h_f}h_f + \overline{v'^{1.5}}h_f\overline{v'^{1.5}}h_f + \overline{v'^{1.5}}h_f\overline{h_f}v'^{1.5} + \overline{v'^{1.5}}h_f\overline{v^{1.5}}v^{1.5} \\ +\overline{h_f}v'^{1.5}\overline{h_f}h_f + \overline{h_f}v'^{1.5}\overline{v'^{1.5}}h_f + \overline{h_f}v'^{1.5}\overline{h_f}v'^{1.5} + \overline{h_f}v'^{1.5}\overline{v^{1.5}}v^{1.5} \\ +\overline{v^{1.5}}v^{1.5}\overline{h_f}h_f + \overline{v^{1.5}}v^{1.5}\overline{v'^{1.5}}h_f + \overline{v^{1.5}}v^{1.5}\overline{h_f}v'^{1.5} + \overline{v^{1.5}}v^{1.5}\overline{v^{1.5}}v^{1.5} \end{pmatrix}. \tag{5.5}$$

This contains sixteen (16) products of a scalar number $\overline{h_f}h_f$, $\overline{v'^{1.5}}h_f$, $\overline{h_f}v'^{1.5}$ or $\overline{v^{1.5}}v^{1.5} = v^3$ with another like scalar number. Any one of these scalars may be commuted with any other, so we can immediately consolidate some of the terms in (5.5) as such:

$$\overline{\psi}\psi\overline{\psi}\psi = \begin{pmatrix} +\overline{h_f}h_f\overline{h_f}h_f + \overline{v'^{1.5}}h_f\overline{v'^{1.5}}h_f + \overline{h_f}v'^{1.5}\overline{h_f}v'^{1.5} + v^6 \\ +2\overline{v'^{1.5}}h_f\overline{h_f}h_f + 2\overline{h_f}v'^{1.5}\overline{h_f}h_f + 2v'^3\overline{h_f}h_f + 2\overline{h_f}v'^{1.5}\overline{v'^{1.5}}h_f + 2v^3\overline{v'^{1.5}}h_f + 2v^3\overline{h_f}v'^{1.5} \end{pmatrix}. \quad (5.6)$$

Then we use (5.4) and (5.6) in (5.2) for fermions with $h_f \rightarrow h_\psi$, $v^{1.5} \rightarrow {v_+}^{1.5}$ to obtain:

$$\begin{aligned} V(\psi) &= \lambda_f\left(-2\overline{v^{1.5}}v^{1.5}\overline{\psi}\psi + \overline{\psi}\psi\overline{\psi}\psi\right) \\ &= \lambda_f\begin{pmatrix} -2{v_+}^3\overline{h_\psi}h_\psi - 2{v_+}^3\overline{v_+'^{1.5}}h_\psi - 2{v_+}^3\overline{h_\psi}v_+'^{1.5} - 2{v_+}^6 \\ +\overline{h_\psi}h_\psi\overline{h_\psi}h_\psi + \overline{v_+'^{1.5}}h_\psi\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\overline{h_\psi}v_+'^{1.5} + {v_+}^6 \\ +2\overline{v_+'^{1.5}}h_\psi\overline{h_\psi}h_\psi + 2\overline{h_\psi}v_+'^{1.5}\overline{h_\psi}h_\psi + 2{v_+}^3\overline{h_\psi}h_\psi + 2\overline{h_\psi}v_+'^{1.5}\overline{{v_+}^{1.5}}h_\psi + 2{v_+}^3\overline{v_+'^{1.5}}h_\psi + 2{v_+}^3\overline{h_\psi}v_+'^{1.5} \end{pmatrix} \end{aligned} \quad (5.7)$$

and in (5.3) for antifermions with $h_f \rightarrow h_\varphi$, $v^{1.5} \rightarrow {v_-}^{1.5}$ to obtain:

$$\begin{aligned} V(\varphi) &= \lambda_f\left(+2\overline{v^{1.5}}v^{1.5}\overline{\psi}\psi + \overline{\psi}\psi\overline{\psi}\psi\right) \\ &= \lambda_f\begin{pmatrix} +2{v_-}^3\overline{h_\psi}h_\psi + 2{v_-}^3\overline{v_-'^{1.5}}h_\psi + 2{v_-}^3\overline{h_\psi}v_-'^{1.5} + 2{v_-}^6 \\ +\overline{h_\psi}h_\psi\overline{h_\psi}h_\psi + \overline{v_-'^{1.5}}h_\psi\overline{v_-'^{1.5}}h_\psi + \overline{h_\psi}v_-'^{1.5}\overline{h_\psi}v'^{1.5} + {v_-}^6 \\ +2\overline{v_-'^{1.5}}h_\psi\overline{h_\psi}h_\psi + 2\overline{h_\psi}v_-'^{1.5}\overline{h_\psi}h_\psi + 2{v_-}^3\overline{h_\psi}h_\psi + 2\overline{h_\psi}v_-'^{1.5}\overline{v_-'^{1.5}}h_\psi + 2{v_-}^3\overline{v_-'^{1.5}}h_\psi + 2{v_-}^3\overline{h_\psi}v_-'^{1.5} \end{pmatrix} \end{aligned} \quad (5.8)$$

We have shown this expansion explicitly to highlight the impact of the negative and positive vacua for fermions and antifermions. Now consolidating, (5.7) and (5.8) respectively become:

$$\begin{aligned} V(\psi) &= \lambda_f\left(-2\overline{v^{1.5}}v^{1.5}\overline{\psi}\psi + \overline{\psi}\psi\overline{\psi}\psi\right) \\ &= \lambda_f\begin{pmatrix} +\overline{v_+'^{1.5}}h_\psi\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\overline{h_\psi}v_+'^{1.5} + 2\overline{h_\psi}v_+'^{1.5}\overline{v_+'^{1.5}}h_\psi + 2\overline{v_+'^{1.5}}h_\psi\overline{h_\psi}h_\psi + 2\overline{h_\psi}v_+'^{1.5}\overline{h_\psi}h_\psi + \overline{h_\psi}h_\psi\overline{h_\psi}h_\psi \\ -{v_+}^6 \end{pmatrix} \end{aligned}, \quad (5.9)$$

$$\begin{aligned} V(\varphi) &= \lambda_f\left(+2\overline{v^{1.5}}v^{1.5}\overline{\psi}\psi + \overline{\psi}\psi\overline{\psi}\psi\right) \\ &= \lambda_f\begin{pmatrix} +\overline{v_-'^{1.5}}h_\varphi\overline{v_-'^{1.5}}h_\varphi + \overline{h_\varphi}v_-'^{1.5}\overline{h_\varphi}v_-'^{1.5} + 2\overline{h_\varphi}v_-'^{1.5}\overline{v_-'^{1.5}}h_\varphi + 2\overline{v_-'^{1.5}}h_\varphi\overline{h_\varphi}h_\varphi + 2\overline{h_\varphi}v_-'^{1.5}\overline{h_\varphi}h_\varphi + \overline{h_\varphi}h_\varphi\overline{h_\varphi}h_\varphi \\ +3{v_-}^6 + 4{v_-}^3\overline{v_-'^{1.5}}h_\varphi + 4{v_-}^3\overline{h_\varphi}v'^{1.5} + 4{v_-}^3\overline{h_\varphi}h_\varphi \end{pmatrix} \end{aligned}. \quad (5.10)$$

In the above we clearly see an inherent broken symmetry between fermions and antifermions. All of the terms in top line of the final expression in (5.9) and (5.10) are the same, but in the bottom lines, they are not. This should be contrasted to the analogous result (2.24) and (2.25) for scalars. This is where the Dirac version of the $\pm v$ symmetry breaking based on $\mathrm{diag}\left(\gamma^0\right)=\left(+1,-1\right)$ manifests most directly, because as we see, this breaks (or, hides) the symmetry between the particle and antiparticle potentials which is not hidden in the original "seed" potential $V\left(\psi\right)=\frac{1}{2}e\left(\phi-\mathbf{A}^2/\phi\right)\overline{\psi}\psi+\lambda_f\left(\overline{\psi}\psi\right)^2$ of (5.2).

Before proceeding to develop these further, it is useful to do one final consolidation of some terms in (5.9) and (5.10) above, including more scalar commutation, into the form:

$$\begin{aligned} V\left(\psi\right) &= \lambda_f\left(-2\overline{v^{1.5}}v^{1.5}\overline{\psi}\psi+\overline{\psi}\psi\overline{\psi}\psi\right) \\ &= \lambda_f\begin{pmatrix} +\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)+2\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)\overline{h_\psi}h_\psi+\overline{h_\psi}h_\psi\overline{h_\psi}h_\psi \\ -v_+{}^6 \end{pmatrix}, \end{aligned} \tag{5.11}$$

$$\begin{aligned} V\left(\varphi\right) &= \lambda_f\left(+2\overline{v^{1.5}}v^{1.5}\overline{\psi}\psi+\overline{\psi}\psi\overline{\psi}\psi\right) \\ &= \lambda_f\begin{pmatrix} +\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)+2\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)\overline{h_\varphi}h_\varphi+\overline{h_\varphi}h_\varphi\overline{h_\varphi}h_\varphi \\ +3v_-{}^6+4v_-{}^3\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)+4v_-{}^3\overline{h_\varphi}h_\varphi \end{pmatrix}. \end{aligned} \tag{5.12}$$

Finally we return to the Lagrangian density (3.9) which we now recognize will have two separate appearances for fermions versus antifermions. We use (5.1) and (5.11) to write the fermion (particle) Lagrangian density as:

$$\begin{aligned} \mathfrak{L}\left(\psi\right) &= \overline{\psi}\left(i\partial+eA\right)\psi-\frac{1}{2}e\phi\overline{\psi}\psi+\frac{1}{2}e\frac{\mathbf{A}^2}{\phi}\overline{\psi}\psi-\lambda_f\left(\overline{\psi}\psi\right)^2=T\left(\psi\right)-V\left(\psi\right) \\ &= \overline{h_\psi}\pi h_\psi+\overline{v_+'^{1.5}}\pi h_\psi+\overline{h_\psi}\pi v_+'^{1.5}+\overline{v_+{}^{1.5}}\pi v_+{}^{1.5} \\ &\quad -\lambda_f\begin{pmatrix} +\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)+2\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)\overline{h_\psi}h_\psi+\overline{h_\psi}h_\psi\overline{h_\psi}h_\psi \\ -v_+{}^6 \end{pmatrix} \end{aligned} \tag{5.13}$$

and (5.1) and (5.12) to write for antiparticles:

$$\mathfrak{L}(\varphi)=\overline{\psi}(i\partial+eA)\psi-\frac{1}{2}e\phi\overline{\psi}\psi+\frac{1}{2}e\frac{\mathbf{A}^2}{\phi}\overline{\psi}\psi-\lambda_f\left(\overline{\psi}\psi\right)^2=T(\psi)-V(\psi)$$

$$=\overline{h_\varphi}\pi h_\varphi+\overline{v_-'^{1.5}}\pi h_\varphi+\overline{h_\varphi}\pi v_-'^{1.5}+\overline{v_-^{1.5}}\pi v_-^{1.5}$$

$$-\lambda_f\begin{pmatrix}+\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)+2\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)\overline{h_\varphi}h_\varphi+\overline{h_\varphi}h_\varphi\overline{h_\varphi}h_\varphi\\+3v_-^{6}+4v_-^{3}\left(\overline{v_-'^{1.5}}h_\varphi+\overline{h_\varphi}v_-'^{1.5}\right)+4v_-^{3}\overline{h_\varphi}h_\varphi\end{pmatrix} \quad (5.14)$$

Now we have enough information to reveal a fermion rest mass. Comparing to the canonical Dirac Lagrangian $\mathfrak{L}=\overline{\psi}i\partial\psi-m\overline{\psi}\psi=\mathfrak{L}=\overline{\psi}p\psi-m\overline{\psi}\psi$, we seek to identify the coefficients of (5.13) and (5.14) against the template $\mathfrak{L}=\overline{h_f}\pi h_f-m_f\overline{h_f}h_f$ for the Higgs fields fermions $h_f=h_\psi$ in (5.13) and antifermions $h_f=h_\varphi$ in (5.14). That is, we specifically look for coefficients of terms $\overline{h_f}h_f$ which are second order in the Higgs field.

6. Revealing Fermion Rest Mass

From here we focus on the positive energy Higgs fermion particles. The fermion mass term we are seeking must have the form $-m_f\overline{h_f}h_f$. One of the terms in (5.13) is $-\lambda_f\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)^2$ which is of the desired second order in the Higgs field but with $v_+'^{1.5}$ interspersed in a way that is not trivially commuted. Another term is $-2\lambda_f\left(\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}\right)\overline{h_\psi}h_\psi$ which does have the desired $\overline{h_f}h_f$ but is of third order in the Higgs fields. To convert these into the form of $-m_f\overline{h_f}h_f$ we will need to do some explicit calculations along the lines of (4.24), using the $v_+^{1.5}$ spinor in (4.19) and the h_ψ spinors in (4.22).

First we calculate $\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}$ and use $v_+^{3}=\frac{1}{4}e\left(\mathbf{A}^2/\phi-\phi\right)/\lambda_f>0$ from (4.24). Using a photon momentum $q^\mu\equiv(p'-p)^\mu$ and keeping in mind that $\pi^\mu=p^\mu+eA^\mu$, we shall regard p^μ as the four-momentum of $h_\psi(p)=h_\psi(\pi)$ and $p'^\mu=p^\mu+q^\mu$ as the four-momentum of $\overline{h_\psi}(p')=\overline{h_\psi}(p+q)=\overline{h_\psi}(\pi')$ where $\pi'^\mu=p'^\mu+eA^\mu=p^\mu+q^\mu+eA^\mu$. Thus, we associate q^μ with the gauge field (e.g., photon) momentum vector of a transition current $J^\mu=\overline{\psi}(p')\Gamma^\mu\psi(p)$, see for example, [18] pages 343-345.

In a more complicated variant of the calculation (4.24), with $h_\psi(\pi)$ we may obtain:

$$
\begin{aligned}
\overline{v_+'^{1.5}} h_\psi &= \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \chi^{(s)T} & -\chi^{(s)T} \dfrac{\phi\sigma^i A^i}{\mathbf{A}^2} \end{bmatrix} \left(\mu\sqrt{\pi^0+\mu} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\sigma^j \pi^j}{\pi^0+\mu}\chi^{(s)} \end{bmatrix} - \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \end{bmatrix} \right) \\
&= \frac{1}{2}\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \left(1 - \chi^{(s)T} \frac{\phi\sigma^i A^i}{\mathbf{A}^2} \frac{\sigma^j \pi^j}{\pi^0+\mu}\chi^{(s)} \right) - \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi} \left(1 - \chi^{(s)T} \frac{\phi\sigma^i A^i}{\mathbf{A}^2} \frac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \right) \quad (6.1) \\
&= \frac{1}{2}\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} - \frac{1}{2}\frac{\mu}{\sqrt{\pi^0+\mu}}\sqrt{\frac{e}{\lambda_f \phi \mathbf{A}^2}} \chi^{(s)T} \left(\phi\sigma^i A^i\right)\left(\sigma^j \pi^j\right)\chi^{(s)} - v_+{}^3
\end{aligned}
$$

Similarly with $\overline{h_\psi}(\pi')$ we may obtain:

$$
\begin{aligned}
\overline{h_\psi} v_+'^{1.5} &= \left(\mu\sqrt{\pi'^0+\mu} \begin{bmatrix} \chi^{(s)T} & -\chi^{(s)T} \dfrac{\sigma^i \pi'^i}{\pi'^0+\mu} \end{bmatrix} - \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \chi^{(s)T} & -\chi^{(s)T} \dfrac{\phi\sigma^i A^i}{\mathbf{A}^2} \end{bmatrix} \right) \frac{1}{2}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \begin{bmatrix} \chi^{(s)} \\ \dfrac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \end{bmatrix} \\
&= \frac{1}{2}\mu\sqrt{\pi'^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \left(1 - \chi^{(s)T} \frac{\sigma^i \pi'^i}{\pi'^0+\mu} \frac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \right) - \frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f \phi} \left(1 - \chi^{(s)T} \frac{\phi\sigma^i A^i}{\mathbf{A}^2} \frac{\phi\sigma^j A^j}{\mathbf{A}^2}\chi^{(s)} \right) \quad (6.2) \\
&= \frac{1}{2}\mu\sqrt{\pi'^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} - \frac{1}{2}\frac{\mu}{\sqrt{\pi'^0+\mu}}\sqrt{\frac{e}{\lambda_f \phi \mathbf{A}^2}} \chi^{(s)T} \left(\sigma^i \pi'^i\right)\left(\phi\sigma^j A^j\right)\chi^{(s)} - v_+{}^3
\end{aligned}
$$

Adding (6.2) and (6.4) we obtain (pay close attention to the π versus π'):

$$
\begin{aligned}
\overline{v_+'^{1.5}} h_\psi + \overline{h_\psi} v_+'^{1.5} &= \frac{1}{2}\mu\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} \left(\sqrt{\pi'^0+\mu} + \sqrt{\pi^0+\mu} \right) - 2v_+{}^3 \\
&- \frac{1}{2}\mu\sqrt{\frac{e}{\lambda_f \phi \mathbf{A}^2}} \left(\frac{1}{\sqrt{\pi'^0+\mu}} \chi^{(s)T} \left(\sigma^i \pi'^i\right)\left(\phi\sigma^j A^j\right)\chi^{(s)} + \frac{1}{\sqrt{\pi^0+\mu}} \chi^{(s)T} \left(\phi\sigma^i A^i\right)\left(\sigma^j \pi^j\right)\chi^{(s)} \right). \quad (6.3)
\end{aligned}
$$

In the $q^\mu \to 0$, thus $\pi'^\mu \to \pi^\mu$ limit, with $\left\{\sigma^i \pi^i, \phi\sigma^j A^j\right\} \equiv \left(\sigma^i \pi^i\right)\left(\phi\sigma^j A^j\right) + \left(\phi\sigma^i A^i\right)\left(\sigma^j \pi^j\right)$, this reduces to:

$$
\overline{v_+'^{1.5}} h_\psi + \overline{h_\psi} v_+'^{1.5} = \mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}} - \frac{1}{2}\frac{\mu}{\sqrt{\pi^0+\mu}}\sqrt{\frac{e}{\lambda_f \phi \mathbf{A}^2}} \chi^{(s)T} \left\{\sigma^i \pi^i, \phi\sigma^j A^j\right\}\chi^{(s)} - 2v_+{}^3, \quad (6.4)
$$

We see that the dominant distinction between (6.1) and (6.2) when $q^\mu \to 0$ boils down to the reversed order of terms in the anticommutator $\left\{\sigma^i \pi^i, \phi\sigma^j A^j\right\}$. This anticommutator and similar expressions will become very important in the later development, because it contains the fermion magnetic moment. But for the moment, we are focused on the fermion mass.

Given that we have employed $q^\mu \to 0$ to arrive at (6.4) from (6.3), we should note that with S_F designating the fermion propagator which at the lowest order is ${S_F}^{-1} = \not{p} - m$ for a "bare mass" *m*, the Ward identity for ${S_F}^{-1} \to S_F'^{-1}$ at any order is given by $\partial S_F'^{-1} / \partial p_\mu = \Gamma^\mu(p,0,p)$ where with $\Gamma^\mu(p,q,p+q) = \gamma^\mu + \Lambda^\mu(p,q,p+q)$ is the *total vertex* of the transition current $J^\mu = \overline{\psi}(p')\Gamma^\mu\psi(p)$. We also note that $q_\mu\Gamma^\mu(p,q,p+q) = S_F'(p+q) - S_F'(p)$ which is the Ward-Takahashi identity, reduces to the Ward identity in the $q^\mu \to 0$ limit. In this context, going back to $\mathfrak{L}(\psi)$ in (5.13), by taking $q^\mu \to 0$ hence $\pi'^\mu \to \pi^\mu$ in (6.4), we are really first in (6.3) calculating a term $\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}$ in $\mathfrak{L}\left(\psi(p), q, \overline{\psi}(p+q)\right) \equiv \mathfrak{L}(p,q,p+q)$, then in (6.4) taking $\mathfrak{L}(p,q,p+q) \to \mathfrak{L}(p,0,p)$. Viewed in this light, and by adopting the notation $\mathfrak{L}(p,q,p+q) \to \mathfrak{L}(p,0,p)$ we align our parameterization of the vacuum-expanded Lagrangian (5.13) (and (5.14) for antiparticles) with how one comes to parameterize the Ward and Ward-Takahashi identities which are at the heart of renormalization theory. We also note that the Ward identity at *all orders* is $\Lambda^\mu(p,0,p) = -\partial\Sigma / \partial p_\mu$ with $\Sigma(p)$ denoting the fermion self-energy, for which $q^\mu \to 0$ is an "essential condition" (see [18], final full paragraph on page 267). Thus we see that by going from $\mathfrak{L}(p,q,p+q) \to \mathfrak{L}(p,0,p)$ between (6.3) and (6.4), we are examining the behavior of the Lagrangian density under the essential condition $q^\mu \to 0$ through which that Ward identity works at all orders during renormalization. In the subsequent development, rather than calculate analogs to (6.3) for $\mathfrak{L}(p,q,p+q)$ and then take the $q^\mu \to 0$ limit to arrive at $\mathfrak{L}(p,0,p)$ analogs to (6.4), we shall go straight to (6.4) analogs by calculating the $\mathfrak{L}(p,0,p)$ Lagrangian density, thus aligning the Lagrangian density with the operative essential condition of the Ward identity as used for high order renormalization. This will pay off in section 14, when we show how to renormalize while maintaining an invariant rest mass by absorbing any variation in mass at different renormalization scales into a gauge transformation. For further background, the reader is referred especially to sections 7.4 and 9.6 of [18].

Next, with this in mind, let us calculate $\overline{h_\psi}h_\psi(p,0,p)$, i.e., $\overline{h_\psi}h_\psi\left(q^\mu = 0\right)$. Here we use (4.22) and $\overline{h_\psi} = {h_\psi}^\dagger\gamma^0$ (and $\boldsymbol{\pi}^2 = \sigma^i\pi^i\sigma^j\pi^j$) in the $q^\mu \to 0$ limit to obtain:

$$\overline{h_\psi}h_\psi(p,0,p)=\mu^2(\pi^0+\mu)\left[\chi^{(s)T}\quad -\chi^{(s)T}\frac{\sigma^i\pi^i}{\pi^0+\mu}\right]\left[\begin{matrix}\chi^{(s)}\\ \frac{\sigma^j\pi^j}{\pi^0+\mu}\chi^{(s)}\end{matrix}\right]+\frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\left[\chi^{(s)T}\quad -\chi^{(s)T}\frac{\phi\sigma^iA^i}{\mathbf{A}^2}\right]\left[\begin{matrix}\chi^{(s)}\\ \frac{\phi\sigma^jA^j}{\mathbf{A}^2}\chi^{(s)}\end{matrix}\right]$$
$$-\frac{1}{2}\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\left(\left[\chi^{(s)T}\quad -\chi^{(s)T}\frac{\sigma^i\pi^i}{\pi^0+\mu}\right]\left[\begin{matrix}\chi^{(s)}\\ \frac{\phi\sigma^jA^j}{\mathbf{A}^2}\chi^{(s)}\end{matrix}\right]+\left[\chi^{(s)T}\quad -\chi^{(s)T}\frac{\phi\sigma^iA^i}{\mathbf{A}^2}\right]\left[\begin{matrix}\chi^{(s)}\\ \frac{\sigma^j\pi^j}{\pi^0+\mu}\chi^{(s)}\end{matrix}\right]\right)$$
$$=\mu^2(\pi^0+\mu)\left(1-\chi^{(s)T}\frac{\sigma^i\pi^i}{\pi^0+\mu}\frac{\sigma^j\pi^j}{\pi^0+\mu}\chi^{(s)}\right)+\frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\left(1-\chi^{(s)T}\frac{\phi\sigma^iA^i}{\mathbf{A}^2}\frac{\phi\sigma^jA^j}{\mathbf{A}^2}\chi^{(s)}\right) \quad .(6.5)$$
$$-\frac{1}{2}\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}\left(\left(1-\chi^{(s)T}\frac{\sigma^i\pi^i}{\pi^0+\mu}\frac{\phi\sigma^jA^j}{\mathbf{A}^2}\chi^{(s)}\right)+\left(1-\chi^{(s)T}\frac{\phi\sigma^iA^i}{\mathbf{A}^2}\frac{\sigma^j\pi^j}{\pi^0+\mu}\chi^{(s)}\right)\right)$$
$$=\mu^2\frac{(\pi^0+\mu)^2-\boldsymbol{\pi}^2}{\pi^0+\mu}-\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}+\frac{1}{2}\frac{\mu}{\sqrt{\pi^0+\mu}}\sqrt{\frac{e}{\lambda_f\phi\mathbf{A}^2}}\chi^{(s)T}\left\{\sigma^i\pi^i,\phi\sigma^jA^j\right\}\chi^{(s)}+v_+{}^3$$

Before proceeding further, let's simplify the form of both (6.4) and (6.5) by defining the anticommutator term A with mass dimension $D=+3$ in $\hbar=c=1$ units as:

$$A(p,0,p)\equiv\frac{1}{2}\frac{\mu}{\sqrt{\pi^0+\mu}}\sqrt{\frac{e}{\lambda_f\phi\mathbf{A}^2}}\chi^{(s)T}\left\{\sigma^i\pi^i,\phi\sigma^jA^j\right\}\chi^{(s)}. \tag{6.6}$$

Then (6.4) and (6.5) simplify for $(p,0,p)$ to:

$$\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}=\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}-A-2v_+{}^3, \tag{6.7}$$

$$\overline{h_\psi}h_\psi=\mu^2\frac{(\pi^0+\mu)^2-\boldsymbol{\pi}^2}{\pi^0+\mu}-\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}+A+v_+{}^3. \tag{6.8}$$

From the combination of (6.7) and (6.8) we then see that for $(p,0,p)$:

$$\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}+2v_+{}^3=\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}-A=+v_+{}^3-\overline{h_\psi}h_\psi+\mu^2\frac{(\pi^0+\mu)^2-\boldsymbol{\pi}^2}{\pi^0+\mu}, \tag{6.9}$$

or more directly:

$$\overline{h_\psi}h_\psi(p,0,p)=-\overline{v_+'^{1.5}}h_\psi-\overline{h_\psi}v_+'^{1.5}+\mu^2\frac{(\pi^0+\mu)^2-\boldsymbol{\pi}^2}{\pi^0+\mu}-v_+{}^3, \tag{6.10}$$

This gives us what we need to go back to the fermion Lagrangian (5.13) and recast the terms $\left(\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\right)^2$ and $-2\lambda_f\left(\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\right)\overline{h_\psi}h_\psi$ into an $\overline{h_\psi}h_\psi$ term. First, all in the $(p,0,p)$ parameterization, squaring (6.7):

$$\begin{aligned}&\left(\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\right)^2 \\ &= \mu^2\left(\pi^0+\mu\right)\frac{e\mathbf{A}^2}{\lambda_f\phi} + A^2 - 2A\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}} - 4{v_+}^3\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}} + 4A{v_+}^3 + 4{v_+}^6\end{aligned} \quad (6.11)$$

On the other hand, multiplying (6.8) by the scalar number $4{v_+}^3$, we find that:

$$4{v_+}^3\overline{h_\psi}h_\psi = 4{v_+}^3\mu^2\frac{\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2}{\pi^0+\mu} - 4{v_+}^3\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}} + 4A{v_+}^3 + 4{v_+}^6. \quad (6.12)$$

Combining (6.11) and (6.12) now yields with $(p,0,p)$:

$$\begin{aligned}&-4{v_+}^3\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}} + 4A{v_+}^3 + 4{v_+}^6 \\ &= \left(\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\right)^2 - \mu^2\left(\pi^0+\mu\right)\frac{e\mathbf{A}^2}{\lambda_f\phi} - A^2 + 2A\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}. \\ &= 4{v_+}^3\overline{h_\psi}h_\psi - 4{v_+}^3\mu^2\frac{\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2}{\pi^0+\mu}\end{aligned} \quad (6.13)$$

More directly, from the second and third lines:

$$\begin{aligned}&\left(\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\right)^2(p,0,p) \\ &= 4{v_+}^3\overline{h_\psi}h_\psi - 4{v_+}^3\mu^2\frac{\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2}{\pi^0+\mu} + \mu^2\left(\pi^0+\mu\right)\frac{e\mathbf{A}^2}{\lambda_f\phi} - 2A\mu\sqrt{\pi^0+\mu}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}} + A^2\end{aligned} \quad (6.14)$$

Moreover, from (6.9) multiplied through by $2\overline{h_\psi}h_\psi$, still with $(p,0,p)$:

$$2\left(\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}\right)\overline{h_\psi}h_\psi = -2{v_+}^3\overline{h_\psi}h_\psi - 2\overline{h_\psi}h_\psi\overline{h_\psi}h_\psi + 2\mu^2\frac{\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2}{\pi^0+\mu}\overline{h_\psi}h_\psi, \quad (6.15)$$

Then, using (6.14) and (6.15) in the fermion Lagrangian (5.13) and reducing yields:

$$
\begin{aligned}
\mathfrak{L}\left(h_\psi\right) &= \mathfrak{L}\left(p,0,p\right) = T\left(h_\psi\right) - V\left(h_\psi\right) \\
&= \overline{h_\psi}\not{\pi} h_\psi + \overline{v_+'^{1.5}}\not{\pi} h_\psi + \overline{h_\psi}\not{\pi} v_+'^{1.5} + \overline{v_+^{1.5}}\not{\pi} v_+^{1.5} \\
&\quad -\lambda_f \left(
\begin{array}{l}
+2v_+^3 \overline{h_\psi} h_\psi - \overline{h_\psi} h_\psi \overline{h_\psi} h_\psi + 2\mu^2 \dfrac{\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2}{\pi^0+\mu} \overline{h_\psi} h_\psi \\
-v_+^6 - 4v_+^3 \mu^2 \dfrac{\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2}{\pi^0+\mu} + \mu^2 \left(\pi^0+\mu\right) \dfrac{e\mathbf{A}^2}{\lambda_f \phi} - 2\mu\sqrt{\pi^0+\mu}\sqrt{\dfrac{e\mathbf{A}^2}{\lambda_f \phi}} A + A^2
\end{array}
\right)
\end{aligned}
, \qquad (6.16)
$$

This has been our goal. Now we have found a term $-\lambda_f 2v_+^3 \overline{h_\psi} h_\psi$ which has the form of a "revealed" fermion mass. But there is a term $-2\lambda_f \mu^2 \left(\left(\pi^0+\mu\right)^2 - \boldsymbol{\pi}^2\right) / \left(\pi^0+\mu\right) \overline{h_\psi} h_\psi$ in the above that also contains $\overline{h_\psi} h_\psi$. So how do we pinpoint the mass? Contrasting with (2.18) from scalar field theory, this latter term is analogous to the term $-\frac{1}{2} e^2 \left(\pm 2vh + h^2 + \xi^2\right) A_\sigma A^\sigma$ which has both h^2 and $A_\sigma A^\sigma$, yet is *not* used to determine the mass. Rather, in (2.18) through (2.20), we identify the boson mass from $m_A{}^2 = (ev)^2$ and the Higgs mass from $m_h{}^2 = 2\lambda v^2$. In both cases, we identify the mass from *only* a vacuum vev *v* and a coupling which is *e* for m_A and λ for m_h. To repeat: *the mass is constructed only from a vacuum and a coupling*, and nothing else. The same applies here, so comparing $-2\lambda_f v_+^3 \overline{h_\psi} h_\psi$ to the expected form $-m\overline{h}h$ for a fermion mass term, that is, contrasting $-m\overline{h}h \Leftrightarrow -2\lambda_f v_+^3 \overline{h_\psi} h_\psi$, and also combining with (4.3) and (3.7), we see that the coupling is λ_f and the vacuum is v_+^3. So the mass (rest energy) of the fermion with *c*=1 restored is revealed to be:

$$
\boxed{mc^2 = 2\lambda_f v_+^3 = \frac{1}{2} e\left(\frac{\mathbf{A}^2}{\phi} - \phi\right) = \frac{1}{2} e\left(\frac{\mathbf{A}^2 - \phi^2}{\phi}\right) = -\mu c^2 > 0} \qquad (6.17)
$$

This fundamental result has been the goal of the entire development thus far. It is the direct analog of (2.20) in which the mass of the scalar Higgs field – which is what is believed to have been detected at CERN in 2012 – is revealed in scalar Higgs theory. But this is now a *fermion* mass and may be taken to correspond to the observed masses of the observed fermions. And, as we see, this mass is constructed entirely from the energy of the gauge potentials A^μ and so is in the nature of a *self-energy* $\Sigma = mc^2$ which should be very helpful for renormalization. But, because this mass is specified in terms of the gauge potentials, the question now arises: How do we interpret this result? Keep in mind that a gauge potential by itself has no physical meaning. All that is measurable is a "voltage drop," i.e. a *difference* between two potential energies at two different points in space.

While neither ϕ nor $\mathbf{A}^2$ has a physical meaning by itself, the electron does possess an "intrinsic" spin which Ohanian explains [15] as an entirely classical circular energy flow in the electron (fermion) wave field. Further, we are reminded by the Gordon decomposition $2mJ^\mu = 2m\psi\gamma^\mu\psi = i\left(\overline{\psi}\partial^\mu\psi - \left(\partial^\mu\overline{\psi}\right)\psi\right) + \partial_\nu\left(\overline{\psi}\sigma^{\mu\nu}\psi\right)$ for a free electron that this spin results in a circulating flow of charge *even for an electron at rest*. We see this from the space components $2m\mathbf{J} = i\left(\overline{\psi}\partial^k\psi - \left(\partial^k\overline{\psi}\right)\psi\right) + \partial_\nu\left(\overline{\psi}\sigma^{k\nu}\psi\right)$, which show that the spin current density $\partial_\nu\left(\overline{\psi}\sigma^{k\nu}\psi\right)/2m$ is non-zero *even in the electron rest frame,* and even if the convection / orbital angular momentum current density $\overline{\psi}\partial^k\psi - \left(\partial^k\overline{\psi}\right)\psi = 0$. So because of its spin, an electron "at rest" is not really, fully "at rest." There is always an inherent, kinetic, circulating flow of energy and charge.

Therefore, if consider that $\mathbf{A}^2$ in (6.17) fundamentally captures the *kinetic* aspects of the energies brought about by the electron spin, then equation (6.17) is fundamentally a mirror of the Gordon decomposition at the fermion mass level. It tells us that the fermion mass is no more and no less than the *difference* between the two intrinsic potentials ϕ and $\mathbf{A}^2$ of the electron which always has a spin and thus kinetic activity *even at rest*, and that the specific measure of this difference in potential is given by a "voltage drop" $\frac{1}{2}e\mathbf{A}^2/\phi - \frac{1}{2}e\phi$ between a potential $\frac{1}{2}e\mathbf{A}^2/\phi$ which reflects self-energies arising from the kinetic properties of a circulating (spinning) charge, and a non-kinetic scalar voltage $\frac{1}{2}e\phi$ attributable solely to the charge without any spin-related motion. So the electron mass arises from its own electromagnetic self-energies as the *difference* between a potential $\frac{1}{2}e\mathbf{A}^2/\phi$ arising from the circulating flow of its charge and a potential $\frac{1}{2}e\phi$ arising from the charge itself absent any kinetics.

Now the question emerges which we shall study at length in section 14: at what event point or points in spacetime is this $\frac{1}{2}e\mathbf{A}^2/\phi - \frac{1}{2}e\phi$ difference in potential taken, and does the mass in (6.17) vary when these potentials are taken at *different* event points? We do not at the moment have enough information developed to answer these questions, but will return to answer them in section 14. There we shall see that this difference is taken *at a single point in space*, or in renormalization language, that this difference is taken *at a single, given renormalization scale*. And, of fundamental importance, as we shall also see in section 14, *this rest mass can be made to remain invariant over all renormalization scales, because any variation in mass from one scale to the next can be gauged away by a simple gauge transformation of the gauge fields A^μ from which the mass in (6.17) is constructed.* This is what we refer to as "Invariant Mass, Variable Gauge Renormalization," and it will be detailed in section 14.

Given (6.17), we may go back to all the other equations developed thus far and substitute a rest mass *m* wherever any of the other expressions in (6.17) may appear. This includes setting $\mu \to -m$ wherever the former appears. First, of course, we may go back to the Lagrangian (6.16) and use (6.17) as well as $\pi^0 = E + e\phi$ and $\boldsymbol{\pi}^2 = (\mathbf{p} + e\mathbf{A})^2$ to write:

$$
\begin{aligned}
\mathfrak{L}\left(h_\psi\right) &= \mathfrak{L}\left(p,0,p\right) = T\left(h_\psi\right) - V\left(h_\psi\right) \\
&= \overline{h_\psi}\pi h_\psi + \overline{v_+'^{1.5}}\pi h_\psi + \overline{h_\psi}\pi v_+'^{1.5} + \overline{v_+^{1.5}}\pi v_+^{1.5} \\
&\quad - m\overline{h_\psi}h_\psi + \frac{1}{2}\frac{\overline{h_\psi}h_\psi}{v_+^3} m\overline{h_\psi}h_\psi - m^2\frac{\left(E+e\phi-m\right)^2 - \left(\mathbf{p}+e\mathbf{A}\right)^2}{E+e\phi-m} m\frac{\overline{h_\psi}h_\psi}{v_+^3} \\
&\quad + \frac{1}{2}m\overline{v_+^{1.5}}v_+^{1.5} + 2m^3\frac{\left(E+e\phi-m\right)^2 - \left(\mathbf{p}+e\mathbf{A}\right)^2}{E+e\phi-m} - \left(m\sqrt{E+e\phi-m}\sqrt{\frac{e\mathbf{A}^2}{\phi}} + \sqrt{\lambda_f}A\right)^2
\end{aligned}
\quad . \qquad (6.18)
$$

We have in the above eliminated all appearances of λ_f and μ in favor of m and $v_+^{1.5}$ except for the λ_f still remaining in the final term involving the anticommutator term A as defined in (6.6), which will cancel out in the end because A is defined so as to include a $1/\sqrt{\lambda_f}$.

As to the remaining terms, keep in mind that the above describes $\mathfrak{L}$ *in any frame of motion*, relativistic or not. To get a better handle in these terms, we now reduce (6.18) to the fermion's own rest frame, in which the only remaining kinetic activity arises from spin. To do this we set **p**=0 thus $E=m$ via $m^2 = p_\sigma p^\sigma$, and we also reapply $-2m = e\left(\phi^2 - \mathbf{A}^2\right)/\phi$ of (6.17) in two places. This reduces (6.18) at rest to:

$$
\begin{aligned}
\mathfrak{L}\left(h_\psi\right) &= \mathfrak{L}\left(p,0,p,\mathbf{p}=0\right) = T\left(h_\psi\right) - V\left(h_\psi\right) \\
&= \overline{h_\psi}\pi h_\psi + \overline{v_+'^{1.5}}\pi h_\psi + \overline{h_\psi}\pi v_+'^{1.5} + \overline{v_+^{1.5}}\pi v_+^{1.5} \\
&\quad - \left(1 - \frac{1}{2}\frac{\overline{h_\psi}h_\psi}{v_+^3} - 2\frac{m^3}{v_+^3}\right) m\overline{h_\psi}h_\psi + \frac{1}{2}m\overline{v_+^{1.5}}v_+^{1.5} - 4m^4 - \left(me\sqrt{\mathbf{A}^2} + \sqrt{\lambda_f}A\right)^2
\end{aligned}
\quad . \qquad (6.19)
$$

It is especially informative to now contrast the above term for $m\overline{h_\psi}h_\psi$ with the analogous term for $m_h^2h^2$ in the scalar Lagrangian density (2.24), that is, to contrast:

$$
-\left(1 - \frac{1}{2}\frac{\overline{h_\psi}h_\psi}{v_+^3} - 2\frac{m^3}{v_+^3}\right) m\overline{h_\psi}h_\psi \Leftrightarrow -\frac{1}{2}\left(1 + \frac{h}{v} + \frac{h^2}{4v^2}\right) m_h^2 h^2 . \qquad (6.20)
$$

We see that in each case, the mass terms $-\frac{1}{2}m_h^2h^2$ and $-m\overline{h_\psi}h_\psi$ also have further non-linear interactions, with h/v and $\left(h/v\right)^2$ in the case of scalars and with $\overline{h_\psi}h_\psi/v_+^3$ and with its own mass-to-vev ratio m^3/v_+^3 in the case of fermions. The remaining term in (6.19), in the rest frame, is $-\left(em\sqrt{\mathbf{A}^2} + \sqrt{\lambda_f}A\right)^2$, which contains a magnetic moment term that we shall develop further in the next section.

Put into context, in the scalar Higgs theory reviewed in section 2, we revealed both a scalar Higgs mass from the potential portion of the Lagrangian density and a "bonus" gauge boson mass from the kinetic portion. Above, we have revealed a fermion Higgs mass from the potential portion $-V(h_\psi)$ of the Lagrangian density and not yet touched upon the kinetic portion $T(h_\psi)$. We should expect a similar "bonus" to emerge there. We know from Dirac theory that the kinetic terms $T(\psi)=i\overline{\psi}\gamma^\mu\partial_\mu\psi+e\overline{\psi}\gamma^\mu\psi A_\mu$ already contain the four-current density $J^\mu=e\overline{\psi}\gamma^\mu\psi$, and that this is precisely the term that may be Gordon-decomposed into a convection (orbital angular momentum) and a spin current density which includes the magnetic moment and the gyromagnetic ratio. So as we shall see starting in section 8, the "bonus" that we obtain in fermion Higgs theory is *not* the gauge boson and its mass. We already have that from the scalar Higgs theory. Rather, we shall obtain new information about how this gauge field interacts with the spinning electron via the physics of the intrinsic magnetic moment, and in particular, shall show how the so-called "anomalies" in the magnetic g-factor are revealed *directly from Dirac's equation itself* when one considers the Higgs fermions h_ψ rather than the seed fermions ψ to be the *observable* fermions.

Before concluding this section, it has already been noted that (6.17) may now be used to place the revealed fermion mass *m* throughout the earlier relationships developed in this paper. Let us now explicitly do this for four particularly noteworthy cases. First, we return to (3.4) in which we first defined mass parameter μ. With $\mu\to-m$ may rewrite this as:

$$m^2\equiv\rho_\mu\rho^\mu=(-m-e\phi,-e\mathbf{A})(-m-e\phi,+e\mathbf{A})=(-m-e\phi)^2-e\mathbf{A}^2=m^2+2me\phi+e^2\phi^2-e^2\mathbf{A}^2 . \tag{6.21}$$

This may now be used to write down an "apples-to-apples" relationship to the relativistic mass/energy relationship (3.2), namely:

$$m^2=(E+e\phi)^2-(\mathbf{p}+e\mathbf{A})^2=m^2+2me\phi+e^2\phi^2-e^2\mathbf{A}^2 . \tag{6.22}$$

Expanding the latter two expressions then reducing with some parenthetical emphasis of corresponding terms leaves us with the relativistic relationship:

$$\left(E^2-\mathbf{p}^2\right)+2\left(Ee\phi-\mathbf{p}e\mathbf{A}\right)=m^2+2\left(me\phi\right). \tag{6.23}$$

In the rest frame $E=m$, $\mathbf{p}=0$ this reduces to the identity $m^2+2me\phi=m^2+2me\phi$, with $E^2-\mathbf{p}^2\to m^2$ and $E\phi-\mathbf{p}\mathbf{A}\to m\phi$. This displays how the momentum four-vector $p^\mu=(E,\mathbf{p})$ and the potential four vector $A^\mu=(\phi,\mathbf{A})$ both Lorentz transform in a proper manner.

Second, we return to (3.6) which in light of $\mu\to-m$ from (6.17) now becomes:

$$e\phi = m\left(\mp\sqrt{1+\frac{e^2\mathbf{A}^2}{m^2}}-1\right) \tag{6.24}$$

We continue to defer making a choice of sign from the quadratic solution until we can do so with experimental data comparisons. But in light of (6.24) a negative sign will necessarily yield a $\phi<0$ in all cases while a positive sign will produce $\phi>0$ so long as $\mathbf{A}$ is real and not imaginary.

Third, we can now rewrite (6.17) as:

$$e^2\mathbf{A}^2 = 2me\phi + e^2\phi^2\,, \tag{6.25}$$

which is the quadratic in $e\phi$ for which (6.24) is the solution. This is also (3.5) with $\mu \to -m$.

Finally, we return to the seed fermions (4.21) and use $\mu \to -m$ to rewrite those as:

$$\psi = -m\sqrt{\pi^0 - m}\begin{bmatrix}\chi^{(s)} \\ \dfrac{\sigma^i\pi^i}{\pi^0 - m}\chi^{(s)}\end{bmatrix}e^{-i\pi_\sigma x^\sigma}; \quad \varphi = -m\sqrt{\pi^0 - m}\begin{bmatrix}\dfrac{\sigma^i\pi^i}{\pi^0 - m}\chi^{(s)} \\ \chi^{(s)}\end{bmatrix}e^{-i\pi_\sigma x^\sigma}\,, \tag{6.26}$$

Now we turn to the magnetic moments.

7. Development of the Magnetic Moment Term in the Potential Portion *V* of the Lagrangian Density, and the Emergence of the Maxwell Equation Field Terms

It was mentioned in the last section that the anticommutator term *A* which was defined in (6.6) contains a magnetic moment, which term *A* enters the at rest Lagrangian density (6.19) as part of the term $-\left(me\sqrt{\mathbf{A}^2}+\sqrt{\lambda_f}A\right)^2$. Specifically, in (6.6) which is for $\mathfrak{L}\left(p,0,p\right)$, we uncovered an anticommutator term:

$$\left\{\sigma^i\pi^i,\phi\sigma^j A^j\right\} = \left(\sigma^i\pi^i\right)\left(\phi\sigma^j A^j\right)+\left(\phi\sigma^j A^j\right)\left(\sigma^i\pi^i\right) = \sigma^i\sigma^j\left(\pi^i\right)\left(\phi A^j\right)+\sigma^i\sigma^j\left(\phi A^i\right)\left(\pi^j\right). \tag{7.1}$$

We now develop this. This discussion is of interest in its own right, but it will illustrate two broader aspects of fermion Higgs theory which are of even more consequence than the term in (7.1): first, how terms such as the magnetic moment arises out of a variety of commutation relationships and second, closely related, how time and space dependencies in *observed* physics are revealed not *ab initio*, but via Heisenberg-type commutations. We shall also see where non-Abelian Yang-Mills theory such as that of strong and weak interactions comes into play.

We first observe that $\pi^i = p^i + eA^i \Leftrightarrow iD^i = i\partial^i + eA^i$ contains $p^i = \mathbf{p} \Leftrightarrow i\partial^i = i\nabla$ which is the gradient operator. Thus, we must pay close attention to how this operator is commuted.

And, when we "multiply" something else with this operator, for example, when we take $p^i\left(\phi A^j\right)$, we do not just multiply as if it is an ordinary number, but rather, we apply it via a product rule just as we would for ∂^i. Therefore, $p^i\left(\phi A^j\right)=p^i\phi A^j+\phi p^i A^j$, and *not* $p^i\left(\phi A^j\right)=p^i\phi A^j$. The latter would only be a correct equality if p^i was an ordinary vector without the character of a gradient. Similarly, from the right, $\left(\phi A^i\right)p^j=\phi A^i p^j+\phi p^j A^i$, and not $\left(\phi A^i\right)p^j=\phi A^i p^j$. As a result, the careful, precise and correct way to develop (7.1) via $\pi^i=p^i+eA^i$ is to write:

$$\begin{aligned}\left\{\sigma^i\pi^i,\phi\sigma^j A^j\right\}&=\left(\sigma^i\pi^i\right)\left(\phi\sigma^j A^j\right)+\left(\phi\sigma^j A^j\right)\left(\sigma^i\pi^i\right)=\sigma^i\sigma^j\left(\pi^i\right)\left(\phi A^j\right)+\sigma^i\sigma^j\left(\phi A^i\right)\left(\pi^j\right)\\&=\sigma^i\sigma^j\left(p^i+eA^i\right)\left(\phi A^j\right)+\sigma^i\sigma^j\left(\phi A^i\right)\left(p^j+eA^j\right)\\&=\sigma^i\sigma^j\left(p^i\phi A^j+\phi p^i A^j+e\phi A^i A^j\right)+\sigma^i\sigma^j\left(\phi A^i p^j+\phi p^j A^i+e\phi A^i A^j\right)\\&=\sigma^i\sigma^j\left(\phi p^i A^j+\phi A^i p^j+p^i\phi A^j+\phi p^j A^i+2e\phi A^i A^j\right)\end{aligned}\quad. \tag{7.2}$$

Now we make use of the mathematical identity (see, e.g., [18] page 54):

$$\sigma^i\sigma^j=\delta^{ij}+i\varepsilon^{ijk}\sigma^k \tag{7.3}$$

where ε^{ijk} is the totally-antisymmetric Levi-Civita tensor in a $\varepsilon^{123}=1$ basis, to advance (7.2) to:

$$\begin{aligned}\left\{\sigma^i\pi^i,\phi\sigma^j A^j\right\}&=\left(\delta^{ij}+i\varepsilon^{ijk}\sigma^k\right)\left(\phi p^i A^j+\phi A^i p^j+p^i\phi A^j+\phi p^j A^i+2e\phi A^i A^j\right)\\&=\phi p^i A^i+\phi A^i p^i+p^i\phi A^i+\phi p^i A^i+2e\phi A^i A^i+i\varepsilon^{ijk}\sigma^k\left(\phi p^i A^j+\phi A^i p^j+p^i\phi A^j+\phi p^j A^i+2e\phi A^i A^j\right).\\&=\phi\mathbf{p}\cdot\mathbf{A}+\phi\mathbf{A}\cdot\mathbf{p}+\mathbf{p}\phi\cdot\mathbf{A}+\phi\mathbf{p}\cdot\mathbf{A}+2e\phi\mathbf{A}^2+i\varepsilon^{ijk}\sigma^k\left(\phi p^i A^j+\phi A^i p^j+p^i\phi A^j+\phi p^j A^i+2e\phi A^i A^j\right)\end{aligned} \tag{7.4}$$

The term we now focus upon is the one with the leading factor $i\varepsilon^{ijk}\sigma^k$. This has three parts. It is easily shown that $\varepsilon^{ijk}\sigma^k A^i A^j=\boldsymbol{\sigma}\cdot\left(\mathbf{A}\times\mathbf{A}\right)=0$ by identity, *but only if we assume that the gauge fields commute according to* $\left[A^\mu,A^\nu\right]=0$. If they are non-commuting which is the hallmark of non-Abelian Yang-Mills theory, i.e., if $\left[A^\mu,A^\nu\right]\neq 0$, then $\varepsilon^{ijk}\sigma^k A^i A^j\neq 0$, and this is where one would embark upon the Yang-Mills version of all that is being developed here. To keep things relatively simple, we shall set $\varepsilon^{ijk}\sigma^k A^i A^j=0$ here and so remain focused on Abelian gauge theories such as electrodynamics. But, it is important to be aware that non-Abelian gauge theory naturally emerges from this development as well, in this way.

Abelian or non-Abelian, the other two parts of (7.4) are not zero, and are of great interest. Consolidated with $\left[p^j,A^k\right]\equiv p^j A^k-A^k p^j$ and $\left[p^i,\phi\right]\equiv p^i\phi-\phi p^i$, it is readily shown that:

$$\begin{aligned}
&i\varepsilon^{ijk}\sigma^k\left(\phi p^i A^j + \phi A^i p^j + p^i \phi A^j + \phi p^j A^i + 2e\phi A^i A^j\right)\\
&= i\sigma^1\left(\left[p^2,\phi\right]A^3 - \left[p^3,\phi\right]A^2 + \phi\left[p^2,A^3\right] - \phi\left[p^3,A^2\right]\right)\\
&\ \ + i\sigma^2\left(\left[p^3,\phi\right]A^1 - \left[p^1,\phi\right]A^3 + \phi\left[p^3,A^1\right] - \phi\left[p^1,A^3\right]\right)\\
&\ \ + i\sigma^3\left(\left[p^1,\phi\right]A^2 - \left[p^2,\phi\right]A^1 + \phi\left[p^1,A^2\right] - \phi\left[p^2,A^1\right]\right)
\end{aligned} \qquad (7.5)$$

We now have two fields $\phi(x)$ and $A^k(x)$ commuting with the canonical momentum p^i. Thus, we may employ the commutator relationships $\left[p^i,\phi\right] = -i\partial^i\phi$ and $\left[p^i,A^j\right] = -i\partial^i A^j$, see, e.g., [18] just after (2.164) to reduce (7.5) to:

$$\begin{aligned}
&i\varepsilon^{ijk}\sigma^k\left(\phi p^i A^j + \phi A^i p^j + p^i \phi A^j + \phi p^j A^i + 2e\phi A^i A^j\right)\\
&= \sigma^1\left(\partial^2\phi A^3 - \partial^3\phi A^2 + \phi\left(\partial^2 A^3 - \partial^3 A^2\right)\right)\\
&\ \ + \sigma^2\left(\partial^3\phi A^1 - \partial^1\phi A^3 + \phi\left(\partial^3 A^1 - \partial^1 A^3\right)\right)\\
&\ \ + \sigma^3\left(\partial^1\phi A^2 - \partial^2\phi A^1 + \phi\left(\partial^1 A^2 - \partial^2 A^1\right)\right)\\
&= \sigma^i\left(-\nabla\phi\times\mathbf{A}\right)^i + \phi\sigma^i\left(-\nabla\times\mathbf{A}\right)^i = \sigma^i\varepsilon^{ijk}\partial^j\phi A^k + \phi\sigma^i\varepsilon^{ijk}\partial^j A^k = \sigma^i\varepsilon^{ijk}\partial^j\phi A^k + \tfrac{1}{2}\phi\sigma^i\varepsilon^{ijk}F^{jk}\\
&= \varepsilon^{ijk}\sigma^i\partial^j\phi A^k - \phi\sigma^i B^i = \boldsymbol{\sigma}\cdot\left(-\nabla\phi\times\mathbf{A}\right) - \phi\boldsymbol{\sigma}\cdot\mathbf{B} = \boldsymbol{\sigma}\cdot\left(\mathbf{E} + \partial\mathbf{A}/\partial t\right) - \phi\boldsymbol{\sigma}\cdot\mathbf{B}
\end{aligned} \quad . \qquad (7.6)$$

In the above, we have made use of $\left(-\nabla\phi\times\mathbf{A}\right)^i \equiv \varepsilon^{ijk}\partial^j\phi A^k$, $\left(-\nabla\times\mathbf{A}\right)^i = \varepsilon^{ijk}\partial^j A^k$, and $\tfrac{1}{2}\varepsilon^{ijk}F^{jk} = \varepsilon^{ijk}\partial^j A^k$ where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic field strength tensor and $\mathbf{B} = B^i = -\tfrac{1}{2}\varepsilon^{ijk}F^{jk}$ is the magnetic field vector, and $\mathbf{E} = E^i = F^{i0} = \partial^i A^0 - \partial^0 A^i = -\nabla\phi - \partial\mathbf{A}/\partial t$ where **E** is the electric field vector. (Keep in mind that $\partial_\mu \equiv \left(\partial_0,\partial_1,\partial_2,\partial_3\right) = \left(\partial_t,\partial_x,\partial_y,\partial_3\right)$ is defined in covariant (lower index) form so that $\partial^\mu = \left(\partial^0,\partial^1,\partial^2,\partial^3\right) = \left(\partial^t,-\partial^x,-\partial^y,-\partial^z\right)$, which accounts for the minus sign in $-\nabla$ via raising with $\mathrm{diag}\left(\eta^{\mu\nu}\right) = \left(1,-1,-1,-1\right)$.)

Now we see a specific instance (the first of what will be several) of what we introduced in the discussion leading to (4.6) and again prior to (4.9) when we said that space and time dependencies which we observe in the physical universe are not those which appear in a Lagrangian such as $\mathfrak{L} = \overline{\psi}\left(i\partial\!\!\!/ + eA\!\!\!/ - \mu\right)\psi$ but rather are those space and time dependencies that arise out of quantum mechanical Heisenberg commutations. Keep in mind the progression of development: in (4.7) and (4.8) we began to use a kinetic Fourier kernel $e^{-i\pi_\sigma x^\sigma}$ containing $p_\sigma \to \pi_\sigma \equiv p_\sigma + eA_\sigma$ to solve Dirac's equation and obtain the free spinor solutions along geodesics in the curved gauge space for $\left(iD\!\!\!\!/ - \mu\right)\psi = 0$ in which $\partial_\sigma \to D_\sigma \equiv \partial_\sigma - ieA_\sigma$. But to do so, the gauge fields A_σ had to "check at the door" any explicit time dependency which they may have had. So as these A_σ appear in (4.19), and (4.21) to (4.23), these A_σ have no explicit time or space dependency except insofar as they might acquire such dependency via a commutation such as $\left[p^i,\phi\right]=-i\partial^i\phi$ or $\left[p^i,A^j\right]=-i\partial^i A^j$ with a canonical momentum.

Now, in (7.6), that is exactly what has happened! The field strength tensor $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$ contains both the time and space dependency of the gauge field, and was "revealed" in (7.6) because of the commutation relationships which presented themselves in (7.5). So while the gauge fields had "checked at the door" their time and space dependencies when they entered the spinors of section 4, *they now retrieve these time and space dependencies on the way back out the door because of a canonical commutation*. And not only are these time and space dependencies retrieved at the door, but they show up in the form of a $\boldsymbol{\sigma}\cdot\mathbf{B}$ term which is a magnetic moment term which contains a magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$ which involves the curl (a spatial dependency) of the gauge field vector potential **A**. And they further show up in the form of a $-\nabla\phi=\mathbf{E}+\partial\mathbf{A}/\partial t$ which is the gradient of the scalar potential and which is equal to the electric field plus a *time-dependency* of the gauge field **A**.

So not only does the anticommutator $\left\{\sigma^i\pi^i,\phi\sigma^j A^j\right\}$ found in (9.6) contain a magnetic moment term, which we now see is of the form $-\phi\boldsymbol{\sigma}\cdot\mathbf{B}$, but it also implicitly embeds both the electric and magnetic fields because of the commutation relationships that it produces. In the subsequent development of the kinetic terms in the Lagrangian density, staring in section 9, we will uncover not only electric and magnetic fields from the time and space-dependencies of the gauge fields, but will further uncover the time and space dependencies of the electric and magnetic fields themselves in the explicit forms $\nabla\cdot\mathbf{E}$, $\nabla\times\mathbf{E}$ and $\nabla\times\mathbf{B}$ as they appear in three of the four Maxwell equations, and in the form of the magnetic moment term $\boldsymbol{\sigma}\cdot\mathbf{B}$ in lieu of the fourth Maxwell equation $\nabla\cdot\mathbf{B}=0$. And this fourth equation $\nabla\cdot\mathbf{B}=0$ only because we chose not to pursue Yang-Mills theory by setting $\varepsilon^{ijk}\sigma^k A^i A^j=0$ at (7.3). As is well known and as the author has developed in several other papers [19], [20], [21], [22], $\nabla\cdot\mathbf{B}\neq 0$ in Yang-Mills theory, magnetic monopoles do exist, and these Yang-Mills monopole can be used to understand not only QCD, but the very existence of baryons including protons and neutrons, and to explain the proton and neutron masses and nuclear binding energies.

For the moment, however, let us continue on, using (7.6) in (7.4) to yield the complete anticommutator:

$$\begin{aligned}&\left\{\sigma^i\pi^i,\phi\sigma^j A^j\right\}=\left(\delta^{ij}+i\varepsilon^{ijk}\sigma^k\right)\left(\phi p^i A^j+\phi A^i p^j+p^i\phi A^j+\phi p^j A^i+2e\phi A^i A^j\right)\\&=\phi p^i A^i+\phi A^i p^i+p^i\phi A^i+\phi p^i A^i+2e\phi A^i A^i+\sigma^i\varepsilon^{ijk}\partial^j\phi A^k-\phi\sigma^i B^i\\&=\phi\mathbf{p}\cdot\mathbf{A}+\phi\mathbf{A}\cdot\mathbf{p}+\mathbf{p}\phi\cdot\mathbf{A}+\phi\mathbf{p}\cdot\mathbf{A}+2e\phi\mathbf{A}^2+\boldsymbol{\sigma}\cdot\left(-\nabla\phi\times\mathbf{A}\right)-\phi\boldsymbol{\sigma}\cdot\mathbf{B}\\&=\phi\mathbf{p}\cdot\mathbf{A}+\phi\mathbf{A}\cdot\mathbf{p}+\mathbf{p}\phi\cdot\mathbf{A}+\phi\mathbf{p}\cdot\mathbf{A}+2e\phi\mathbf{A}^2+\boldsymbol{\sigma}\cdot\left(\mathbf{E}+\partial\mathbf{A}/\partial t\right)-\phi\boldsymbol{\sigma}\cdot\mathbf{B}\end{aligned}\quad . \tag{7.7}$$

We then insert (7.7) into the anticommutator term A defined in (6.6) along with $\mu\rightarrow -m$ from (6.17), and apply $\chi^{(s)T}\chi^{(s)}=1$ where possible to obtain:

$$A\left(p,0,p\right)=-\frac{1}{2}\frac{m}{\sqrt{\pi^0-m}}\sqrt{\frac{e}{\lambda_f\phi\mathbf{A}^2}}\left(\phi p^i A^i+\phi A^i p^i+p^i\phi A^i+\phi p^i A^i+2e\phi A^i A^i\right)$$
$$-\frac{1}{2}\frac{m}{\sqrt{\pi^0-m}}\sqrt{\frac{e}{\lambda_f\phi\mathbf{A}^2}}\chi^{(s)T}\left(\varepsilon^{ijk}\sigma^i\partial^j\phi A^k-\phi\sigma^i B^i\right)\chi^{(s)} \quad . \tag{7.8}$$

Using $\pi^0=E+e\phi$, in the rest frame **p**=0 thus $E=m$ thus $\pi^0-m=e\phi$, this becomes greatly simplified down to:

$$A\left(p,0,p,\mathbf{p}=0\right)=-m\frac{1}{\sqrt{\lambda_f}}e\sqrt{\mathbf{A}^2}-\frac{1}{2}m\frac{1}{\sqrt{\lambda_f}}\frac{1}{\phi\sqrt{\mathbf{A}^2}}\chi^{(s)T}\left(\varepsilon^{ijk}\sigma^i\partial^j\phi A^k-\phi\sigma^i B^i\right)\chi^{(s)}\ . \tag{7.9}$$

Next we use this in the rest frame Lagrangian density (6.19). Two terms cancel identically via $me\sqrt{\mathbf{A}^2}-me\sqrt{\mathbf{A}^2}$, and λ_f drops out via the $\sqrt{\lambda_f}\,/\sqrt{\lambda_f}=1$. We factor out the lead coefficient, an overall negative sign washes out in the square, and we then obtain:

$$\begin{aligned}\mathfrak{L}\left(h_\psi\right)&=\mathfrak{L}\left(p,0,p,\mathbf{p}=0\right)=T\left(h_\psi\right)-V\left(h_\psi\right)\\&=\overline{h_\psi}\pi h_\psi+\overline{v_+^{1.5}}\pi h_\psi+\overline{h_\psi}\pi v_+^{1.5}+\overline{v_+^{1.5}}\pi v_+^{1.5}\\&\quad-\left(1-\frac{1}{2}\frac{\overline{h_\psi}h_\psi}{v_+^3}-2\frac{m^3}{v_+^3}\right)m\overline{h_\psi}h_\psi+\frac{1}{2}m\overline{v_+^{1.5}}v_+^{1.5}-4m^4\ \cdot\\&\quad-\frac{1}{4}\frac{m^2}{\phi^2\mathbf{A}^2}\left(\chi^{(s)T}\sigma^i\left(\varepsilon^{ijk}\partial^j\phi A^k-\phi B^i\right)\chi^{(s)}\right)^2\end{aligned} \tag{7.10}$$

A second order magnetic moment term $\left(\sigma^i B^i\right)^2=\left(\boldsymbol{\sigma}\cdot\mathbf{B}\right)^2$ has explicitly entered through the potential terms $V\left(h_\psi\right)$, as well as the electric field via $\partial^j\phi=-\nabla\phi=\mathbf{E}+\partial\mathbf{A}/\partial t$. Depending on circumstance, we can recast (7.10) using the equivalent formulations:

$$\sigma^i\left(\varepsilon^{ijk}\partial^j\phi A^k-\phi B^i\right)=\boldsymbol{\sigma}\cdot\left(-\nabla\phi\times\mathbf{A}\right)-\phi\boldsymbol{\sigma}\cdot\mathbf{B}=\boldsymbol{\sigma}\cdot\left(\mathbf{E}+\partial\mathbf{A}/\partial t\right)-\phi\boldsymbol{\sigma}\cdot\mathbf{B}\ . \tag{7.11}$$

The discussion so far has been centered in the potential sector *V* (3.11) of the Lagrangian (3.9). Now, we move over to focus on the kinetic sector *T* (3.10), and with it, an exploration of magnetic moments.

PART II: REVEALING FERMION MAGNETIC MOMENTS, AND THE RENORMALIZATION OF FERMION MASSES – LAGRANGIAN KINETICS

8. Gordon Decomposition of the "Seed Fermion" Electric Current Density

It is well-known that Dirac's equation predicts a gyromagnetic g-factor $g=2$ unless supplemented by the work of Schwinger [23] and others who have calculated the one-loop and higher anomalous corrections to this magnetic moment. By calculating higher-order loops, one finds an explanation for the *deviation* of the observed g-factor $g=2.00231930436153$ [24] from Dirac's $g=2$. However, it will also be recognized in the present context that the Dirac equation for a seed fermion ψ in the form of $(i\not{D}-m)\psi=0$ is the equation from which one obtains the result $g=2$. If, however, ψ is *not* an observed fermion, and if the observed fermion is in fact the Higgs field h_ψ which expands ψ about the vacuum (just as *h* but not ϕ is observed in the Higgs theory of scalars), then one might also supplement the loop approach by also understanding the observed $g=2.00231930436153$ to be the g-factor *for the observed Higgs field* h_ψ, with $g=2$ being the g-factor of the unobserved seed field ψ.

To begin a detailed exploration of the magnetic moment of the Higgs fermions h_ψ, it will be helpful to review the Gordon decomposition. In the same way that the Lagrangian density term $-m\overline{\psi}\psi$ guided us in section 6 to look for terms with $\overline{h_\psi}h_\psi$, we will use the Gordon decomposition to provide guidance as to how to look for the g-factor of the Higgs fermions. Mathematically, the heart of the Gordon decomposition is the identity $\gamma^\mu\gamma^\nu=\eta^{\mu\nu}-i\sigma^{\mu\nu}$ which is easily obtained by combining the definitions $2\eta^{\mu\nu}\equiv\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu$ and $-2i\sigma^{\mu\nu}\equiv\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu$. As to the space components of this identity $\gamma^\mu\gamma^\nu=\eta^{\mu\nu}-i\sigma^{\mu\nu}$, it is easily shown in the Dirac representation of the γ^i that $\gamma^i\gamma^j=-\sigma^i\sigma^j$. Therefore, given that the group structure relationship for the Pauli matrices is $\frac{1}{2}\left[\sigma^i,\sigma^j\right]=i\varepsilon^{ijk}\sigma^k$ with $\varepsilon^{ijk}\equiv+1$, it is also easily shown that $-i\sigma^{ij}=-i\varepsilon^{ijk}\sigma^k$. And of course, $\eta^{ij}=-\delta^{ij}$ with $\mathrm{diag}(\eta^{\mu\nu})=(1,-1,-1,-1)$. Therefore, the spacetime identity $\gamma^\mu\gamma^\nu=\eta^{\mu\nu}-i\sigma^{\mu\nu}$ leads directly to the (negative of) the space-only identity $\sigma^i\sigma^j=\delta^{ij}+i\varepsilon^{ijk}\sigma^k$. This of course, is (7.3) which we already used to develop the magnetic moment terms in the last section, so in fact we have already done a Gordon decomposition to reveal the magnetic moment term $-\phi\boldsymbol{\sigma}\cdot\mathbf{B}$ which first appeared in (7.6) along with other terms for the time-dependency of the gauge field.

With this background, let us approach the Gordon decomposition using

$$\mathfrak{L}=i\overline{\psi}\gamma^\mu\partial_\mu\psi+e\overline{\psi}\gamma^\mu\psi A_\mu-\mu\overline{\psi}\psi-\lambda_f\left(\overline{\psi}\psi\right)^2, \tag{8.1}$$

which is our original (3.8) to which we appended the $-\lambda_f\left(\overline{\psi}\psi\right)^2$ term in (3.9). The mass parameter μ was first defined in (3.4) and later developed into $m=\frac{1}{2}e\left(\mathbf{A}^2/\phi-\phi\right)=-\mu>0$ in (6.17). The Dirac equation corresponding to the first three terms in (8.1) is $i\gamma^\mu\partial_\mu\psi+e\gamma^\mu\psi A_\mu-\mu\psi=0$, and the related adjoint equation is $i\partial_\mu\overline{\psi}\gamma^\mu+e\overline{\psi}\gamma^\mu A_\mu+\mu\overline{\psi}=0$. We then take this pair of equations and rewrite them for a free field ($A_\mu=0$) as:

$$\begin{cases}\frac{1}{2\mu}\left(i\gamma^\mu\partial_\mu\psi\right)=\frac{1}{2}\psi\\ \frac{1}{2\mu}\left(-i\partial_\mu\overline{\psi}\gamma^\mu\right)=\frac{1}{2}\overline{\psi}\end{cases}. \tag{8.2}$$

We then use the identity $\gamma^\mu\gamma^\nu=\eta^{\mu\nu}-i\sigma^{\mu\nu}$ just discussed while multiplying the first equation from the left by $\overline{\psi}\gamma^\nu$ and the second equation from the right by $\gamma^\nu\psi$ to obtain:

$$\begin{cases}\frac{1}{2\mu}\left(i\overline{\psi}\gamma^\nu\gamma^\mu\partial_\mu\psi\right)=\frac{1}{2\mu}\left(i\overline{\psi}\left(\eta^{\mu\nu}-i\sigma^{\nu\mu}\right)\partial_\mu\psi\right)=\frac{i}{2\mu}\overline{\psi}\partial^\nu\psi-\frac{1}{2\mu}\overline{\psi}\sigma^{\mu\nu}\partial_\mu\psi=\frac{1}{2}\overline{\psi}\gamma^\nu\psi\\ \frac{1}{2\mu}\left(-i\partial_\mu\overline{\psi}\gamma^\mu\gamma^\nu\psi\right)=\frac{1}{2\mu}\left(-i\partial_\mu\overline{\psi}\left(\eta^{\nu\mu}-i\sigma^{\mu\nu}\right)\psi\right)=-\frac{i}{2\mu}\partial^\nu\overline{\psi}\psi-\frac{1}{2\mu}\partial_\mu\overline{\psi}\sigma^{\mu\nu}\psi=\frac{1}{2}\overline{\psi}\gamma^\nu\psi\end{cases}. \tag{8.3}$$

Then, combining, we arrive at the Gordon decomposition:

$$\overline{\psi}\gamma^\nu\psi=\frac{i}{2\mu}\left(\overline{\psi}\partial^\nu\psi-\partial^\nu\overline{\psi}\psi\right)-\frac{g}{2}\frac{1}{2\mu}\partial_\mu\left(\overline{\psi}\sigma^{\mu\nu}\psi\right). \tag{8.4}$$

where we have also inserted $g/2=1$ in the appropriate position, knowing that with the spin defined as $\mathbf{S}^{\mu\sigma}\equiv\frac{1}{2}\sigma^{\mu\sigma}$, the coefficients of $\left(1/2\mu\right)\partial_\sigma\left(\overline{\psi}\sigma^{\mu\sigma}\psi\right)=2\left(1/2\mu\right)\partial_\sigma\left(\overline{\psi}\mathbf{S}^{\mu\sigma}\psi\right)$ represent $g/2$ and g, respectively, see [18], eq. [9.138]. We now use this to replace the current density $J^\mu=e\overline{\psi}\gamma^\mu\psi$ in (8.1) and so write:

$$\mathfrak{L}=i\overline{\psi}\gamma^\mu\partial_\mu\psi+i\frac{e}{2\mu}\left(\overline{\psi}\partial_\mu\psi-\partial_\mu\overline{\psi}\psi\right)A^\mu+\frac{g}{2}\frac{e}{2\mu}\partial_\sigma\left(\overline{\psi}\sigma^{\mu\sigma}\psi\right)A_\mu-\mu\overline{\psi}\psi-\lambda_f\left(\overline{\psi}\psi\right)^2=T-V, \tag{8.5}$$

As noted just before (8.1), $\sigma^{ij}=\varepsilon^{ijk}\sigma^k$, so in three space dimensions the spin matrix is $\frac{1}{2}\sigma^{ij}=\mathbf{S}^{ij}=\frac{1}{2}\varepsilon^{ijk}\sigma^k=\varepsilon^{ijk}\mathbf{S}^k$. There are now three main kinetic terms in the Lagrangian density. First is $i\overline{\psi}\gamma^\mu\partial_\mu\psi$ for the translational motion of the electron. This will generally go to zero at rest. Second is $i\left(e/2\mu\right)\left(\overline{\psi}\partial^\mu\psi-\partial^\mu\overline{\psi}\psi\right)$ which is the convection current and contains any orbital angular momentum associated with the electron. Third is $\left(e/2\mu\right)\partial_\sigma\left(\overline{\psi}\sigma^{\mu\sigma}\psi\right)A_\mu$ which is for the magnetic moment.

The fact that $g/2=1$ in (8.5) is what leads us to state that Dirac's equation predicts a g-factor $g=2$. The fact that the observed g-factors are somewhat "north" of 2 by a small amount was first thought to be an "anomaly," but was then explained by Schwinger [23] followed by others as resulting from loop diagrams at various orders. But (8.5) is for "seed" fermions. If the observed fermions are Higgs fields expanded about the vacuum as developed for the mass terms in Part I, then the observed g-factor should be found in the coefficient – not of a term $\partial_\sigma\left(\overline{\psi}\sigma^{\mu\sigma}\psi\right)$ which contains the seed fields as in (8.5) – but of a like-term that contains the Higgs fermions h_ψ. One may also anticipate that such a term for the Higgs fields will reveal the "anomaly" to actually be an inherent feature of Dirac's equation, which then maps over to the usual high-loop approaches in an orderly manner and may help at the same time to gain a better handle on renormalization issues.

The next few steps from (8.5) are relatively basic, but they need to be done carefully so we take them step by step. First, focusing on the kinetic terms *T* while applying the result (6.17) to set $\mu=-m$, we write:

$$T = i\overline{\psi}\gamma^\mu\partial_\mu\psi - i\frac{e}{2m}\left(\overline{\psi}\partial_\mu\psi - \partial_\mu\overline{\psi}\psi\right)A^\mu - \frac{g}{2}\frac{e}{2m}\partial_\sigma\left(\overline{\psi}\sigma^{\mu\sigma}\psi\right)A_\mu\,. \tag{8.6}$$

Keeping $g/2=1$ as a "placeholder," we first separate the space and time components as such:

$$\begin{aligned}T &= i\overline{\psi}\gamma^0\partial_0\psi + i\overline{\psi}\gamma^k\partial_k\psi - i\frac{e}{2m}\left(\overline{\psi}\partial_0\psi - \partial_0\overline{\psi}\psi\right)A^0 - i\frac{e}{2m}\left(\overline{\psi}\partial_k\psi - \partial_k\overline{\psi}\psi\right)A^k \\ &\quad - \frac{g}{2}\frac{e}{2m}\partial_0\left(\overline{\psi}\sigma^{\mu 0}\psi\right)A_\mu - \frac{g}{2}\frac{e}{2m}\partial_k\left(\overline{\psi}\sigma^{\mu k}\psi\right)A_\mu\end{aligned}\,. \tag{8.7}$$

Next we convert (8.7) into momentum space with $\psi(p)$ and $\overline{\psi}(p')$ where as earlier at the start of section 6, $p'^\mu = p^\mu + q^\mu$. For the moment, therefore, we will consider $T(p,q,p+q)$. This means that we use apply the product rule and then use $i\partial_\mu\psi = p_\mu\psi$, $i\partial_\mu\overline{\psi} = -p'_\mu\overline{\psi}$ and then raise all indexes with $\operatorname{diag}\left(\eta^{\mu\nu}\right) = (1,-1,-1,-1)$ to arrive at:

$$\begin{aligned}T(p,q,p+q) &= \overline{\psi}\gamma^0 p^0\psi - \overline{\psi}\gamma^k p^k\psi - \frac{e}{2m}\left(\overline{\psi}p^0\psi + p'^0\overline{\psi}\psi\right)A^0 + \frac{e}{2m}\left(\overline{\psi}p^k\psi + p'^k\overline{\psi}\psi\right)A^k \\ -\frac{g}{2}\frac{e}{2m}\left(ip'^0\overline{\psi}\sigma^{\mu 0}\psi\right)A_\mu &+ \frac{g}{2}\frac{e}{2m}\left(\overline{\psi}\sigma^{\mu 0}ip^0\psi\right)A_\mu + \frac{g}{2}\frac{e}{2m}\left(ip'^k\overline{\psi}\sigma^{\mu k}\psi\right)A_\mu - \frac{g}{2}\frac{e}{2m}\left(\overline{\psi}\sigma^{\mu k}ip^k\psi\right)A_\mu\end{aligned}\,. \tag{8.8}$$

Finally, we fully separate $\sigma^{\mu\nu}A_\mu$ into its time and space component and the raise the index on the components of A_μ. In the process we lose two terms because of $\sigma^{00}=0$. We now obtain:

$$T\left(p,q,p+q\right)=\overline{\psi}\gamma^0 p^0\psi-\overline{\psi}\gamma^k p^k\psi-\frac{e}{2m}\left(\overline{\psi}p^0\psi+p'^0\overline{\psi}\psi\right)A^0+\frac{e}{2m}\left(\overline{\psi}p^k\psi+p'^k\overline{\psi}\psi\right)A^k$$
$$+\frac{g}{2}\frac{e}{2m}\left(ip'^0\overline{\psi}\sigma^{j0}\psi\right)A^j-\frac{g}{2}\frac{e}{2m}\left(\overline{\psi}\sigma^{j0}ip^0\psi\right)A^j \quad .(8.9)$$
$$+\frac{g}{2}\frac{e}{2m}\left(ip'^k\overline{\psi}\sigma^{0k}\psi\right)A^0-\frac{g}{2}\frac{e}{2m}\left(ip'^k\overline{\psi}\sigma^{jk}\psi\right)A^j-\frac{g}{2}\frac{e}{2m}\left(\overline{\psi}\sigma^{0k}ip^k\psi\right)A^0+\frac{g}{2}\frac{e}{2m}\left(\overline{\psi}\sigma^{jk}ip^k\psi\right)A^j$$

Now, in the bottom line of (8.8), using the Feynman slash notation $\not{A}\equiv\gamma^\mu A_\mu$, we have terms of the general form $\sigma^{\mu\nu}A_\mu=\tfrac{1}{2}i\left[\gamma^\mu,\gamma^\nu\right]A_\mu=\tfrac{1}{2}i\left[\not{A},\gamma^\nu\right]$. The point is that $\sigma^{\mu\nu}$ is an "anchor" which holds the commutation position of A_μ. At the same time we have terms of the form $p'^\nu\sigma^{\mu\nu}$ whereby p' enters to the left of $\sigma^{\mu\nu}$ and thus to the left of A_μ, and of $\sigma^{\mu\nu}p^\nu$ whereby p enters to the right of $\sigma^{\mu\nu}$ and thus to the right of A_μ. Carefully maintaining this ordering among p, p' and A_μ, we "pin" the A^μ immediately to the right of $\sigma^{\mu\nu}$ and keep the p, p' in their order relative to $\sigma^{\mu\nu}$, to restructure (8.9) into:

$$T\left(p,q,p+q\right)=\overline{\psi}\gamma^0 p^0\psi-\overline{\psi}\gamma^k p^k\psi-\frac{e}{2m}\overline{\psi}\left(p'^0+p^0\right)\psi A^0+\frac{e}{2m}\overline{\psi}\left(p'^k+p^k\right)\psi A^k$$
$$+i\frac{g}{2}\frac{e}{2m}\left\{\overline{\psi}\left[p'^0\sigma^{j0}A^j-\sigma^{j0}A^j p^0\right]\psi+\overline{\psi}\left[p'^k\sigma^{0k}A^0-\sigma^{0k}A^0p^k\right]\psi-\overline{\psi}\left[p'^k\sigma^{jk}A^j-\sigma^{jk}A^jp^k\right]\psi\right\} \quad .(8.10)$$

Note in the above that we write $\overline{\psi}\left(p'^\mu+p^\mu\right)\psi$ with $p'^\mu+p^\mu$ between the fermion wavefunctions recognizing that p'^μ operates on (and is to the right of) $\overline{\psi}$ and p^μ operates on (and is to the left of) ψ. This is pertinent to being able to identify the orbital angular momentum via commutator relationship $\left[H,\mathbf{L}\right]=-i\left(\boldsymbol{\alpha}\times\mathbf{p}\right)$, which we review in section 15. Finally, we are ready to take the $p=p'$ and $q=0$ limit to obtain the $T\left(p,0,p\right)$ which aligns with the Ward identity. We also now move the $\sigma^{\mu\nu}$ to the left of each expression in which it appears to emphasize the commutator of p^μ and A^μ. This turns (8.10) into:

$$T\left(p,0,p\right)=\overline{\psi}\gamma^0 p^0\psi-\overline{\psi}\gamma^k p^k\psi-\frac{e}{m}\overline{\psi}\left(p^0A^0-p^kA^k\right)\psi$$
$$+i\frac{g}{2}\frac{e}{2m}\left\{\overline{\psi}\sigma^{j0}\left[p^0,A^j\right]\psi+\overline{\psi}\sigma^{0k}\left[p^k,A^0\right]\psi-\overline{\psi}\sigma^{jk}\left[p^k,A^j\right]\psi\right\} . \quad (8.11)$$

Had we not been concerned to take special care of the ordering of p^μ and A^μ, which is to say if one treated the commutator $\left[p^\mu,A^\nu\right]$ as if it was zero, then the entire bottom line of (8.11) would go to zero by virtue of taking $p'-p=0$. But with the commutators, we still have a non-zero expression, and this is the place from which the magnetic moment arises together with the time-dependencies of the electric and magnetic fields to reveal Maxwell's equations amidst the Dirac equation.

Specifically, with $A^0 = \phi$, we once again apply $\left[p^i, \phi \right] = -i\partial^i \phi$ and $\left[p^i, A^j \right] = -i\partial^i A^j$ which we used to arrive at (7.6). Further, with $p^0 = E$ and $H\psi = E\psi$ where H is the Hamiltonian operator, we apply the Heisenberg picture equation of motion $\partial^0 A^j = i\left[H, A^j \right]$ for an A^j which as noted following (4.21) has no *ab initio* time dependency but acquires a time dependency exclusively from its commutation with the Hamiltonian. With some index renaming this turns (8.11) into:

$$\begin{aligned} T(p,0,p) &= \overline{\psi}\gamma^0 p^0 \psi - \overline{\psi}\gamma^k p^k \psi - \frac{e}{m}\overline{\psi}\left(p^0 A^0 - p^k A^k \right)\psi \\ &\quad + \frac{e}{2m}\frac{g}{2}\left(\overline{\psi}\sigma^{i0}\psi\partial^0 A^i + \overline{\psi}\sigma^{0i}\psi\partial^i \phi + \overline{\psi}\sigma^{ij}\psi\partial^i A^j \right) \end{aligned} . \tag{8.12}$$

We now have a first appearance of various terms $\partial^\mu A^\nu$ which are of course related to the electromagnetic field strength $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, and they contain time and space dependencies of the gauge fields A^μ which are revealed simply from the commutators found through a careful dissection of Dirac's equation.

Now let's work with $\sigma^{\mu\nu}$. From $\sigma^{\mu\nu} = \frac{1}{2} i\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu \right)$ as well as the SU(2) group structure relation $\left[\sigma^j, \sigma^i \right] = 2i\varepsilon^{jik}\sigma^k$ it is straightforward to use the Dirac γ^μ in the Dirac representation in customary manner to deduce that:

$$\sigma^{0i} = i\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \equiv i\alpha^i; \quad \sigma^{ij} = \varepsilon^{ijk}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \equiv \varepsilon^{ijk}\Sigma^k . \tag{8.13}$$

We then may use these to write (8.12) as:

$$\begin{aligned} T(p,0,p) &= \overline{\psi}\gamma^0 p^0 \psi - \overline{\psi}\gamma^i p^i \psi - \frac{e}{m}\overline{\psi}\left(p^0 A^0 - p^i A^i \right)\psi \\ &\quad + \frac{g}{2}\frac{e}{2m}\left(i\overline{\psi}\alpha^i\psi\left(\partial^i\phi - \partial^0 A^i \right) + \varepsilon^{ijk}\overline{\psi}\Sigma^k\psi\partial^i A^j \right) \end{aligned} . \tag{8.14}$$

Now we may explicitly show the components of the electric and magnetic fields. Specifically, one uses the space components of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ as well as $-B^k = \frac{1}{2}\varepsilon^{ijk}F^{ij}$ to show that $\varepsilon^{ijk}\partial^i A^j = \frac{1}{2}\varepsilon^{ijk}F^{ij} = -B^k$. Additionally, reverting $\phi \to A^0$, we have $F^{i0} = \partial^i A^0 - \partial^0 A^i = E^i$. So with this (8.14) becomes:

$$T(p,0,p) = \overline{\psi}\gamma^0 p^0 \psi - \overline{\psi}\gamma^i p^i \psi - \frac{e}{m}\overline{\psi}\left(p^0 A^0 - p^i A^i \right)\psi + \frac{g}{2}\frac{e}{2m}\left(i\overline{\psi}\alpha^i E^i\psi - \overline{\psi}\Sigma^i B^i\psi \right). \tag{8.15}$$

The magnetic moment term thus shows up explicitly in the form $-\left(g/2\right)\left(e/2m\right)\overline{\psi}\Sigma^k B^k\psi$. In 2x2 Dirac component form using (8.13), this becomes:

$$\begin{aligned}T\left(p,0,p\right)&=\overline{\psi}\gamma^0 p^0\psi-\overline{\psi}\gamma^i p^i\psi-\frac{e}{m}\overline{\psi}\left(p^0A^0-p^iA^i\right)\psi+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi\\&=\overline{\psi}\gamma^0 p^0\psi-\overline{\psi}\gamma^i p^i\psi-\frac{e}{m}\overline{\psi}\left(p^0A^0-p^iA^i\right)\psi+\frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix}-\sigma^iB^i & i\sigma^iE^i\\ i\sigma^iE^i & -\sigma^iB^i\end{pmatrix}\psi\end{aligned}. \tag{8.16}$$

So $-\left(g/2\right)\left(e/2m\right)\overline{\psi}\Sigma^k B^k\psi=-\left(1\right)\left(e/2m\right)\overline{\psi}\Sigma^k B^k\psi$ is yet another way of saying that in Dirac theory for a seed field ψ, the g-factor $g=2$.

Now we are ready to expand this about the fermion vacuum via $\psi\left(x\right)\equiv v'^{1.5}+h_f\left(x\right)$ and $\overline{\psi}\left(x\right)\equiv\overline{v'^{1.5}}+\overline{h_f}\left(x\right)$ of (4.17) and (4.18) to obtain the observed Higgs fermions h_f particularly the like term $\overline{h_\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)h_\psi$ rather than $\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi$, and to see what then happens to the g-factor. Via (4.17) and (4.18), we may "dissect" the desired term $\overline{h_\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)h_\psi$ into:

$$\begin{aligned}&\overline{h_\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)h_\psi=\left(\overline{\psi}-\overline{v_+'^{1.5}}\right)\left(i\alpha^iE^i-\Sigma^iB^i\right)\left(\psi-v_+'^{1.5}\right)\\&=\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi+\overline{v_+^{1.5}}\left(i\alpha^iE^i-\Sigma^iB^i\right)v_+'^{1.5}-\overline{v_+'^{1.5}}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi-\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)v_+'^{1.5}\end{aligned}. \tag{8.17}$$

There are very similar in form to the terms $\overline{v_+'^{1.5}}h_\psi+\overline{h_\psi}v_+'^{1.5}$ studied in section 6 and to $\overline{v_+'^{1.5}}v_+'^{1.5}=\overline{v_+^{1.5}}v_+^{1.5}=v_+^3$ developed in section 4, but for the sandwiched term $\left(i\alpha^iE^i-\Sigma^iB^i\right)$. In section 9 we develop $\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi$. In section 10 we develop $\overline{v_+^{1.5}}\left(i\alpha^iE^i-\Sigma^iB^i\right)v_+'^{1.5}$. In section 11 we develop $\overline{v_+'^{1.5}}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi+\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)v_+'^{1.5}$. In section 12 we put everything together to identify the g-factor (there are really three different g-factors) associated with $\overline{h_\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)h_\psi$. In section 13 we use these results for empirical predictions.

9. Magnetic Moments of Seed Fermions

To develop $\overline{\psi}\left(i\alpha^iE^i-\Sigma^iB^i\right)\psi$ we first make use of the fermion particle (positive energy) wavefunction ψ from (6.26) and its adjoint $\overline{\psi}=\psi^\dagger\gamma^0$ under the $\left(p,0,p\right)$ condition of the Ward identity. For the magnetic moment term in (8.16), this gives us:

$$+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)\psi=+\frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\psi$$

$$=+\frac{g}{2}\frac{em}{2}\left(\pi^0-m\right)\begin{bmatrix}\chi^{T(s)} & -\chi^{T(s)}\dfrac{\sigma^i\pi^i}{\pi^0-m}\end{bmatrix}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\begin{bmatrix}\chi^{(s)}\\ \dfrac{\sigma^k\pi^k}{\pi^0-m}\chi^{(s)}\end{bmatrix}, \tag{9.1}$$

$$=+\frac{g}{2}\frac{em}{2}\left(\pi^0-m\right)\begin{pmatrix}-\chi^{T(s)}\sigma^j B^j\chi^{(s)}+\chi^{T(s)}\dfrac{\sigma^i\pi^i}{\pi^0-m}\sigma^j B^j\dfrac{\sigma^k\pi^k}{\pi^0-m}\chi^{(s)}\\ +i\chi^{T(s)}\sigma^j E^j\dfrac{\sigma^k\pi^k}{\pi^0-m}\chi^{(s)}-i\chi^{T(s)}\dfrac{\sigma^i\pi^i}{\pi^0-m}\sigma^j E^j\chi^{(s)}\end{pmatrix}$$

Now let's develop this expression. Using standard rearrangement, being careful not to alter the position of $\pi^i=p^i+eA^i$ which contains a momentum p^i, and segregating the σ^i out front following some index renaming while carefully maintaining their indexed ordering, we obtain:

$$+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)\psi=+\frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\psi$$

$$=+\frac{g}{2}\frac{em}{2}\left(\frac{\chi^{T(s)}\left(-\left(\pi^0-m\right)^2\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\sigma^i\sigma^j\sigma^k\pi^i B^j\pi^k\right)\chi^{(s)}}{\pi^0-m}+i\chi^{T(s)}\sigma^i\sigma^j\left(E^i\pi^j-\pi^i E^j\right)\chi^{(s)}\right). \tag{9.2}$$

Now we turn once again to the identity $\sigma^i\sigma^j=\delta^{ij}+i\varepsilon^{ijk}\sigma^k$ which is a corollary of $\gamma^\mu\gamma^\nu=\eta^{\mu\nu}-i\sigma^{\mu\nu}$ as summarized prior to (8.1). This means that using $\pi^i=p^i+eA^i$, $\chi^{T(s)}\chi^{(s)}=1$ and $\left[E^i,A^i\right]=0$ thus $\varepsilon^{ijk}\sigma^k\left(E^iA^j-A^iE^j\right)=0$, the latter term in (9.2) becomes:

$$\begin{aligned}&i\chi^{T(s)}\sigma^i\sigma^j\left(E^i\pi^j-\pi^iE^j\right)\chi^{(s)}=i\left(E^i\pi^i-\pi^iE^i\right)-\chi^{T(s)}\varepsilon^{ijk}\sigma^k\left(E^i\pi^j-\pi^iE^j\right)\chi^{(s)}\\&=i\left(E^ip^i-p^iE^i+E^ieA^i-eA^iE^i\right)-\chi^{T(s)}\varepsilon^{ijk}\sigma^k\left(E^ip^j-p^iE^j+E^ieA^j-eA^iE^j\right)\chi^{(s)}\\&=-i\left[p^i,E^i\right]+\chi^{T(s)}\varepsilon^{ijk}\sigma^k\left[p^i,E^j\right]\chi^{(s)}\\&=-\partial^iE^i-\chi^{T(s)}i\varepsilon^{ijk}\sigma^k\partial^iE^j\chi^{(s)}\end{aligned}. \tag{9.3}$$

Above, we have now used the commutator $\left[p^i,E^j\right]=-i\partial^iE^j$, which actually starts to unfold terms from the electric and magnetic *current densities* of Maxwell's equations. This is yet another example of a space-dependency – this time for the electric field **E** – remerging through a Heisenberg-type commutation relationship with canonical momentum. Specifically, starting with $J^\mu=\partial_\sigma F^{\sigma\mu}$, we have $\rho\equiv J^0=\partial_iF^{i0}=\partial_iE^i=-\partial^iE^i=\nabla\cdot\mathbf{E}$. Additionally, with

$\nabla = \partial_i = -\partial^i$ hence $\varepsilon^{ijk}\partial^i E^j = (-\nabla\times\mathbf{E})^k$, the $i\varepsilon^{ijk}\sigma^k\partial^i E^j$ term evaluates out to $i\varepsilon^{ijk}\sigma^k\partial^i E^j = i\sigma^k(-\nabla\times\mathbf{E})^k$. Thus, we consolidate (9.3) into:

$$i\chi^{T(s)}\sigma^i\sigma^j\left(E^i\pi^j - \pi^i E^j\right)\chi^{(s)} = \nabla\cdot\mathbf{E} + i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)} = \rho - i\chi^{T(s)}i\boldsymbol{\sigma}\cdot(\partial\mathbf{B}/\partial t)\chi^{(s)}, \tag{9.4}$$

explicitly applying two of four Maxwell equations namely Gauss' law $\nabla\cdot\mathbf{E} = \rho$ and Faraday's law $\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$. So this now reveals a time-dependency $\partial\mathbf{B}/\partial t$ for the magnetic field.

Next let's turn to the term $\chi^{T(s)}\sigma^i\sigma^j\sigma^k\pi^i B^j\pi^k\chi^{(s)}$ in (9.2). This contains the product $\sigma^i\sigma^j\sigma^k$ of *three* spin matrices which we now must deconstruct. Here we use the identity $\sigma^i\sigma^j = \delta^{ij} + i\varepsilon^{ijk}\sigma^k$ in succession as well as $\varepsilon^{ijl}\varepsilon^{kml} = \varepsilon^{ij}\varepsilon^{km} = \delta^{ik}\delta^{jm} - \delta^{im}\delta^{jk}$ to obtain (this can also be validated by explicit use of the σ^i):

$$\begin{aligned}\sigma^i\sigma^j\sigma^k &= \left(\delta^{ij} + i\varepsilon^{ijl}\sigma^l\right)\sigma^k = \delta^{ij}\sigma^k + i\varepsilon^{ijl}\sigma^l\sigma^k = \delta^{ij}\sigma^k + i\varepsilon^{ijl}\left(\delta^{lk} + i\varepsilon^{lkm}\sigma^m\right)\\ &= \delta^{ij}\sigma^k + i\varepsilon^{ijl}\delta^{lk} - \varepsilon^{ijl}\varepsilon^{kml}\sigma^m = \delta^{ij}\sigma^k + i\varepsilon^{ijl}\delta^{lk} - \left(\delta^{ik}\delta^{jm} - \delta^{im}\delta^{jk}\right)\sigma^m\,.\\ &= \delta^{ij}\sigma^k + \delta^{jk}\sigma^i - \delta^{ik}\sigma^j + i\varepsilon^{ijk}\end{aligned} \tag{9.5}$$

We then use (9.5) to write:

$$\begin{aligned}&\chi^{T(s)}\sigma^i\sigma^j\sigma^k\pi^i B^j\pi^k\chi^{(s)} = \chi^{T(s)}\left(\delta^{ij}\sigma^k + \delta^{jk}\sigma^i - \delta^{ik}\sigma^j + i\varepsilon^{ijk}\right)\pi^i B^j\pi^k\chi^{(s)}\\ &= \chi^{T(s)}\left(\sigma^i\pi^j B^j\pi^i + \sigma^i\pi^i B^j\pi^j - \sigma^i\pi^j B^i\pi^j + i\varepsilon^{ijk}\pi^i B^j\pi^k\right)\chi^{(s)}\end{aligned}\,. \tag{9.6}$$

We may write $\sigma^i\pi^j B^j\pi^i + \sigma^i\pi^i B^j\pi^j = (\boldsymbol{\pi}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi}) + (\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\mathbf{B}\cdot\boldsymbol{\pi})$ for the first two terms because σ^i are constants and can be commuted except with other σ^i. But $\boldsymbol{\pi} = \mathbf{p} + e\mathbf{A}$ and the magnetic field $\mathbf{B} = \nabla\times\mathbf{A}$, so with identity $\mathbf{A}\cdot(\nabla\times\mathbf{A})$ for *any* vector **A**, we have the reduction:

$$\pi^j B^j = \boldsymbol{\pi}\cdot\mathbf{B} = (\mathbf{p} + e\mathbf{A})\cdot(\nabla\times\mathbf{A}) = \mathbf{p}\cdot(\nabla\times\mathbf{A}) + \mathbf{A}\cdot(\nabla\times\mathbf{A}) = \mathbf{p}\cdot(\nabla\times\mathbf{A}) = \mathbf{p}\cdot\mathbf{B} = p^j B^j\,. \tag{9.7}$$

Similarly for $B^j\pi^j$. Therefore:

$$\sigma^i\pi^j B^j\pi^i + \sigma^i\pi^i B^j\pi^j = p^j B^j\sigma^i\pi^i + \sigma^i\pi^i B^j p^j = (\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi}) + (\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\mathbf{B}\cdot\mathbf{p})\,. \tag{9.8}$$

In the rest frame, this term thus becomes zero.

As to the term $-\sigma^i\pi^j B^i\pi^j$, we leave this exactly as is, noting that with $\boldsymbol{\pi} = \mathbf{p} + e\mathbf{A}$, in the rest frame, $-\sigma^i\pi^j B^i\pi^j \to -\sigma^i eA^j B^i eA^j = -e^2\mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}$, and so this term is part of the fermion magnetic moment.

Now we return to (9.6) to develop $i\varepsilon^{ijk}\pi^i B^j \pi^k$, using $\pi^k = p^k + eA^k$. In *Abelian* field theory the term with $\varepsilon^{ijk} A^i B^j A^k = 0$ by identity. As noted earlier after (7.4), if one wanted to develop this for a Yang-Mills theory such as the weak or strong interaction, then one would *not* set this term to zero. But here we stick to Abelian theory so terms $A^i B^j A^k - A^k B^j A^i = B^j \left[A^i, A^k \right] = 0$ drop out. Also, we are able to commute $p^i B^j A^k = A^k p^i B^j$. We also use the commutator $i\left[p^i, B^j \right] = \partial^i B^j$ as well as $\nabla = \partial_i = -\partial^i$ and $\left(-\nabla \times \mathbf{B}\right)^i = \varepsilon^{ijk}\partial^j B^k$, and at the end apply Ampere's law $\nabla \times \mathbf{B} = \mathbf{J} + \partial \mathbf{E} / \partial t$. The result is:

$$
\begin{aligned}
& i\varepsilon^{ijk}\pi^i B^j \pi^k = i\varepsilon^{ijk} p^i B^j p^k + i\varepsilon^{ijk} eA^i B^j p^k + i\varepsilon^{ijk} p^i B^j eA^k + i\varepsilon^{ijk} eA^i B^j eA^k \\
& = ip^3 B^1 p^2 - ip^2 B^1 p^3 + ip^1 B^2 p^3 - ip^3 B^2 p^1 + ip^2 B^3 p^1 - ip^1 B^3 p^2 \\
& + ip^3 B^1 eA^2 - ip^2 B^1 eA^3 + ip^1 B^2 eA^3 - ip^3 B^2 eA^1 + ip^2 B^3 eA^1 - ip^1 B^3 eA^2 \\
& + ieA^3 B^1 p^2 - ieA^2 B^1 p^3 + ieA^1 B^2 p^3 - ieA^3 B^2 p^1 + ieA^2 B^3 p^1 - ieA^1 B^3 p^2 \\
& = \left(\partial^1 B^2 - \partial^2 B^1\right) p^3 + \left(\partial^2 B^3 - \partial^3 B^2\right) p^1 + \left(\partial^3 B^1 - \partial^1 B^3\right) p^2 \\
& + i\left(\left[p^1, B^2 \right] - \left[p^2, B^1 \right]\right) eA^3 + i\left(\left[p^2, B^3 \right] - \left[p^3, B^2 \right]\right) eA^1 + i\left(\left[p^3, B^1 \right] - \left[p^1, B^3 \right]\right) eA^2 \\
& = \left(\partial^1 B^2 - \partial^2 B^1\right) \pi^3 + \left(\partial^2 B^3 - \partial^3 B^2\right) \pi^1 + \left(\partial^3 B^1 - \partial^1 B^3\right) \pi^2 \\
& = -\left(\nabla \times \mathbf{B}\right) \cdot \boldsymbol{\pi} = -\left(\mathbf{J} + \partial \mathbf{E} / \partial t\right) \cdot \boldsymbol{\pi}
\end{aligned}
\quad . \qquad (9.9)
$$

This reveals the space dependency $\nabla \times \mathbf{B}$ for the magnetic field, and because $\nabla \cdot \mathbf{B}$ for Abelian gauge theory this term is zero. But as just noted after (9.8), we have a related term $\boldsymbol{\sigma} \cdot \mathbf{B}$ for the magnetic moment.

Having developed these two terms, we now revert to (9.6) and insert (9.8) and (9.9), thus:

$$
\begin{aligned}
& \chi^{T(s)} \sigma^i \sigma^j \sigma^k \pi^i B^j \pi^k \chi^{(s)} = \chi^{T(s)} \left(\sigma^i \pi^j B^j \pi^i + \sigma^i \pi^i B^j \pi^j - \sigma^i \pi^j B^i \pi^j \right) \chi^{(s)} + i\varepsilon^{ijk}\pi^i B^j \pi^k \\
& = \chi^{T(s)} \left((\mathbf{p} \cdot \mathbf{B})(\boldsymbol{\sigma} \cdot \boldsymbol{\pi}) + (\boldsymbol{\sigma} \cdot \boldsymbol{\pi})(\mathbf{B} \cdot \mathbf{p}) - \sigma^i \pi^j B^i \pi^j \right) \chi^{(s)} - \left(\nabla \times \mathbf{B}\right) \cdot \boldsymbol{\pi} \\
& = \chi^{T(s)} \left((\mathbf{p} \cdot \mathbf{B})(\boldsymbol{\sigma} \cdot \boldsymbol{\pi}) + (\boldsymbol{\sigma} \cdot \boldsymbol{\pi})(\mathbf{B} \cdot \mathbf{p}) - \sigma^i \pi^j B^i \pi^j \right) \chi^{(s)} - \left(\mathbf{J} + \partial \mathbf{E} / \partial t\right) \cdot \boldsymbol{\pi}
\end{aligned}
\quad . \qquad (9.10)
$$

We next return to (9.2) and insert (9.4) and (9.10) and use $\chi^{T(s)}\chi^{(s)} = 1$ as appropriate to write:

$$+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi = +\frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix} -\sigma^j B^j & i\sigma^j E^j \\ i\sigma^j E^j & -\sigma^j B^j \end{pmatrix}\psi$$

$$=\frac{g}{2}\frac{em}{2}\left(\begin{array}{l}\dfrac{\chi^{T(s)}\left((\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})+(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\mathbf{B}\cdot\mathbf{p})-\left(\pi^0-m\right)^2\boldsymbol{\sigma}\cdot\mathbf{B}-\sigma^i\pi^j B^i\pi^j\right)\chi^{(s)}-\left(\nabla\times\mathbf{B}\right)\cdot\boldsymbol{\pi}}{\pi^0-m} \\ +\nabla\cdot\mathbf{E}+i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\chi^{(s)}\end{array}\right) \qquad .(9.11)$$

$$=\frac{g}{2}\frac{em}{2}\left(\begin{array}{l}\dfrac{\chi^{T(s)}\left((\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})+(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\mathbf{B}\cdot\mathbf{p})-\left(\pi^0-m\right)^2\boldsymbol{\sigma}\cdot\mathbf{B}-\sigma^i\pi^j B^i\pi^j\right)\chi^{(s)}-\left(\mathbf{J}+\partial\mathbf{E}/\partial t\right)\cdot\boldsymbol{\pi}}{\pi^0-m} \\ +\rho-i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\partial\mathbf{B}/\partial t\right)\chi^{(s)}\end{array}\right)$$

At rest, $\pi^0-m=E+e\phi-m=e\phi$ and $\mathbf{p}=0$ thus $\pi^i=eA^i$ and $\sigma^i\pi^j B^i\pi^j=e^2\mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}$. So this reduces over common denominators which are factored out, in the fermion rest frame, to:

$$+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi = +\frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix} -\sigma^j B^j & i\sigma^j E^j \\ i\sigma^j E^j & -\sigma^j B^j \end{pmatrix}\psi$$

$$=\frac{g}{2}\frac{m}{2\phi}\left(\chi^{T(s)}\left(-\left(e^2\phi^2+e^2\mathbf{A}^2\right)\boldsymbol{\sigma}\cdot\mathbf{B}+ie\phi\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\right)\chi^{(s)}+e\phi\nabla\cdot\mathbf{E}-\left(\nabla\times\mathbf{B}\right)\cdot e\mathbf{A}\right) \qquad . \qquad (9.12)$$

$$=\frac{g}{2}\frac{m}{2\phi}\left(\chi^{T(s)}\left(-\left(e^2\phi^2+e^2\mathbf{A}^2\right)\boldsymbol{\sigma}\cdot\mathbf{B}-ie\phi\boldsymbol{\sigma}\cdot\left(\partial\mathbf{B}/\partial t\right)\right)\chi^{(s)}+e\left(\rho\phi-\mathbf{J}\cdot\mathbf{A}\right)-\left(\partial\mathbf{E}/\partial t\right)\cdot e\mathbf{A}\right)$$

It is intriguing to find that the magnetic moment term $(e/2m)\overline{\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)\psi$ in the fermion Lagrangian density embeds three of the four Maxwell equations in this way, and presumably because $\nabla\cdot\mathbf{B}=0$ (because we have chosen to stick to Abelian theory), has no explicit appearance of a $\nabla\cdot\mathbf{B}$, but rather, shows a $\boldsymbol{\sigma}\cdot\mathbf{B}$. It is also worth noting that the Lagrangian density term $J^\sigma A_\sigma=\rho\phi-\mathbf{J}\cdot\mathbf{A}$ has reconstructed itself the final line above. And, as we have been pointing out throughout the development, while we started out with gauge fields A^μ which were stripped of any space or time dependency when they entered the spinors in section 4, these gauge fields have now regained their space and time dependency strictly via Heisenberg commutation, and furthermore, the particular space and time dependencies revealed are precisely the spacetime field dependencies that appear for the electric and magnetic fields in Maxwell's equations! This reveals an extremely fundamental structural interrelationship among Dirac's equation (note – the result (9.12) makes no use yet of Higgs fields), Heisenberg commutation, and Maxwell's equations. If one of the thrusts of Geometrodynamics is "mass without mass," then here we have revealed "spacetime dependency without spacetime dependency," with of all things, Heisenberg's matrix mechanics sitting right in the middle of this fundamentally geometrodynamic result.

The final developmental step is to note the coefficient in (9.12) of the magnetic moment term $\left(e^2\phi^2+e^2\mathbf{A}^2\right)\boldsymbol{\sigma}\cdot\mathbf{B}$. Making use of our earlier fermion mass result (6.17) written as $e^2\mathbf{A}^2=2me\phi+e^2\phi^2$, and separating terms somewhat differently now yields:

$$
\begin{aligned}
&+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)\psi=+\frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\psi\\
&=\frac{g}{2}\left(-m^2\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{em}{2}\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)}+\nabla\cdot\mathbf{E}\right)-\frac{em}{2\phi}(\nabla\times\mathbf{B})\cdot\mathbf{A}\right)\quad. \qquad (9.13)\\
&=\frac{g}{2}\left(-m^2e\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{em}{2}\left(-i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\partial\mathbf{B}/\partial t)\chi^{(s)}+\rho\right)-\frac{em}{2\phi}\left(e\mathbf{J}+\partial\mathbf{E}/\partial t\right)\cdot\mathbf{A}\right)
\end{aligned}
$$

This the a first appearance of $1+e\phi/m$ factor which will be central to revealing the "anomalous" magnetic moment for the Higgs fermion fields, and which at the Schwinger [23] one loop level will connect $e\phi/m$ to the running coupling number $\alpha/2\pi$.

10. Dissection of Higgs Field Contributions to the Magnetic Moment: Vacuum Self-Interaction

As noted in (8.17), Of course, the expression (9.13) may be "dissected" via (4.17) and (4.18) into:

$$
\begin{aligned}
&+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)\psi\\
&=\frac{g}{2}\frac{e}{2m}\left(\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi+\overline{v_+'^{1.5}}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi+\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)v_+'^{1.5}+\overline{v_+^{1.5}}\left(i\alpha^i E^i-\Sigma^i B^i\right)v_+^{1.5}\right)
\end{aligned}
.(10.1)
$$

In this section we shall obtain $\overline{v_+^{1.5}}\left(i\alpha^i E^i-\Sigma^i B^i\right)v_+^{1.5}$. In the next section we shall obtain $\overline{v_+'^{1.5}}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi+\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)v_+'^{1.5}$. Then taking a difference from (9.13) via (10.1), we will arrive at our target expression for $\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi$.

Analogously to (9.1) we start out by using the positive energy vacuum of (4.19) to form:

$$
\begin{aligned}
&+\frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)\psi=+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}v_+^{1.5}\\
&=+\frac{g}{2}\frac{e}{2m}\frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\left[\chi^{T(s)}\quad -\chi^{T(s)}\frac{\phi\sigma^i A^i}{\mathbf{A}^2}\right]\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\begin{bmatrix}\chi^{(s)}\\ \dfrac{\phi\sigma^k A^k}{\mathbf{A}^2}\chi^{(s)}\end{bmatrix}. \qquad (10.2)\\
&=+\frac{g}{2}\frac{e}{2m}\frac{1}{4}\frac{e\mathbf{A}^2}{\lambda_f\phi}\begin{pmatrix}-\chi^{T(s)}\sigma^j B^j\chi^{(s)}+\chi^{T(s)}\dfrac{\phi\sigma^i A^i}{\mathbf{A}^2}\sigma^j B^j\dfrac{\phi\sigma^k A^k}{\mathbf{A}^2}\chi^{(s)}\\ +i\chi^{T(s)}\sigma^j E^j\dfrac{\phi\sigma^k A^k}{\mathbf{A}^2}\chi^{(s)}-i\chi^{T(s)}\dfrac{\phi\sigma^i A^i}{\mathbf{A}^2}\sigma^j E^j\chi^{(s)}\end{pmatrix}
\end{aligned}
$$

Then corresponding to (9.2)

$$
\begin{aligned}
&+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\left(i\alpha^i E^i-\Sigma^i B^i\right)v_+^{1.5}=+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}v_+^{1.5}\\
&=+\frac{g}{2}\frac{e^2}{8\lambda_f\phi m}\left(\frac{\chi^{T(s)}\left(-\mathbf{A}^2\mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}+\sigma^i\sigma^j\sigma^k\phi A^i B^j\phi A^k\right)\chi^{(s)}}{\mathbf{A}^2}+i\chi^{T(s)}\phi\sigma^i\sigma^j\left(E^iA^j-A^iE^j\right)\chi^{(s)}\right). \qquad (10.3)
\end{aligned}
$$

As in (9.3) we now use the identity $\sigma^i\sigma^j=\delta^{ij}+i\varepsilon^{ijk}\sigma^k$ for $E^iA^j-A^iE^j$ term. Here, $\left[E^i,A^i\right]=0$ because these do commute, and $\varepsilon^{ijk}\sigma^k\left(E^iA^j-A^iE^j\right)=0$ as noted for (9.3). Consequently this entire term becomes zero, that is:

$$
i\chi^{T(s)}\phi\sigma^i\sigma^j\left(E^iA^j-A^iE^j\right)\chi^{(s)}=i\phi\left(E^iA^i-A^iE^i\right)-\chi^{T(s)}\phi\varepsilon^{ijk}\sigma^k\left(E^iA^j-A^iE^j\right)\chi^{(s)}=0, \qquad (10.4)
$$

and (10.3) simplifies to an expression with only magnetic and no electric fields:

$$
\begin{aligned}
&+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\left(i\alpha^i E^i-\Sigma^i B^i\right)v_+^{1.5}=+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}v_+^{1.5}\\
&=+\frac{g}{2}\frac{e^2}{8\lambda_f\phi m\mathbf{A}^2}\chi^{T(s)}\left(-\mathbf{A}^2\mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}+\sigma^i\sigma^j\sigma^k\phi A^i B^j\phi A^k\right)\chi^{(s)} \qquad . \qquad (10.5)
\end{aligned}
$$

Now, as in (9.6) we apply the identity (9.5) and the allowable commutation $\left[A^i,B^j\right]=0$. We also recall from before (9.9) that for *Abelian* gauge theory which is what we choose to develop here, $\varepsilon^{ijk}A^iB^jA^k=0$. And, as in (9.7), we recognize $\mathbf{B}\cdot\mathbf{A}=\left(\nabla\times\mathbf{A}\right)\cdot\mathbf{A}=0$. Therefore:

$$\chi^{T(s)}\sigma^i\sigma^j\sigma^k\phi A^i B^j \phi A^k \chi^{(s)} = \chi^{T(s)}\left(\delta^{ij}\sigma^k + \delta^{jk}\sigma^i - \delta^{ik}\sigma^j + i\varepsilon^{ijk}\right)\phi A^i B^j \phi A^k \chi^{(s)}$$
$$= \phi^2 \chi^{T(s)}\left(\sigma^i A^j B^j A^i + \sigma^i A^i B^j A^j - \sigma^i A^j B^i A^j + i\varepsilon^{ijk} A^i B^j A^k\right)\chi^{(s)} \qquad . \qquad (10.6)$$
$$= \phi^2 \chi^{T(s)}\left(2(\boldsymbol{\sigma}\cdot\mathbf{A})(\mathbf{B}\cdot\mathbf{A}) - \mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}\right)\chi^{(s)} = -\phi^2\mathbf{A}^2\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$$

This should be contrasted to its counterpart (9.10).

We finally insert (10.6) into (10.5) to obtain:

$$+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+^{1.5} = +\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\begin{pmatrix} -\sigma^j B^j & i\sigma^j E^j \\ i\sigma^j E^j & -\sigma^j B^j \end{pmatrix} v_+^{1.5}$$
$$= -\frac{g}{2}\frac{1}{8\lambda_f \phi m}\left(e^2\phi^2 + e^2\mathbf{A}^2\right)\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)} \qquad . \qquad (10.7)$$

This should be contrasted to (9.12) and particularly $-\left(e^2\phi^2 + e^2\mathbf{A}^2\right)\boldsymbol{\sigma}\cdot\mathbf{B}$. As we did for (9.13) we use our earlier fermion mass result (6.17) written as $e^2\mathbf{A}^2 = 2me\phi + e^2\phi^2$ to rewrite (10.7) as:

$$+\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+^{1.5} = +\frac{g}{2}\frac{e}{2m}\overline{v_+^{1.5}}\begin{pmatrix} -\sigma^j B^j & i\sigma^j E^j \\ i\sigma^j E^j & -\sigma^j B^j \end{pmatrix} v_+^{1.5}$$
$$= -\frac{g}{2}\frac{1}{4\lambda_f}\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)} = -\frac{g}{2}\frac{v_+^{\,3}}{2m}\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)} \qquad . \qquad (10.8)$$

where we have also included the vacuum result from (6.17) written as $1/4\lambda_f = v_+^{\,3}/2m$ to eliminate the coupling λ_f. This, finally, should be contrasted to (9.13), and also to (4.24) which shows the calculation for $v_+^{\,3} = \overline{v_+^{1.5}}v_+^{1.5}$ as opposed to $\overline{v_+^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+^{1.5}$ above where the two vacua are sandwiching $i\alpha^i E^i - \Sigma^i B^i$. In fact, making this contrast explicit, we see that:

$$+\overline{v_+^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+^{1.5} = -v_+^{\,3}\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)} = -\overline{v_+^{1.5}}v_+^{1.5}\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}. \qquad (10.9)$$

That is, the term $i\alpha^i E^i - \Sigma^i B^i \to -\left(1+e\phi/m\right)\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$ as a result of being sandwiched between $\overline{v_+^{1.5}}$ and $v_+^{1.5}$, i.e., as a result of its interactions with the pure vacuum. Put differently, if we wish to take the $v_+^{1.5}$ at the right of $+\overline{v_+^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+^{1.5}$ and commute it all the way over to the left to get it right next to $\overline{v_+^{1.5}}$ in the form $\overline{v_+^{1.5}}v_+^{1.5} = v_+^{\,3}$, then as a result of this commutation, one will automatically generate a magnetic moment term of the form $-\left(1+e\phi/m\right)\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$. This will eventually turn into the lead term of the Higgs fermion magnetic moment, and will thus be directly related to the leading one loop terms in the magnetic moment "anomaly."

11. Dissection of Higgs Field Contributions to the Magnetic Moment: Fermion-Vacuum Interaction

In this section we similarly develop $\overline{v_+'^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)h_\psi + \overline{h_\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+'^{1.5}$ in the (10.1) Higgs field dissection. It is important to work with these two terms in tandem, because the symmetric left-right positioning of $v_+'^{1.5} \leftrightarrow h_\psi$ relative to one another is responsible for unveiling some critical commutation relationships involving various fields to reveal further time and space dependencies. This is analogous to the calculation of $\overline{v_+'^{1.5}}h_\psi + \overline{h_\psi}v_+'^{1.5}$ in section 6, but with $i\alpha^i E^i - \Sigma^i B^i$ sandwiched between the spinors.

Noting that from (4.17) and (4.18) that $h_\psi = \psi - v_+'^{1.5}$ and $\overline{h_f} = \overline{\psi} - \overline{v'^{1.5}}$, the specific calculation we wish to develop here is

$$\begin{aligned}&\overline{v_+'^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)h_\psi + \overline{h_\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+'^{1.5}\\ &= \overline{v_+'^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi + \overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+'^{1.5} - 2\overline{v_+{}^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+{}^{1.5}\end{aligned} . \tag{11.1}$$

We just found $\overline{v_+{}^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+{}^{1.5}$ in (10.8) so that calculation need not be repeated. We thus develop $\overline{v_+'^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi + \overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+'^{1.5}$. Using (4.19) for $v_+{}^{1.5}$ with $v'^{1.5} \equiv v^{1.5} e^{i\pi_\sigma x^\sigma}$ and using (6.26) for ψ, as well as suitable adjoint wavefunctions, and in parallel to (9.1) and (10.2) this calculation sets up as follows:

$$\begin{aligned}&+\frac{g}{2}\frac{e}{2m}\left(\overline{v_+'^{1.5}}\left(i\alpha^j E^j - \Sigma^j B^j\right)\psi + \overline{\psi}\left(i\alpha^j E^j - \Sigma^j B^j\right)v_+'^{1.5}\right)\\ &= -\frac{g}{2}\frac{e}{2m}\frac{1}{2}m\sqrt{\pi^0 - m}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}}\begin{bmatrix}\chi^{T(s)} & -\chi^{T(s)}\dfrac{\phi\sigma^i A^i}{\mathbf{A}^2}\end{bmatrix}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\begin{bmatrix}\chi^{(s)}\\ \dfrac{\sigma^k \pi^k}{\pi^0 - m}\chi^{(s)}\end{bmatrix}\\ &\quad -\frac{g}{2}\frac{e}{2m}\frac{1}{2}m\sqrt{\pi^0 - m}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f \phi}}\begin{bmatrix}\chi^{T(s)} & -\chi^{T(s)}\dfrac{\sigma^i \pi^i}{\pi^0 - m}\end{bmatrix}\begin{pmatrix}-\sigma^j B^j & i\sigma^j E^j\\ i\sigma^j E^j & -\sigma^j B^j\end{pmatrix}\begin{bmatrix}\chi^{(s)}\\ \dfrac{\phi\sigma^k A^k}{\mathbf{A}^2}\chi^{(s)}\end{bmatrix}\end{aligned} . \tag{11.2}$$

Corresponding to (9.1) and (10.2), this first becomes:

$$+\frac{g}{2}\frac{e}{2m}\left(\overline{v_{+}'^{1.5}}\left(i\alpha^{j}E^{j}-\Sigma^{j}B^{j}\right)\psi+\overline{\psi}\left(i\alpha^{j}E^{j}-\Sigma^{j}B^{j}\right)v_{+}'^{1.5}\right)$$

$$=-\frac{g}{2}\frac{e}{2m}\frac{1}{2}m\sqrt{\pi^{0}-m}\sqrt{\frac{e\mathbf{A}^{2}}{\lambda_{f}\phi}}\left(\begin{array}{l}-\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\chi^{T(s)}\frac{\phi\sigma^{i}A^{i}}{\mathbf{A}^{2}}\sigma^{j}B^{j}\frac{\sigma^{k}\pi^{k}}{\pi^{0}-m}\chi^{(s)}\\+i\chi^{T(s)}\sigma^{j}E^{j}\frac{\sigma^{k}\pi^{k}}{\pi^{0}-m}\chi^{(s)}-i\chi^{T(s)}\frac{\phi\sigma^{i}A^{i}}{\mathbf{A}^{2}}\sigma^{j}E^{j}\chi^{(s)}\end{array}\right) \quad . \tag{11.3}$$

$$-\frac{g}{2}\frac{e}{2m}\frac{1}{2}m\sqrt{\pi^{0}-m}\sqrt{\frac{e\mathbf{A}^{2}}{\lambda_{f}\phi}}\left(\begin{array}{l}-\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\chi^{T(s)}\frac{\sigma^{i}\pi^{i}}{\pi^{0}-m}\sigma^{j}B^{j}\frac{\phi\sigma^{k}A^{k}}{\mathbf{A}^{2}}\chi^{(s)}\\+i\chi^{T(s)}\sigma^{j}E^{j}\frac{\phi\sigma^{k}A^{k}}{\mathbf{A}^{2}}\chi^{(s)}-i\chi^{T(s)}\frac{\sigma^{i}\pi^{i}}{\pi^{0}-m}\sigma^{j}E^{j}\chi^{(s)}\end{array}\right)$$

This may then be consolidated in the manner of (9.2) and (10.3) to:

$$+\frac{g}{2}\frac{e}{2m}\left(\overline{v_{+}'^{1.5}}\left(i\alpha^{j}E^{j}-\Sigma^{j}B^{j}\right)\psi+\overline{\psi}\left(i\alpha^{j}E^{j}-\Sigma^{j}B^{j}\right)v_{+}'^{1.5}\right)$$

$$=-\frac{g}{2}\frac{1}{4}e\sqrt{\pi^{0}-m}\sqrt{\frac{e\mathbf{A}^{2}}{\lambda_{f}\phi}}\left(\begin{array}{l}-2\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{\phi\chi^{T(s)}\sigma^{i}\sigma^{j}\sigma^{k}\left(A^{i}B^{j}\pi^{k}+\pi^{i}B^{j}A^{k}\right)\chi^{(s)}}{\mathbf{A}^{2}\left(\pi^{0}-m\right)}\\+\frac{i\chi^{T(s)}\sigma^{i}\sigma^{j}\left(E^{i}\pi^{j}-\pi^{i}E^{j}\right)\chi^{(s)}}{\left(\pi^{0}-m\right)}-\frac{i\phi\chi^{T(s)}\sigma^{i}\sigma^{j}\left(A^{i}E^{j}-E^{i}A^{j}\right)\chi^{(s)}}{\mathbf{A}^{2}}\end{array}\right). \tag{11.4}$$

In (10.4) (and (9.3) prior) we found that $i\chi^{T(s)}\phi\sigma^{i}\sigma^{j}\left(E^{i}A^{j}-A^{i}E^{j}\right)\chi^{(s)}=0$. This means that the final term above drops out entirely. The term $i\chi^{T(s)}\sigma^{i}\sigma^{j}\left(E^{i}\pi^{j}-\pi^{i}E^{j}\right)\chi^{(s)}$ has also been previously found in (9.3) and (9.4). With (9.4) and $i\chi^{T(s)}\phi\sigma^{i}\sigma^{j}\left(E^{i}A^{j}-A^{i}E^{j}\right)\chi^{(s)}=0$, (11.4) reduces to:

$$+\frac{g}{2}\frac{e}{2m}\left(\overline{v_{+}'^{1.5}}\left(i\alpha^{j}E^{j}-\Sigma^{j}B^{j}\right)\psi+\overline{\psi}\left(i\alpha^{j}E^{j}-\Sigma^{j}B^{j}\right)v_{+}'^{1.5}\right)$$

$$=-\frac{g}{2}\frac{1}{4}e\sqrt{\pi^{0}-m}\sqrt{\frac{e\mathbf{A}^{2}}{\lambda_{f}\phi}}\left(\begin{array}{l}-2\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{\phi\chi^{T(s)}\sigma^{i}\sigma^{j}\sigma^{k}\left(A^{i}B^{j}\pi^{k}+\pi^{i}B^{j}A^{k}\right)\chi^{(s)}}{\mathbf{A}^{2}\left(\pi^{0}-m\right)}\\+\frac{\nabla\cdot\mathbf{E}+i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\chi^{(s)}}{\left(\pi^{0}-m\right)}\end{array}\right). \tag{11.5}$$

Now in (11.5) we have the term $\phi\chi^{T(s)}\sigma^{i}\sigma^{j}\sigma^{k}\left(A^{i}B^{j}\pi^{k}+\pi^{i}B^{j}A^{k}\right)\chi^{(s)}$. We first use identity (9.5) to expand to (contrast (9.6) and (10.6)):

$$\begin{aligned}&\phi\chi^{T(s)}\sigma^i\sigma^j\sigma^k\left(A^iB^j\pi^k+\pi^iB^jA^k\right)\chi^{(s)}\\&=\phi\chi^{T(s)}\left(\sigma^i\left(\pi^jB^jA^i+A^iB^j\pi^j+A^jB^j\pi^i+\pi^iB^jA^j\right)-2\sigma^iA^jB^i\pi^j\right)\chi^{(s)}+i\phi\varepsilon^{ijk}\left(A^iB^j\pi^k+\pi^iB^jA^k\right)\end{aligned}\qquad .(11.6)$$

Now, for reasons already stated via (9.7), $A^jB^j=\mathbf{A}\cdot\mathbf{B}=\mathbf{A}\cdot(\nabla\times\mathbf{A})=0$ and $\pi^jB^j=p^jB^j$, so (11.6) may be reduced somewhat to (note the parallels to (9.8)):

$$\begin{aligned}&\phi\chi^{T(s)}\sigma^i\sigma^j\sigma^k\left(A^iB^j\pi^k+\pi^iB^jA^k\right)\chi^{(s)}\\&=\phi\chi^{T(s)}\left(p^jB^j\sigma^iA^i+\sigma^iA^iB^jp^j-2\sigma^iA^jB^i\pi^j\right)\chi^{(s)}+i\phi\varepsilon^{ijk}\left(A^iB^j\pi^k+\pi^iB^jA^k\right)\\&=\phi\chi^{T(s)}\left((\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\mathbf{A})+(\boldsymbol{\sigma}\cdot\mathbf{A})(\mathbf{B}\cdot\mathbf{p})-2\sigma^iA^jB^i\pi^j\right)\chi^{(s)}+i\phi\varepsilon^{ijk}\left(A^iB^j\pi^k+\pi^iB^jA^k\right)\end{aligned}\quad .\qquad (11.7)$$

As after (9.8), we note that at rest $-\sigma^i\left(A^jB^i\pi^j+\pi^jB^iA^j\right)\to-2e^2\mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}$, so leave this term exactly as is.

Finally we have the term $i\varepsilon^{ijk}\left(A^iB^j\pi^k+\pi^iB^jA^k\right)$. Keeping in mind that $\pi^k=p^k+eA^k$, as noted before (9.9), $\varepsilon^{ijk}A^iB^jA^k=0$ by identity, *but again, only for an Abelian gauge theory* with $\left[A^\mu,A^\nu\right]=0$. Also noted at (9.9) is that we are allowed to commute $p^iB^jA^k=A^kp^iB^j$. Finally, we make use of part of (9.9):

$$\begin{aligned}&i\phi\varepsilon^{ijk}\left(A^iB^j\pi^k+\pi^iB^jA^k\right)=i\varepsilon^{ijk}\left(A^iB^jp^k+p^iB^jA^k\right)\\&=i\phi\left(B^2p^3A^1+p^1B^2A^3-B^3p^2A^1-p^1B^3A^2\right)\\&+i\phi\left(B^3p^1A^2+p^2B^3A^1-B^1p^3A^2-p^2B^1A^3\right)\\&+i\phi\left(B^1p^2A^3+p^3B^1A^2-B^2p^1A^3-p^3B^2A^1\right)\\&=i\phi\left(\left[p^1,B^2\right]-\left[p^2,B^1\right]\right)A^3+i\left(\left[p^2,B^3\right]-\left[p^3,B^2\right]\right)A^1+i\left(\left[p^3,B^1\right]-\left[p^1,B^3\right]\right)A^2\\&=\phi\left(\partial^1B^2-\partial^2B^1\right)A^3+\phi\left(\partial^2B^3-\partial^3B^2\right)A^1+\phi\left(\partial^3B^1-\partial^1B^3\right)A^2\\&=-\phi(\nabla\times\mathbf{B})\cdot\mathbf{A}=-\phi(\mathbf{J}+\partial\mathbf{E}/\partial t)\cdot\mathbf{A}\end{aligned}\quad .\qquad (11.8)$$

With this we revert to (11.7) and use (11.8) to write:

$$\begin{aligned}&\phi\chi^{T(s)}\sigma^i\sigma^j\sigma^k\left(A^iB^j\pi^k+\pi^iB^jA^k\right)\chi^{(s)}\\&=\phi\chi^{T(s)}\left((\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\mathbf{A})+(\boldsymbol{\sigma}\cdot\mathbf{A})(\mathbf{B}\cdot\mathbf{p})-2\sigma^iA^jB^i\pi^j\right)\chi^{(s)}-\phi(\nabla\times\mathbf{B})\cdot\mathbf{A}\\&=\phi\chi^{T(s)}\left((\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\mathbf{A})+(\boldsymbol{\sigma}\cdot\mathbf{A})(\mathbf{B}\cdot\mathbf{p})-2\sigma^iA^jB^i\pi^j\right)\chi^{(s)}-\phi(\mathbf{J}+\partial\mathbf{E}/\partial t)\cdot\mathbf{A}\end{aligned}\quad .\qquad (11.9)$$

Now we use (11.9) in (11.5) which yields:

$$+\frac{g}{2}\frac{e}{2m}\left(\overline{v_+'^{1.5}}\left(i\alpha^j E^j-\Sigma^j B^j\right)\psi+\overline{\psi}\left(i\alpha^j E^j-\Sigma^j B^j\right)v_+'^{1.5}\right)=-\frac{g}{2}\frac{1}{4}e\frac{1}{\sqrt{\pi^0-m}}\sqrt{\frac{e\mathbf{A}^2}{\lambda_f\phi}}$$
$$\times\left(\begin{array}{l}\dfrac{\chi^{T(s)}\left(\phi(\mathbf{p}\cdot\mathbf{B})(\boldsymbol{\sigma}\cdot\mathbf{A})+\phi(\boldsymbol{\sigma}\cdot\mathbf{A})(\mathbf{B}\cdot\mathbf{p})-2\mathbf{A}^2\left(\pi^0-m\right)\boldsymbol{\sigma}\cdot\mathbf{B}-2\phi\sigma^i A^j B^i\pi^j\right)\chi^{(s)}-\phi(\nabla\times\mathbf{B})\cdot\mathbf{A}}{\mathbf{A}^2}\\ +\nabla\cdot\mathbf{E}+i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)}\end{array}\right).\quad(11.10)$$

We certainly see many parallels to the corresponding relationship (9.11). These are brought to the forefront at rest, where again $\pi^0-m=E+e\phi-m=e\phi$ and $\mathbf{p}=0$ thus $\pi^i=eA^i$ and $2\sigma^i A^j B^i\pi^j=2e^2\mathbf{A}^2\boldsymbol{\sigma}\cdot\mathbf{B}$. Now we have:

$$\begin{aligned}&+\frac{g}{2}\frac{e}{2m}\left(\overline{v_+'^{1.5}}\left(i\alpha^j E^j-\Sigma^j B^j\right)\psi+\overline{\psi}\left(i\alpha^j E^j-\Sigma^j B^j\right)v_+'^{1.5}\right)\\ &=-\frac{g}{2}\frac{1}{4}e\sqrt{\frac{\mathbf{A}^2}{\lambda_f}}\left(-4\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{1}{\phi}\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)}+\nabla\cdot\mathbf{E}\right)-\frac{1}{\mathbf{A}^2}(\nabla\times\mathbf{B})\cdot\mathbf{A}\right)\\ &=-\frac{g}{2}\frac{1}{4}e\sqrt{\frac{\mathbf{A}^2}{\lambda_f}}\left(-4\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{1}{\phi}\left(-i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\partial\mathbf{B}/\partial t)\chi^{(s)}+\rho\right)-\frac{1}{\mathbf{A}^2}\left(e\mathbf{J}+\partial\mathbf{E}/\partial t\right)\cdot\mathbf{A}\right)\end{aligned}\quad.\quad(11.11)$$

12. Formulation of g-Factors for the Charged Higgs Leptons

With (11.11), we now have all the ingredients needed to pinpoint the coefficients of the $\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi$ term as laid out in (8.17) and (10.1). Because the underlying premise of Higgs theory is that the *observed* physical particles are vacuum-expanded Higgs fields (h in scalar field theory and h_ψ presently) and *not* of the seed fields (ϕ in scalar field theory and ψ presently), we shall now wish to combine the results from (9.13), (11.11) and (10.8) respectively, to directly express $\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi$. Doing so in terms of the **E** and **B** fields (recognizing that we can use Maxwell's equations at any time to re-express in terms of sources ρ, **J** and time derivatives $\partial\mathbf{E}/\partial t$, $\partial\mathbf{B}/\partial t$), we obtain:

$$\begin{aligned}&\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi=\left(\overline{\psi}-\overline{v_+'^{1.5}}\right)\left(i\alpha^i E^i-\Sigma^i B^i\right)\left(\psi-v_+'^{1.5}\right)\\ &=\frac{g}{2}\left(-m^2\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{em}{2}\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)}+\nabla\cdot\mathbf{E}\right)-\frac{em}{2\phi}(\nabla\times\mathbf{B})\cdot\mathbf{A}\right)\\ &+\frac{g}{2}\frac{1}{4}e\sqrt{\frac{\mathbf{A}^2}{\lambda_f}}\left(-4\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}+\frac{1}{\phi}\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)}+\nabla\cdot\mathbf{E}\right)-\frac{1}{\mathbf{A}^2}(\nabla\times\mathbf{B})\cdot\mathbf{A}\right)\\ &-\frac{g}{2}\frac{1}{4\lambda_f}\left(1+\frac{e\phi}{m}\right)\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}\end{aligned}\quad.\quad(12.1)$$

Now we restructure this to segregate each of the field with its coefficients. In so doing, we make use of $1/\lambda_f = 2v_+^{\ 3}/m$ from (6.17) to replace the coupling with a vacuum-to-mass ratio. Thus:

$$\begin{aligned}
&\overline{h_\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)h_\psi = \left(\overline{\psi} - \overline{v_+'^{1.5}}\right)\left(i\alpha^i E^i - \Sigma^i B^i\right)\left(\psi - v_+'^{1.5}\right) \\
&= -\frac{g}{2}\left[m^2\left(1+\frac{e\phi}{m}\right) + e\sqrt{\mathbf{A}^2}\sqrt{\frac{2v_+^{\ 3}}{m}} + \frac{v_+^{\ 3}}{2m}\left(1+\frac{e\phi}{m}\right)\right]\chi^{T(s)} e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)} \\
&\quad + \frac{g}{2}\left[\frac{em}{2} + \frac{1}{4}e\sqrt{\mathbf{A}^2}\sqrt{\frac{2v_+^{\ 3}}{m}}\frac{1}{\phi}\right]\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E})\chi^{(s)} + \nabla\cdot\mathbf{E}\right) \\
&\quad - \frac{g}{2}\left[\frac{em}{2\phi} + \frac{1}{4}e\sqrt{\mathbf{A}^2}\sqrt{\frac{2v_+^{\ 3}}{m}}\frac{1}{\mathbf{A}^2}\right](\nabla\times\mathbf{B})\cdot\mathbf{A}
\end{aligned} \qquad . \qquad (12.2)$$

Next, let us fine tune (12.2). Because $\overline{h_\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)h_\psi$ is part of a Lagrangian density in spacetime, it has a mass dimension $D=+4$ in $\hbar = c = 1$ units of mass, length and time. Often, for example in [18] at (2.165) the $\boldsymbol{\sigma}\cdot\mathbf{B}$ term appears as part of a Hamiltonian *H*, which has a mass dimensionality $D=+1$. In a Hamiltonian, the term of interest which "flags" the g-factor $g/2$ is $-(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}$. And of course, the g-factor is a dimensionless number. But in a Lagrangian formulation such as (12.2), we need to pick up an extra mass dimensionality of $D=+3$, to go from a $D=+1$ Hamiltonian formulation to a $D=+4$ Lagrangian formulation. So the question presents: from whence do we pick up an added $D=+3$ mass dimension? And, of course, we also need to pick up the customary $e/2m$ flag. Specifically, the term $\chi^{T(s)} e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$ which has $D=+2$ needs to acquire an additional $D=+2$ to have a total $D=+4$ and thus a dimensionless coefficient which we can relate to the g-factor. Additionally, it needs to show $-(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}$ to connect with the usual Hamiltonian form.

We see that (12.2) has a ready-made answer, namely, the term $v_+^{\ 3}/2m$ which multiplies the latter occurrence of the term $1+e\phi/m$. It naturally contains the extra $D=+2$, and it contains the $1/2m$ denominator to go with the *e* already present in $\chi^{T(s)} e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$. So we factor $v_+^{\ 3}/2m$ out from the coefficient in the $\chi^{T(s)} e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$ line of (2.12) and adjust the balance of the coefficient accordingly to show the desired $-(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}$ term as such:

$$
\begin{aligned}
&\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi=\left(\overline{\psi}-\overline{v_+'^{1.5}}\right)\left(i\alpha^i E^i-\Sigma^i B^i\right)\left(\psi-v_+'^{1.5}\right)\\
&=-\frac{g}{2}\left[\frac{2m^3}{v_+{}^3}\left(1+\frac{e\phi}{m}\right)+2e\sqrt{\mathbf{A}^2}\sqrt{\frac{2m}{v_+{}^3}}+\left(1+\frac{e\phi}{m}\right)\right]v_+{}^3\chi^{T(s)}\left(\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}\right)\chi^{(s)}\\
&\quad+\frac{g}{2}\left[\frac{em}{2}+\frac{1}{4}e\sqrt{\mathbf{A}^2}\sqrt{\frac{2v_+{}^3}{m}}\frac{1}{\phi}\right]\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\chi^{(s)}+\nabla\cdot\mathbf{E}\right)\\
&\quad-\frac{g}{2}\left[\frac{em}{2\phi}+\frac{1}{4}e\sqrt{\mathbf{A}^2}\sqrt{\frac{2v_+{}^3}{m}}\frac{1}{\mathbf{A}^2}\right]\left(\nabla\times\mathbf{B}\right)\cdot\mathbf{A}
\end{aligned}
\quad . \qquad (12.3)
$$

Now let's progress to $i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\chi^{(s)}+\nabla\cdot\mathbf{E}$. This term has $D=+3$, so we need to pick up $D=+1$ to create a dimensionless coefficient. In turn, the $\left(\nabla\times\mathbf{B}\right)\cdot\mathbf{A}$ term has $D=+4$ already, but needs to be reconfigured anyway. Specifically, as we did for $\chi^{T(s)}\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$ let us factor out $v_+{}^3/2m$ from these other coefficients as well. This will "overshoot" the mass dimensionality, but we will momentarily compensate. We now have:

$$
\begin{aligned}
&\overline{h_\psi}\left(i\alpha^i E^i-\Sigma^i B^i\right)h_\psi=\left(\overline{\psi}-\overline{v_+'^{1.5}}\right)\left(i\alpha^i E^i-\Sigma^i B^i\right)\left(\psi-v_+'^{1.5}\right)\\
&=-\frac{g}{2}\left[\frac{2m^3}{v_+{}^3}\left(1+\frac{e\phi}{m}\right)+2e\sqrt{\mathbf{A}^2}\sqrt{\frac{2m}{v_+{}^3}}+\left(1+\frac{e\phi}{m}\right)\right]\frac{v_+{}^3}{2m}\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}\\
&\quad+\frac{g}{2}\left[\frac{em^2}{v_+{}^3}+\frac{1}{2}e\sqrt{\mathbf{A}^2}\sqrt{\frac{2m}{v_+{}^3}}\frac{1}{\phi}\right]\frac{v_+{}^3}{2m}\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\chi^{(s)}+\nabla\cdot\mathbf{E}\right)\\
&\quad-\frac{g}{2}\left[\frac{em^2}{v_+{}^3\phi}+\frac{1}{2}e\sqrt{\mathbf{A}^2}\sqrt{\frac{2m}{v_+{}^3}}\frac{1}{\mathbf{A}^2}\right]\frac{v_+{}^3}{2m}\left(\nabla\times\mathbf{B}\right)\cdot\mathbf{A}
\end{aligned}
\quad . \qquad (12.4)
$$

The next cue comes from comparing the $\sqrt{\mathbf{A}^2}$ terms on all three lines, which we now need to harmonize. At the same time we need to reduce the $\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times\mathbf{E}\right)\chi^{(s)}+\nabla\cdot\mathbf{E}\right)$ term by $D=-1$ and the $\left(\nabla\times\mathbf{B}\right)\cdot\mathbf{A}$ term by $D=-2$. That too is ready-made, for both of these objects are achieved by factoring out $1/\phi$ and $1/\mathbf{A}^2$ respectively, then adjusting accordingly. We also explicitly associate the charge strength e with each pertinent field, to make clear all of the dimensional and charge balancing that is embedded in (12.4). With this final tune up, and creating dimensionally balanced ratios in the all terms, (12.4) becomes:

$$\overline{h_\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)h_\psi = \left(\overline{\psi} - \overline{v_+'^{1.5}}\right)\left(i\alpha^i E^i - \Sigma^i B^i\right)\left(\psi - v_+'^{1.5}\right)$$
$$= -\frac{g}{2}\left[\frac{2m^3}{v_+{}^3}\left(1+\frac{e\phi}{m}\right)+2\sqrt{\frac{2m^3}{v_+{}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}+\left(1+\frac{e\phi}{m}\right)\right]\frac{v_+{}^3}{2m}\chi^{T(s)}\boldsymbol{\sigma}\cdot e\mathbf{B}\chi^{(s)}$$
$$+\frac{g}{2}\left[\frac{m^3}{v_+{}^3}\frac{e\phi}{m}+\frac{1}{2}\sqrt{\frac{2m^3}{v_+{}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}\right]\frac{v_+{}^3}{2me^2\phi^2}\left(i\chi^{T(s)}\boldsymbol{\sigma}\cdot\left(\nabla\times e\mathbf{E}\right)\chi^{(s)}+\nabla\cdot e\mathbf{E}\right)e\phi$$
$$-\frac{g}{2}\left[\frac{m^3}{v_+{}^3}\frac{e\phi}{m}\frac{e^2\mathbf{A}^2}{e^2\phi^2}+\frac{1}{2}\sqrt{\frac{2m^3}{v_+{}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}\right]\frac{v_+{}^3}{2me^2\mathbf{A}^2}\left(\nabla\times e\mathbf{B}\right)\cdot e\mathbf{A} \quad . \qquad (12.5)$$

The term common to all, is now $\sqrt{2m^3/v_+{}^3}\sqrt{e^2\mathbf{A}^2/m^2}$. It should be clear that (12.5) is precisely the same as (12.1), but in a form that will be very useful for carrying out calculations that can be compared with experimental data. Now let's take a close look at (12.5), starting with the $\boldsymbol{\sigma}\cdot e\mathbf{B}$ term for the magnetic moment.

Aside from the $v_+{}^3$ required to go from $D=+1$ in a Hamiltonian to $D=+4$ in a Lagrangian (and the $\chi^{(s)}$ as well), we see precisely the $(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}=(e\hbar/2m)\boldsymbol{\sigma}\cdot\mathbf{B}$ term for which the coefficient is understood to be $g/2$. Further, in the event that $m, \sqrt{\mathbf{A}^2} << v_+$, the dominant factor will be $1+e\phi/m$ which if we trace this back originated in the vacuum-to-vacuum self-interactions developed in (10.9). Might this parenthetic term in fact be the g-factor of the Higgs fermion field h_ψ? And, if we only observe Higgs fields and *not* the seed fields from which these are expanded about the vacuum, *might this in reality be the expression for the g-factor which we do observe experimentally*?

We have carried $g/2=1$ in all of our equations as a placeholder ever since equation (8.4) for the Gordon decomposition. But this is just that, a placeholder which is equal to 1. In reality, the g-factor is *whatever dimensionless number* we end up finding as the coefficient of $-(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}$ in a Hamiltonian formulation, or of $-v_+{}^3\chi^{T(s)}(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)}$ in a Lagrangian formulation. Thus, we now discard the $g/2=1$ placeholder and identify (define) the g-factor for the observed Higgs fermion to be:

$$\frac{g}{2}\equiv\left(1+\frac{e\phi}{m}\right)+2\sqrt{\frac{2m^3}{v_+{}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}+\frac{2m^3}{v_+{}^3}\left(1+\frac{e\phi}{m}\right). \qquad (12.6)$$

with a reordering of terms from most-to-least dominant for $m, \sqrt{\mathbf{A}^2} << v_+$. So starting to now think about physical particles, we expect the first term to be especially dominant for the electron if it should happen that $v_+ = 246.219650794137$ GeV is the same vacuum associated with Fermi's weak coupling constant $G_F = 1.1663787\times10^{-5}$ GeV^{-1} [25] that is used elsewhere in particle physics, most notably, to arrive at the electroweak vector boson masses and the strength of the weak interactions. And, we expect the other terms to come more into play for the mu and especially tau leptons.

To work with (12.6) (and also (12.5)), let us we write the fermion mass result (6.17) as $e^2\mathbf{A}^2 = 2me\phi + e^2\phi^2$ (see (6.25), or better yet, $e^2\mathbf{A}^2 + e^2\phi^2 = 2me\phi + 2e^2\phi^2$. From there one derives the useful identity:

$$1+\frac{e\phi}{m} = \frac{1}{2me}\left(\frac{e^2\mathbf{A}^2 + e^2\phi^2}{\phi}\right) \tag{12.7}$$

which may be rewritten also as (see yet another alternative in (6.25)):

$$e^2\mathbf{A}^2 = 2me\phi\left(1+\frac{e\phi}{m}\right) - e^2\phi^2 . \tag{12.8}$$

Using (12.8) in (12.6) then yields:

$$\frac{g}{2} \equiv \left(1+\frac{e\phi}{m}\right) + 2\sqrt{\frac{2m^3}{{v_+}^3}}\sqrt{2\frac{e\phi}{m}\left(1+\frac{e\phi}{m}\right) - \frac{e^2\phi^2}{m^2}} + \frac{2m^3}{{v_+}^3}\left(1+\frac{e\phi}{m}\right). \tag{12.9}$$

which we re-factor into (again compare (6.25)):

$$\boxed{\frac{g}{2} = \left(1+\frac{e\phi}{m}\right)\left(1+\frac{2m^3}{{v_+}^3}\right) + 2\sqrt{\frac{2m^3}{{v_+}^3}}\sqrt{2\frac{e\phi}{m} + \left(\frac{e\phi}{m}\right)^2}}. \tag{12.10}$$

The benefit of this formulation, which is one or several, alternatives, is that the **A** is removed and everything is expressed as a function of the dimensionless ratios m/v_+ or $e\phi/m$. This is what we shall now use for a variety of g-factor related calculations.

Let us now use Maxwell's equations to rewrite (12.5) in terms of sources rather than field densities, as:

$$\begin{aligned}
&\overline{h_\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)h_\psi = \left(\overline{\psi} - \overline{v_+'^{1.5}}\right)\left(i\alpha^i E^i - \Sigma^i B^i\right)\left(\psi - v_+'^{1.5}\right) \\
&= -\left[\frac{2m^3}{{v_+}^3}\left(1+\frac{e\phi}{m}\right) + 2\sqrt{\frac{2m^3}{{v_+}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}} + \left(1+\frac{e\phi}{m}\right)\right]\frac{{v_+}^3}{2m}\chi^{T(s)}e\boldsymbol{\sigma}\cdot\mathbf{B}\chi^{(s)} \\
&\quad + \left[\frac{m^3}{{v_+}^3}\frac{e\phi}{m} + \frac{1}{2}\sqrt{\frac{2m^3}{{v_+}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}\right]\frac{{v_+}^3}{2me^2\phi^2}\left(-i\chi^{T(s)}e\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t\chi^{(s)} + e\rho\right)e\phi \\
&\quad - \left[\frac{m^3}{{v_+}^3}\frac{e\phi}{m}\frac{e^2\mathbf{A}^2}{e^2\phi^2} + \frac{1}{2}\sqrt{\frac{2m^3}{{v_+}^3}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}\right]\frac{{v_+}^3}{2me^2\mathbf{A}^2}\left(e\mathbf{J} + e\partial\mathbf{E}/\partial t\right)\cdot e\mathbf{A}
\end{aligned} . \tag{12.11}$$

This formulation is very interesting from a range of viewpoints including the fact that all of these spacetime dependencies was revealed following a canonical Heisenberg commutation of a field with the canonical momentum, but the one we shall focus upon here is the fact that this reveals an expression $\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$ for the magnetic moment of the electron *in a time-dependent magnetic field*. Specifically, analogously to (12.6) where we specified *g*, let us define another dimensionless ratio:

$$\frac{g'}{2}\equiv\frac{m^3}{v_+^{\ 3}}\frac{e\phi}{m}+\frac{1}{2}\sqrt{\frac{2m^3}{v_+^{\ 3}}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}} \tag{12.12}$$

to be the coefficient of the $-(e/2m)\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$ term. Why is this of particular interest? For the $-(e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}$ term for a stationary (time-independent) magnetic field, the dominant term for $m,\sqrt{\mathbf{A}^2}<<v_+$ is $1+e\phi/m$, which is independent of the m/v_+ ratio. But for the time-dependent term $-(e/2m)\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$, this coefficient does not exist, and so the mass ratio m/v_+ does come into play in the leading order effects. This means while the electron and the mu and tau leptons all respond more or less similarly to a *stationary* **B** because of the dominance of $1+e\phi/m$, this will not be the case for a time-varying $\partial\mathbf{B}/\partial t$. Here, we expect to see the mu and especially the tau leptons responding with much more sensitivity to a magnetic field that varies in time. In fact, g' as defined above is best thought of as the g-factor for the response of a fermion in a time-dependent magnetic field. This will provide the basis for momentarily making quantitative predictions which it should be possible to confirm or contradict with experiments to measure and compare the behaviors of all three charged leptons in a given, time-dependent magnetic field.

Finally, for good measure, let also define a third dimensionless ratio:

$$\frac{g''}{2}\equiv\frac{m^3}{v_+^{\ 3}}\frac{e\phi}{m}\frac{e^2\mathbf{A}^2}{e^2\phi^2}+\frac{1}{2}\sqrt{\frac{2m^3}{v_+^{\ 3}}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2}}\,. \tag{12.13}$$

which is the dimensionless factor in front of the term containing $\nabla\times\mathbf{B}=\mathbf{J}+\partial\mathbf{E}/\partial t$. The second term is identical to that in (2.12) but the first term differs by the ratio $\mathbf{A}^2/\phi^2$. This tells us about the response of the electron in a time-dependent electric field.

So, with the foregoing, as regards the charged leptons e,μ,τ, it should be possible to use each of the known experimental g-factors $g_{e,\mu,\tau}$ and each of the known masses $m_{e,\mu,\tau}$ in (12.10), together with a vev which we shall presume is $v_+=246.219650794137\ \mathrm{GeV}$ as it is for electroweak and scalar Higgs theory, to derive definitive "values" ϕ_e, ϕ_μ and ϕ_τ for each lepton and then, via (12.8), the magnitude $|\mathbf{A}|=\left|\sqrt{\mathbf{A}^2}\right|$ for each of e,μ,τ. Because these numbers are potentials, which presumably vary in space, it will also be important to understand that they actually mean when measured at various locales in relation to the charged lepton, which as we shall see is closely related to charge screening and loop diagram calculations and renormalization, which will be the focus of section 14. But this discussion will be easier to have with specific numbers as a backdrop for explanation.

Then, knowing ϕ and $|\mathbf{A}|$ for each lepton, we can calculate (predict) their various $g'/2$ factors which express their response to a time-dependent magnetic field. Should it turn out that the heavier leptons exhibit a more sensitive response in the predicted manner, this would be a source of experimental validation.

We take note that (12.10), (2.12) and (2.13) for the three g-factors are, for Higgs fermion theory, the "bonus" that emerges that is analogous to the gauge boson mass (2.19) that emerges from scalar Higgs field. Scalar Higgs theory reveals a mass for a scalar field and a "bonus" gauge field along with its mass. Fermion Higgs theory reveals a mass for the fermion and a "bonus" gauge field along with its magnetic moment which represents how a particular spatial dependencies of the gauge field, namely those given by $\boldsymbol{\sigma}\cdot\mathbf{B}$, $\nabla\times\mathbf{E}$, $\nabla\cdot\mathbf{E}$ and $\nabla\times\mathbf{B}$ in (12.5).

Let us now turn to experimental data.

13. Numerical Results Based on Empirical Data (and a Prediction for the Impact of a Time-Dependent Magnetic field on the Charged Lepton g-Factors)

Our starting point for experimental comparisons will be equation (12.10). The experimental data from [26], [27], [28] which we shall use for the three charged leptons are their masses $m_e = 0.510998928 \text{ MeV}$, $m_\mu = 105.6583715 \text{ MeV}$ and $m_\tau = 1776.82 \text{ MeV}$ and their g-factors $g_e/2 = 1.00115965218076$, $g_\mu/2 = 1.0011659209$ and $g_\tau/2 = 1.0011772100$. We shall also work on the supposition that $v_+ = 246.219650794137 \text{ GeV}$, though this is a supposition that can also be tested in the event that the data we are about to review points toward there being a different vev for Higgs fermions than for Higgs scalars, for some reason. And of course, we shall use the low-energy running coupling $\alpha = 1/137.0359990740 = 0.007297352570$ [25], which is related to the charge strength via $e = \sqrt{4\pi\alpha} = 0.302822120883$. This data set via (12.10) will *uniquely* determine the potentials ϕ_e, ϕ_μ and ϕ_τ for each of these charged leptons. In this section we shall simply do the numeric calculations. In the next section we shall discuss the meaning and interpretation of these results.

Now, the first inclination one has is to try to cast (12.10) as a quadratic in $e\phi/m$. But the term $\sqrt{2(e\phi/m)+(e\phi/m)^2}$ makes this appear to not be possible, as this function becomes imaginary of the range $-2 < e\phi/m < 0$. One could of course expand this square root as an infinite series and use only a $e\phi/m \geq 0$ domain to yield a real range, but that would entail an approximation that is best not done given that we are trying to match up g-factor data which is known with extremely high experimental precision. So instead we shall evaluate (12.10) numerically, inserting various values for $e\phi/m$ until (12.10) produces an exact match to all experimentally-known digits for each g-factor and each mass to all known experimental digits. The masses enter (12.10) via the three mass ratios for the e, μ, τ leptons respectively, namely:

$$\frac{m}{v_+} = \begin{cases} 0.0000020754 = 1/481{,}839.8578 \\ 0.0004291224 = 1/2330.337363 \\ 0.0072164021 = 1/138.57320989 \end{cases} . \tag{13.1}$$

It is of interest to note that the tau ratio is not too different from $\alpha = 1/137.0359990740$ which is suggestive that the tau mass is in some way a first order screening effect from the vacuum itself, but we shall not pursue this further right here. Plugging all of this data into (12.10) enables us to find that the mass and g-factor data is fitted to all known decimal places (which are better known by several decimal places for the electron than for the mu and tau leptons) by the following potentials which we represent with a 2π coefficient for reasons that will immediately be apparent when also comparing $\alpha = 1/137.0359990740$, namely:

$$2\pi \frac{e\phi}{m} = \begin{cases} 0.0072863069840 = 1/137.2437370810 \\ 0.0073180690 = 1/136.64807 \\ 0.0068818625 = 1/145.30950 \end{cases} . \tag{13.2}$$

Writing (6.25) with $|\mathbf{A}| = \left|\sqrt{\mathbf{A}^2}\right|$ as:

$$\frac{e|\mathbf{A}|}{m} = \sqrt{2\frac{e\phi}{m} + \left(\frac{e\phi}{m}\right)^2} , \tag{13.3}$$

we may also obtain the vector potential magnitudes:

$$2\pi \frac{e|\mathbf{A}|}{m} = \begin{cases} 0.3026805646884 \\ 0.3033399428 \\ 0.2941553923 \end{cases} . \tag{13.4}$$

The close comparison of the numbers above to $e = \sqrt{4\pi\alpha} = 0.302822120883$ is understood by writing an equation *analogous to* (13.3) multiplied by 2π, namely:

$$2\pi\sqrt{\frac{\alpha}{\pi} + \frac{\alpha^2}{4\pi^2}} = \sqrt{4\pi^2\frac{\alpha}{\pi} + 4\pi^2\frac{\alpha^2}{4\pi^2}} = \sqrt{4\pi\alpha + \alpha^2} \cong e = \sqrt{4\pi\alpha} = 0.302822120883 . \tag{13.5}$$

It is very telling (and closely related to Schwinger's one-loop result [23] $g/2 = 1 + \alpha/2\pi + \ldots$) that $2\pi(e\phi/m)$ behaves roughly like a running coupling $\alpha \simeq 2\pi(e\phi/m)$, and that $2\pi(e|\mathbf{A}|/m)$

behaves roughly like the associated running charge strength $e \simeq 2\pi\left(e\left|\mathbf{A}\right|/m\right)$. We shall take a closer look at this momentarily.

Next, we may directly find the energy magnitudes of each of the ϕ and $\left|\mathbf{A}\right|$, namely:

$$\begin{cases} \phi_e = 0.0019568610 \text{ MeV} \\ \phi_\mu = 0.4063805796 \text{ MeV}, \\ \phi_\tau = 6.4266102 \text{ MeV} \end{cases} \tag{13.6}$$

$$\begin{cases} \left|\mathbf{A}_e\right| = 0.0812899880 \text{ MeV} \\ \left|\mathbf{A}_\mu\right| = 16.8448073365 \text{ MeV} \\ \left|\mathbf{A}_\tau\right| = 274.6962830 \text{ MeV} \end{cases}, \tag{13.7}$$

The experimentally-based finding in (13.6) that all three potentials $\phi > 0$ finally answers for us, the question which arose originally at (3.6), then again at (6.24), as to the appropriate sign choice in the quadratic solution for ϕ. With the empirical data telling us that the potentials $\phi > 0$ are all positive numbers, we now see that the sign choice must be "+", that is, that (6.24) is now finally to be written as:

$$\frac{e\phi}{m} = \left(\sqrt{1+\frac{e^2\mathbf{A}^2}{m^2}} - 1\right) \tag{13.8}$$

It is also instructive to return to the fermion mass result (6.17) and use (13.6) and (13.7) to decompose the three lepton masses into their respective contributions from the scalar (voltage) and vector potentials ϕ and $\mathbf{A}$. These turn out to be:

$$m = \frac{1}{2}e\frac{\mathbf{A}^2}{\phi} - \frac{1}{2}e\phi = \begin{cases} 0.511295218 \text{ MeV} - 0.000296290 \text{ MeV} = 0.510998928 \text{ MeV} \\ 105.7199020 \text{ MeV} - 0.0615305 \text{ MeV} = 105.6583715 \text{ MeV} \\ 1777.79 \text{ MeV} - 0.97 \text{ MeV} = 1776.82 \text{ MeV} \end{cases} \tag{13.9}$$

This brings us back to the point made after (6.17) that gauge potentials alone are not measurable numbers, and that physically *observable* energies are those which involve a *difference* between two potentials. Usually, measurable energies reflect a difference between *one type of potential* (often a voltage $\phi = V$) *at two different spatial points* $\mathbf{x}_1$ and $\mathbf{x}_2$. But in (13.9) we see that the observed fermion masses are functions of a difference between *two different types of potential V at a single spatial point* **x**, namely, $V_1(\mathbf{x}) \equiv \tfrac{1}{2}e\mathbf{A}^2/\phi$ and $V_2(\mathbf{x}) \equiv \tfrac{1}{2}e\phi$, such that $m = V_1(\mathbf{x}) - V_2(\mathbf{x})$. This still leaves the open question: what / where is **x**? This will be an important part of the renormalization discussion in the next section. Notice also that we deliberately did *not* write the mass here as $m(\mathbf{x})$ but rather wrote the mass as a constant *m* which is *invariant* as a function of space (or time). This will all be central to and greatly expanded upon in the next section as we study what the result (13.9) says about running couplings and charge screening and renormalization. This all relates to the fact that *m* in this development was not introduced by hand, but was naturally revealed in (6.17) as the coefficient of the term Lagrangian density term $-2\lambda_f v_+^{\ 3}\overline{h_\psi}h_\psi$ (6.17) for the Higgs fermion h_ψ which represented the expansion $\psi(x)=v'+h(x)$ of the Dirac wavefunction about the vacuum. So this mass is not a typical "bare mass," but rather is a renormalized "self-energy" mass Σ that is built exclusively and naturally out of the gauge potentials $\phi, \mathbf{A}$.

We may also use (13.1), (13.2) and (13.4) in (12.12) to calculate the g-factor impact of a *time-dependent* magnetic field term $\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$ in (12.11) to be:

$$\frac{g'}{2}\equiv\frac{m^3}{v_+^{\ 3}}\frac{e\phi}{m}+\frac{1}{2}\sqrt{\frac{2m^3}{v_+^{\ 3}}}\frac{e|\mathbf{A}|}{m}=\begin{cases}1.0184403241\times10^{-10}\\3.0346364402\times10^{-7}\\0.0000202942\end{cases}. \tag{13.10}$$

Comparing with the well-known experimental g-factors $g_e/2=0.001159652[18076]$, $g_\mu/2=1.001165[9209]$ and $g_\tau/2=1.0011[772100]$ which we write to enclose in brackets the (known) digits at which g' comes into play (this is *not* an experimental error notation), we see that a time-dependent magnetic field can produce an effect which can be seen well within the experimental ranges of the g-factor for all three charged leptons, and that the $\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$ effect is most pronounced for the tauon. *This <u>prediction</u> that the g-factor of the heaviest tau lepton is much more significantly impacted by a time-dependent magnetic field than that of the electron or muon, as well as the magnitude of the impact as predicted in (13.10), would appear to be a leading candidate for experimentally validating or contradicting the results derived here.* The same facilities which study and establish g-factors for $\boldsymbol{\sigma}\cdot\mathbf{B}$, should be able to discern these effects for $\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$, as they are well within experimentally-detectable ranges.

Similarly, (12.13) for the $\mathbf{A}\cdot\partial\mathbf{E}/\partial t$ term of (12.11) is found to be:

$$\frac{g''}{2}\equiv\frac{m^3}{v_+^{\ 3}}\frac{e\phi}{m}\frac{\mathbf{A}^2}{\phi^2}+\frac{1}{2}\sqrt{\frac{2m^3}{v_+^{\ 3}}}\frac{e|\mathbf{A}|}{m}=\begin{cases}1.0184405028\times10^{-10}\\3.0362168641\times10^{-7}\\0.0000210458\end{cases}. \tag{13.11}$$

Clearly the right-most term dominates so this is close to (13.10), but the ratio

$$\frac{\mathbf{A}^2}{\phi^2}=\begin{cases}1725.6556646595\\1718.1702913569\\1827.0130307872\end{cases} \tag{13.12}$$

does substantially enhance the less-dominant term. So there may be a discernible effect for the mu lepton and there appears to be a definitely discernible effect for the tau lepton. However, unlike $\boldsymbol{\sigma}\cdot\mathbf{B}$ or $\boldsymbol{\sigma}\cdot\partial\mathbf{B}/\partial t$, the $\mathbf{A}\cdot\partial\mathbf{E}/\partial t$ term contains the scalar product with **A** rather than a product with the spin matrices $\boldsymbol{\sigma}$ and so the $\mathbf{A}\cdot\partial\mathbf{E}/\partial t$ g''-factor is unconnected to the spin of the charged leptons.

Now let us discuss these various numeric results.

14. Invariant Mass, Variable Gauge Renormalization

What warrants immediate attention in the numeric data of the last section are the ratios in (13.2) being very close to the low energy electromagnetic "fine structure" coupling $\alpha = 1/137.0359990740$, especially for the electron and the muon. It is independently known that Schwinger [23], when considering the "one loop" contribution to the magnetic moment, first described the small deviation of the electron's $g_e/2 = 1.00115965218076$ from the $g/2 = 1$ of Dirac's equation by finding that:

$$\frac{g}{2} \simeq 1 + \frac{\alpha}{2\pi} + \ldots \; . \tag{14.1}$$

Since Schwinger, painstaking calculations have been done for some higher-order loops, but the basic result that the one-loop correction equals $\alpha/2\pi$ remains valid to this day.

Because we shall be discussing loops and renormalization in some detail here, let us use the notation $0,1$ in subscripted form to denote a physical quantity that is based on only 0- plus 1-loop perturbative calculations. Let us also use $2 \to \infty$ to denote the sum total of all corrections to a $0,1$ quantity that occur as a result of all perturbative loop calculations from 2 loops all the way up to an infinite number of loops. A simple 1 as a subscript will denote the 1-loop contribution only, absent the 0 loop or the $2 \to \infty$ loop contributions. Using this notation, we may summarize what is being said in (13.2) *for all three charged leptons* by writing:

$$\frac{e\phi_1}{m} = \frac{\alpha}{2\pi} = \frac{1}{2\pi \cdot 137.0359990740} \tag{14.2}$$

and

$$\frac{e\phi_{0,1}}{m} = 1 + \frac{\alpha}{2\pi} = 1 + \frac{1}{2\pi \cdot 137.0359990740} . \tag{14.3}$$

In other words, the one loop contribution to the potential, ϕ_1, when made dimensionless via the combination $e\phi_1/m$, is *synonymous with* Schwinger's 1 loop-only correction. More generally, this means that that the potential ϕ at each order bears some very close relation to the loop calculations at each order. We wish to explore more closely the manner in which this is so.

The origin of this concurrence is to be found in the fact that the g-factor (12.10), in the leading order where the mass to vacuum ratio $m/v_+ \to 0$ which is especially pertinent to the electron and muon but less so to the tau lepton, reduces to:

$$\frac{g}{2}\left(m/v_+ \to 0\right) = 1 + \frac{e\phi}{m} + \ldots, \tag{14.4}$$

and that comparison to Schwinger's (14.1) therefore produces an immediate connection $\alpha/2\pi \leftrightarrow e\phi/m$. Thus, without any *a priori* expectation that the voltage / scalar potential would connect in some way to the running coupling α, we find that such a connection does exist precisely because the Higgs fermions, in leading order of low mass, produce the relationship (14.4), while Schwinger gives us (14.1). If we trace back through the development here, we see that the term $1 + e\phi/m$ in the above originated in (10.8), which describes the self-interaction of the fermion vacuum via $\overline{v_+^{1.5}}\left(i\alpha^i E^i - \Sigma^i B^i\right)v_+^{1.5}$, as part of the Higgs expansion of fermion fields via $\psi(x) \equiv v'^{1.5} + h_f(x)$ of (4.17). So what we see is that although Dirac's equation yields $g/2 = 1$ when analyzed in the form of (8.4) or (8.16) in relation to "seed fermions" ψ, it actually yields a $g/2 = 1 + e\phi/m + \ldots$ when ψ is dissected as in sections 10 and 11 into an expansion of a Higgs field about the vacuum and when the Higgs field rather than the seed fermion is understood to be the *observable* fermion. Once we are able to use Dirac's equation via a Higgs field dissection to identify $g/2 = 1 + e\phi/m + \ldots$ rather than $g/2 = 1$ as the leading-order g-factor for $h_\psi(x)$ rather than $\psi(x)$, and given Schwinger, we are able to uncover the very important 0,1 loop connections (14.2), (14.3) between the scalar potential ϕ and the running coupling α.

With (14.2) and (14.3), we immediately may use the complete expression (12.10) for $g/2$ together with the specific numeric result (13.2) to tell us the value of $\phi_{2\to\infty}$ for each of these three fermions, by simply subtracting off the α from each ratio in (13.2). This result is:

$$2\pi\frac{e\phi_{2\to\infty}}{m} = \begin{cases} 1/137.2437370810 - 1/137.0359990740 = -0.0000110455858 = -1/90{,}533.90325 \\ 1/136.64807 - 1/137.0359990740 = 0.0000207164 = 1/48{,}270.95 \\ 1/145.30950 - 1/137.0359990740 = -0.0004154901 = -1/2406.7964 \end{cases} .(14.5)$$

To be clear: This is the total contribution to the potential which is to be *expected* to arise from the $2 \to \infty$ loop diagrams, based on 1) theoretical relationships (14.2), (14.3), 2) theoretical relationship (12.10) for $g/2$ to which the mass ratio m/v_+ also makes a contribution which starts to noticeably impact these numbers for the heaviest tau lepton, and 3) the known empirical data for the three lepton g-factors *g*, the three lepton masses *m*, and the vev v_+ which we take to be $246.219650794137\text{ GeV}$ derived from the Fermi weak coupling constant G_F.

Now, the results (14.2), (14.3) connecting the scalar potentials (voltages) ϕ with the running coupling α deliver us directly into a discussion of renormalization, and especially, of *mass renormalization*. We start this discussion by picking up where we left off following (13.9) where we noted that the fermion rest masses are revealed to be the *difference* $m = V_1(\mathbf{x}) - V_2(\mathbf{x})$ between two potentials $V_1(\mathbf{x}) \equiv \frac{1}{2} e\mathbf{A}^2 / \phi$ and $V_2(\mathbf{x}) \equiv \frac{1}{2} e\phi$ *at the same point in space*, and where we regarded this mass *m* to be a constant which does not depend on **x** or time *t*. Following up on this, we now ask two sets of questions. First: at what point in space? At $\mathbf{x} = 0$, i.e., at the center of the charged lepton? At $\mathbf{x} = \infty$ all the way out to infinity? Or somewhere else? We pose this question because any time one talks about a potential, one is talking about a field, i.e., a variable function of space and time, and about a field which is not observable except as a *difference* between one potential and another potential. Second, if the rest mass is a function $m = V_1(\mathbf{x}) - V_2(\mathbf{x})$ of a difference between two potentials taken at a single point in space, will this mass be the same when this potential difference is taken after we translate over to a *different* point in space, $\mathbf{x} \to \mathbf{x}'$. That is, if $m = V_1(\mathbf{x}) - V_2(\mathbf{x})$ and $m' = V_1(\mathbf{x}') - V_2(\mathbf{x}')$, will we have $m = m'$, or will we have $m \neq m'$? Is *m* an invariant at all points in space (and events in spacetime), or is it a function $m(\mathbf{x})$ of spatial position (and time)? We have regarded *m* as an invariant this far, but this supposition needs to be justified.

The critical ingredient needed to address these questions – and the reason we have not been able to address them until this moment – comes from finding in (14.2), which we rewrite using $4\pi\alpha = e^2 / \hbar c$ in $\hbar = c = 1$ units, that:

$$\phi_1(m,e) = \frac{m}{8\pi^2} e . \tag{14.6}$$

In other words, the potential ϕ, at the 1-loop level, *assuming constant mass which is part of what we are now opening up for discussion*, runs in direct proportion to the running electric charge strength $e(\mathbf{x} = \infty) = 0.3028221209$, which is based on $\alpha(\mathbf{x} = \infty) = 1/137.0359990740$ via $4\pi\alpha = e^2$, and which are both expressions for the running of charge and coupling only for a probe taken *entirely outside the charge screening* of the electron, muon or tauon, i.e., at spatial "infinity." As we probe more deeply into the electron at a probe energy / renormalization scale μ (this is not the same μ which was introduced as a mass parameter back in (3.3)), we know that these two numbers $e(\mathbf{x} \propto 1/\mu)$ and $\alpha(\mathbf{x} \propto 1/\mu)$ will grow larger because we have penetrated past some of the charge screening / vacuum polarization which surrounds the "bare" electron (and we shortly return to examine "bareness") and thus modified the *observed* charge strength. It is also worth noting that if *m* is a constant, then (14.6) does *not* generalize to higher $2 \to \infty$ loops, that is, that $\phi \neq (m/8\pi^2)e$ in general. Why? Because *if* $\phi = (m/8\pi^2)e$ were to be generally true, then one could replace $e\phi / m \to \alpha / 2\pi$ everywhere it appears in (12.10). But if we do so, we would have:

$$\frac{g}{2} = \left(1 + \frac{\alpha}{2\pi}\right)\left(1 + \frac{2m^3}{v_+^{\ 3}}\right) + 2\sqrt{\frac{2m^3}{v_+^{\ 3}}}\sqrt{2\frac{\alpha}{2\pi} + \left(\frac{\alpha}{2\pi}\right)^2} . \tag{14.7}$$

Then, if we were to use $\alpha = 1/137.0359990740$ and the known *m* for each lepton, we would not get the experimentally observed g-factors. That (14.7) is *not* true, and is contradicted by empirical data, is evidenced by the empirically-based result (13.2) in which all of the numbers would be $1/137.0359990740$ if (14.7) were true. So we do know that $\phi(m,e)$ in general takes on a more complicated form in relation to the running charge *e* than (14.6), and is only a first loop relationship.

Still referring to (14.6), if the one loop potential ϕ_1 is the same as the electric charge strength *e* up to a factor $m/8\pi^2$ which is a constant factor *if we assume that m is constant* (the topic of detailed exploration in a moment), then although the higher loop relations $\phi_{2\to\infty}(m,e)$ must be more complicated relationships than (14.6), it is safe to reach the *qualitative* conclusion that the overall potential ϕ runs as a function of $\mathbf{x} \propto 1/\mu$ in the same sort of fashion as do *e* and α. So because $e(\mathbf{x}=\infty) = 0.3028221209$ and $\alpha(\mathbf{x}=\infty) = 1/137.0359990740$ are both valid numeric relationships *only outside the charged lepton at a spatial perch which we formally set to* $\mathbf{x}=\infty$, or in different words to say the same thing, at a renormalization scale $\mu \to 0$, we are able to answer the first question: The potentials $V_1(\mathbf{x}) \equiv \frac{1}{2}e\mathbf{A}^2/\phi$ and $V_2(\mathbf{x}) \equiv \frac{1}{2}e\phi$ in $m = V_1(\mathbf{x}) - V_2(\mathbf{x})$ of (13.9), and more generally the potentials numerically characterized in (13.2), *are all potentials taken at* $\mathbf{x}=\infty$, *i.e., at a position formally identified with spatial infinity, or in equivalent terms, at a low probe energy / renormalization scale* $\mu \to 0$.

Now we get to the second question as to whether $m(\mathbf{x}' \neq \infty) = m(\mathbf{x}=\infty)$, where $\mathbf{x}'$ is a locale other than a formal spatial infinity, and in particular, is a locale at which the charge screen of the lepton is deeply probed with a definitively observable difference. To explore this question, we start at spatial infinity, i.e., at a renormalization scale / probe energy $\mu = 0$, and regard each of *e*, *m*, ϕ and $|\mathbf{A}|$ a to have $\mu = 0$ values of $e(0) = \sqrt{4\pi\alpha} = 0.302822120883$ based on $\alpha(0) = 1/137.0359990740$, $m_e(0) = 0.510998928\text{ MeV}$, $m_\mu(0) = 105.6583715\text{ MeV}$ and $m_\tau(0) = 1776.82\text{ MeV}$, $\phi(0)$ given by (13.6) and $|\mathbf{A}|(0)$ given by (13.7). The relationship (13.3) is a generally-covariant relationship among *e*, *m*, **A** and ϕ which is independent of locale, i.e., independent of renormalization scale. So with all of this in mind, we may write (13.3) as:

$$\left(\frac{e(0)|\mathbf{A}(0)|}{m(0)}\right)^2 = 2\frac{e(0)\phi(0)}{m(0)} + \left(\frac{e(0)\phi(0)}{m(0)}\right)^2, \tag{14.8}$$

Let us now transform over to a different renormalization scale $\mu' \neq \mu = 0$. Let us posit that each and every one of the objects with values v appearing in (14.8) undergoes a change to a different value v', i.e., that at the new scale μ', (14.8) becomes:

$$\left(\frac{e'(\mu')|\mathbf{A}'(\mu')|}{m'(\mu')}\right)^2 = 2\frac{e'(\mu')\phi'(\mu')}{m'(\mu')} + \left(\frac{e'(\mu')\phi'(\mu')}{m'(\mu')}\right)^2. \tag{14.9}$$

So in the above, even the rest masses are presumed to have changed. But suppose we would like to keep these masses invariant even under this change in renormalization scale. That is, suppose we wish to have $m' = m$ regardless of the change $\mu \to \mu'$. Can we do this? And if so, what are the conditions which would allow us to do this?

To see, let us take (14.9) and change m' back to *m*, and then transform to a different $\phi'(\mu') \to \phi''(\mu')$, $\mathbf{A}'(\mu') \to \mathbf{A}''(\mu')$ and $e'(\mu') \to e''(\mu')$ *such that* we are allowed to have $m'(\mu') = m(\mu') = m(0)$. Take careful note: we are still at the same scale μ', but we are setting $m'(\mu')$ back to $m(0)$ and then changing every other value in (14.9) to make this happen, i.e., to be able to make *m* invariant. So now, still at μ', (14.9) becomes:

$$\left(\frac{e''(\mu')|\mathbf{A}''(\mu')|}{m(\mu')}\right)^2 = 2\frac{e''(\mu')\phi''(\mu')}{m(\mu')} + \left(\frac{e''(\mu')\phi''(\mu')}{m(\mu')}\right)^2. \tag{14.10}$$

The two relationships (14.9) and (14.10) are totally equivalent relationships. They just involve some shifting among *e*, *m*, ϕ and $|\mathbf{A}|$, and we keep in mind that ϕ and $|\mathbf{A}|$ are both gauge potentials and so are not observable except as a difference between potentials. Because these are equivalent, setting (14.9) to equal (14.10) yields the following *simultaneous* relationships:

$$\frac{e''(\mu')\phi''(\mu')}{m(\mu')} = \frac{e'(\mu')\phi'(\mu')}{m'(\mu')}; \quad \frac{e''(\mu')|\mathbf{A}''(\mu')|}{m(\mu')} = \frac{e'(\mu')|\mathbf{A}'(\mu')|}{m'(\mu')}. \tag{14.11}$$

This is turn is easily rewritten as:

$$\frac{e''(\mu')\phi''(\mu')}{e'(\mu')\phi'(\mu')} = \frac{e''(\mu')|\mathbf{A}''(\mu')|}{e'(\mu')|\mathbf{A}'(\mu')|} = \frac{m(\mu')}{m'(\mu')}, \tag{14.12}$$

which means the transformations we must use to maintain an invariant $m(\mu') = m(\mu)$ regardless of renormalization scale are:

$$\begin{cases} e'(\mu')\phi'(\mu') \to e''(\mu')\phi''(\mu') = \dfrac{m(\mu')}{m'(\mu')} e'(\mu')\phi'(\mu') \\ e'(\mu')|\mathbf{A}'(\mu')| \to e''(\mu')|\mathbf{A}''(\mu')| = \dfrac{m(\mu')}{m'(\mu')} e'(\mu')|\mathbf{A}'(\mu')| \end{cases}. \tag{14.13}$$

But ϕ and $\mathbf{A} = A^i$ are the components of the four-vector $A^\mu = (\phi, A^1, A^2, A^3)$, and the transformations in (14.13) are of identical form for each component. So we consolidate (14.13) into a single transformation on A^μ, and because everything is taken at the same scale μ', we remove that part of the notation and leave that understood so that the form of the transformation is highlighted without additional clutter. So at the scale μ', the transformation we do to maintain a constant, renormalization scale-invariant mass is:

$$e'A'^\mu \rightarrow e''A'' = \frac{m}{m'} e'A'^\mu . \tag{14.13}$$

Lo and behold, this looks like a gauge transformation on a gauge field! Let us see.

Now, in (14.13), both the charge strength is transformed, $e' \rightarrow e''$ and the gauge field is transformed $A'^\mu \rightarrow A''^\mu$ in order to return $m' \rightarrow m$. But we saw in (14.6) that although A^μ and e both run with the renormalization scale, and while they are related by (14.6) at the one loop level, for higher loops they do have some independence, or more to the point, there is a more complicated relationship between them than (14.6). And in general, we know that in renormalization theory, mass renormalization is not the same as charge renormalization, i.e., mass and charge renormalize differently. Specifically, the mass get renormalized via $S'_F \rightarrow Z_2 S_F$ where S_F is the bare and S'_F is the complete, total observed propagator, while the charge strength / running coupling gets renormalized via $\Gamma^\mu(p,0,p) \rightarrow (1/Z_1)\gamma^\mu$ where Γ^μ are the complete vertex operators and γ^μ are the bare vertex Dirac gamma operators, and where $Z_1 = Z_2$ are infinite renormalization constants and are equal to one another because of the Ward identity $\partial S'^{-1}_F / \partial p_\mu = \Gamma^\mu(p,0,p)$. (See [18], section 7.4.) So while e in (14.13) has so far been caught up in our effort to maintain a constant mass at all scales, let us now cure that by leaving e alone and letting the entire job of keeping the constant mass fall to the gauge field A^μ. That is, still at μ', we now set $e'(\mu') = e''(\mu')$. Then, writing what looks like a gauge transformation in (14.13) to explicitly show that this is a gauge transformation, the above becomes:

$$A'^\mu \rightarrow A'' = \frac{m}{m'} A'^\mu \equiv A'^\mu + \partial^\mu \theta . \tag{14.14}$$

Being as explicit as can be, the above means that:

$$\partial^\mu \theta = \left(\frac{m}{m'} - 1 \right) A'^\mu \text{ !!!} \tag{14.15}$$

So, we can maintain an invariant fermions rest mass over *all scales of renormalization*, simply via a suitable gauge transformation transforming the gauge field A^μ! With $A'' = (m/m')A'^\mu$ informing us that A'^μ scales up or down slightly in magnitude to keep the mass constant, this is truly a "gauge" transformation the Weyl's original sense of the word before he changed the e^{a} exponential transformation factor of his original theory [9], [10] into the later $e^{i\theta}$ phase factor of modern gauge (really, "phase-invariant") theory [11]. What is ordinarily a rest mass that varies with renormalization scale, becomes an invariant mass at *all scales*, because the gauge field becomes a proxy to absorb the changes in mass brought about by renormalization, and we exploit the gauge freedom to allow the gauge field to do this. What we have just shown above, whereby we maintain a constant rest mass at all renormalization scales by using the gauge freedom of A^{μ} to instead make A^{μ} scale-dependent, we shall refer to as "invariant mass, variable gauge renormalization," the title of this section, or more simply, "Invariant Mass Renormalization."

So now the question arises, what allows us to do this? Ordinarily, the bare fermion propagator inverse $S_F^{-1} = \gamma^{\mu} p_{\mu} - m$, where m is a so-called "bare mass," while the complete, observed propagator is $S_F'^{-1} = \gamma^{\mu} p_{\mu} - m - \Sigma = S_F^{-1} - \Sigma$, where Σ is the fermion "self-energy." But here there is no bare mass. The bare mass is what shows up in the Dirac Lagrangian density $\mathfrak{L} = \overline{\psi}(i\partial - m)\psi$, but here we started with $\mathfrak{L} = \overline{\psi}(i\partial - \mu)\psi$ in (3.8) and a mass parameter μ (here this is not the renormalization scale symbol) defined in (3.4) and then saw in (6.16) and (6.17) how this led to a mass $m = -\mu$ for the Higgs fermion h_{ψ}. But this mass (6.17) is defined totally as a function of the gauge field A^{μ}, with no "bareness" at all. The bare mass is zero, and the revealed, observed mass is (6.17). And because (13.3) and (14.8) et al. are just variants of (6.17), this is what led to the ability shown in (4.20) and (4.21) to maintain a renormalization scale-invariant mass simply by a gauge transformation. So we should really now write (6.17), not as a "mass" in the sense of the bare mass in $\mathfrak{L} = \overline{\psi}(i\partial - m)\psi$, but as a self-energy, that is, as:

$$\Sigma = m = \frac{1}{2} e \left(\frac{\mathbf{A}^2 - \phi^2}{\phi} \right) = -\mu \tag{14.16}$$

The mass, i.e., the self-energy bubble, is a pure gauge field and charge / coupling bubble. The mass no longer has an independent existence, except as a creature of the gauge field and the running coupling. So now, how do we distinguish the bare propagator inverse S_F^{-1} from the complete inverse $S_F'^{-1}$? With $\Sigma = m$, if we write $S_F'^{-1} = S_F^{-1} - \Sigma = S_F^{-1} - m$, but if $S_F^{-1} = \gamma^{\mu} p_{\mu} - m$ as usual, then $S_F'^{-1} = \gamma^{\mu} p_{\mu} - 2m$. That simply makes no sense. What really happens is that the bare propagator is now $S_F^{-1} = \gamma^{\mu} p_{\mu}$ *with no mass at all*, (i.e., this is the propagator for a luminous fermion), but the complete propagator is:

$$S_F'^{-1} = \gamma^{\mu} p_{\mu} - \Sigma = \gamma^{\mu} p_{\mu} - \frac{1}{2} e \left(\frac{\mathbf{A}^2 - \phi^2}{\phi} \right), \tag{14.17}$$

and the gauge fields bring about a subluminous, material fermion. The Ward identity $\partial S_F'^{-1} / \partial p_{\mu} = \Gamma^{\mu}(p, 0, p)$ then acts on the above expression. Renormalization then occurs via:

$$S'_F = \left[\gamma^\mu p_\mu - \frac{1}{2} e \left(\frac{\mathbf{A}^2 - \phi^2}{\phi} \right) \right]^{-1} \rightarrow Z_2 S_F = Z_2 \left(\gamma^\mu p_\mu \right)^{-1}, \tag{14.18}$$

So we have taken care of the propagators and the masses, but what about the charge / coupling via the vertex operator renormalization $\Gamma^\mu(p,0,p) \rightarrow (1/Z_1)\gamma^\mu$? That is, how do we now obtain the vertex $\Gamma^\mu(p,0,p)$? Here, we return to equation (8.5) which is the Lagrangian density containing the Gordon decomposition of the seed field ψ and it includes a "placeholder" $g/2$. But now we know $g/2$ for the Higgs fermion from any of the alternative formulations (12.6), (12.9), (12.10). So we insert $g/2$ from, say, (12.6) into (8.5) to obtain:

$$\begin{aligned}
\mathfrak{L} &= T - V = i\overline{\psi}\gamma^\mu \partial_\mu \psi - \mu\overline{\psi}\psi - \lambda_f \left(\overline{\psi}\psi \right)^2 \\
&+ i\frac{e}{2\mu}\left(\overline{\psi}\partial^\mu \psi - \partial^\mu \overline{\psi}\psi \right) A_\mu + \left(1 + \frac{e\phi}{m} + 2\sqrt{\frac{2m^3}{v_+^{\ 3}}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2} + \frac{2m^3}{v_+^{\ 3}}\left(1 + \frac{e\phi}{m} \right)} \right) \frac{e}{2\mu}\partial_\sigma \left(\overline{\psi}\sigma^{\mu\sigma}\psi \right) A_\mu
\end{aligned} . \tag{14.19}$$

On the second line, contrast [9.138] in [18], using (6.17) to set $\mu \rightarrow -m$, we see the complete vertex $\Gamma^\mu = \gamma^\mu + \Lambda^\mu$ embedded in:

$$\begin{aligned}
&\overline{\psi}\Gamma^\mu \psi A_\mu = \overline{\psi}\left(\gamma^\mu + \Lambda^\mu \right)\psi A_\mu \\
&= -i\frac{e}{2m}\left(\overline{\psi}\partial^\mu \psi - \partial^\mu \overline{\psi}\psi \right) A_\mu - \left(1 + \frac{e\phi}{m} + 2\sqrt{\frac{2m^3}{v_+^{\ 3}}}\sqrt{\frac{e^2\mathbf{A}^2}{m^2} + \frac{2m^3}{v_+^{\ 3}}\left(1 + \frac{e\phi}{m} \right)} \right) \frac{e}{2m}\partial_\sigma \left(\overline{\psi}\sigma^{\mu\sigma}\psi \right) A_\mu
\end{aligned} . \tag{14.20}$$

Now, the above does for the moment mix apples and oranges, because it contains the seed field ψ at the same time that it contains $g/2$ for the Higgs fermion h_ψ with $T(p,0,p)$, and because it is in spacetime ($i\partial^\mu \psi$, etc.) not momentum space ($p^\mu \psi$ etc.). But we may then use (14.20) as the starting point to convert into momentum space as we did from (8.7) to (8.16). Thereafter we may expand about the vacuum as we did in sections 9 to 11 to isolate $h_\psi = \psi - v_+^{\ 1.5}$ rather than ψ. And as result of this exercise (which we leave as an exercise), one may extract the vertex $\Gamma^\mu(p,0,p)$ which goes into the renormalization equation $\Gamma^\mu(p,0,p) \rightarrow (1/Z_1)\gamma^\mu$. Then we use the Ward identity on (14.18) and the vertex derived from (14.20) to write:

$$\frac{\partial S'^{-1}_F}{\partial p_\mu} = \frac{\partial}{\partial p_\mu}\left[\gamma^\mu p_\mu - \frac{1}{2} e \left(\frac{\mathbf{A}^2 - \phi^2}{\phi} \right) \right]^{-1} = \Gamma^\mu(p,0,p). \tag{14.21}$$

which yields a first order differential equation for *e*, ϕ, **A** and *m* as a running function of p_μ. Because *m* may be held constant at all scales by the utilizing the gauge transformation (4.14), we are enabled via (14.21) to renormalize with the mass $m = \frac{1}{2} e\left(\mathbf{A}^2 - \phi^2\right)/\phi$ held constant, that is, mathematically speaking, treated as a constant number in the differential equation (14.21).

This is the upshot of "Invariant Mass Renormalization." Renormalization of rest mass is no more and no less than a re-gauging of gauge fields. The ability to renormalize using gauge symmetry during renormalization to absorb any variation of the mass into the gauge freedom of the gauge field A^μ, is another reason for having made definition (3.4), (3.5) in the first place.

15. Orbital Angular Momentum

Let us now briefly touch upon the orbital angular momentum of the Higgs fermion h_ψ. As the starting point for this discussion, we turn to Dirac's equation $i\gamma^\mu \partial_\mu \psi + e\gamma^\mu \psi A_\mu - \mu\psi = 0$ used after (8.1) for a free field $A_\mu = 0$. Now, in contrast to (8.2) which enabled us to pinpoint the "intrinsic" spin, we write Dirac's equation multiplied from the left by γ^0 and developed in a customary manner with $\gamma^0\gamma^i = \alpha^i$, see (8.13), as:

$$\begin{aligned} 0 &= \left(i\gamma^\mu \partial_\mu - \mu\right)\psi = \left(i\gamma^0\gamma^\mu \partial_\mu - \gamma^0\mu\right)\psi = \left(\gamma^0\gamma^\mu p_\mu - \gamma^0\mu\right)\psi = \left(E - \gamma^0\gamma^i p^i - \gamma^0\mu\right)\psi \\ &= \left(E - \gamma^0\gamma^i p^i - \gamma^0\mu\right)u = \begin{pmatrix} E-\mu & -\sigma^i p^i \\ -\sigma^i p^i & E+\mu \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} \end{aligned} . \tag{15.1}$$

We then restructure this is the usual way into (see, e.g., [8] at [5.21]):

$$Eu = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \mu & \sigma^i p^i \\ \sigma^i p^i & -\mu \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \left(\alpha^i p^i + \mu\gamma^0\right)u = Hu \tag{15.2}$$

while defining the Dirac Hamiltonian via $Hu = Eu$, and specifically, as the *operator*:

$$H \equiv \alpha^i p^i + \mu\gamma^0 . \tag{15.3}$$

Then, because it is sandwiched between (operating on) Dirac spinors as in (15.2), we return to (8.16) and use (15.2) to replace each occurrence of $E = p^0$ above with $E \to H + \mu\gamma^0 = \alpha^i p^i + \mu\gamma^0$. Then, with $\mu \to -m$ via (6.17), this yields:

$$\begin{aligned} T(p,0,p) &= \overline{\psi}\gamma^0 p^0 \psi - \overline{\psi}\gamma^i p^i \psi - \frac{e}{m}\overline{\psi}\left(\alpha^i p^i \phi - p^i A^i\right)\psi + \frac{g}{2}\frac{e}{2m}\overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi + e\overline{\psi}\gamma^0\psi\phi \\ &= \overline{\psi}\gamma^0 p^0 \psi - \overline{\psi}\gamma^i p^i \psi - \frac{e}{m}\overline{\psi}\begin{pmatrix} -p^i A^i & \sigma^i p^i \phi \\ \sigma^i p^i \phi & -p^i A^i \end{pmatrix}\psi + \frac{g}{2}\frac{e}{2m}\overline{\psi}\begin{pmatrix} -\sigma^i B^i & i\sigma^i E^i \\ i\sigma^i E^i & -\sigma^i B^i \end{pmatrix}\psi + e\overline{\psi}\gamma^0\psi\phi \end{aligned} . \tag{15.4}$$

This is equivalent to (8.16) in all respects. We then develop the term $\overline{\psi}\left(\alpha^i p^i \phi - p^i A^i\right)\psi$ in (15.4) *in exactly the same way* via Higgs expansion as we developed the term $\overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi$ by Higgs expansion in sections 9, 10 and 11. In the process of this development, originating from $\alpha^i p^i$, we will come across the cross product $\boldsymbol{\alpha}\times\mathbf{p} = i[H,\mathbf{L}]$ which commutes the Hamiltonian with the orbital angular momentum just as we earlier came across terms based in $i\alpha^i E^i$ which ended up yielding both the $\nabla\cdot\mathbf{E}$ and $\nabla\times\mathbf{E}$ terms from Maxwell's equations. And, because the lead coefficient for $\overline{\psi}\left(\alpha^i p^i \phi - p^i A^i\right)\psi$ is e/m rather than the $e/2m$ which precedes $\overline{\psi}\left(i\alpha^i E^i - \Sigma^i B^i\right)\psi$, the orbital angular momentum will inherently have twice the magnitude as the spin angular momentum, that is, it will come in whole increments of $\hbar$ rather than only in a $\frac{1}{2}\hbar$ packet. We leave this full development of orbital angular momentum to a future exercise.

16. Conclusion

As observed throughout this paper, Higgs theory is fundamentally a theory about the particles and fields that we do and do not observe in nature. It informs us that the fields we write down in our Lagrangians or Hamiltonians are not the fields we observe, and that only after we have expanded these "seed" fields about a non-trivial non-zero vacuum do we obtain the particles and fields that are actually observed. But to date, Higgs theory has only been fully developed for scalars, and the experimental data for Higgs scalars is very thin compared to the wealth of data that is available and known for fermions. It is intended that the application of Higgs theory to fermions as developed in this paper will provide additional avenues through which this fundamentally-important theory of what we observe in the physical universe might be experimentally validated.

One possible avenue for validation, at (13.10), involves detecting the impact of a time-dependent magnetic field on the known g-factors of the charged leptons. This is a specific numeric prediction as to the magnitude of the g-prime-factor associated with a time-dependent magnetic field, as well as a qualitative prediction that a time-dependent magnetic field will cause a much greater response for the tau lepton than for the other two charged leptons, and in the muon more than in the electron, progressively by about 2 to 3 orders of magnitude from one generation to the next. In general, all three of the g-factor types (12.10), (12.12) and (12.13) could with some ingenuity provide paths to further validation of the Higgs thesis.

Another possible venue for validation is (13.9) which decomposes the mass of each fermion into the two constituent gauge potentials for which the observed mass is the *difference* between potentials. Of course, gauge fields are not observables as absolute numbers; the only thing that has physical meaning is a difference in potential. So one expects the separate energy contributions in (13.9) will not be directly observable. Yet at the same time, there can be little doubt that the scalar potential ϕ is fundamentally an indicator of the fact that an electrically-charged fermion has a charge, while the vector potential $\mathbf{A}$ fundamentally indicates that that charge is spinning and thus has "intrinsic" kinetic aspects as well. Whether there is some alternate, observable way to discern how much of the mass-energy associated with a charged lepton comes from its charge and how much comes from its spin is left as an open question, but (13.9) is the "food for thought" as to this possible avenue for validation.

Another avenue to consider originates when contrasting the particle Lagrangian (5.13) with the antiparticle Lagrangian (5.14). Clearly, there is a broken symmetry as between fermions and antifermions. Beyond (5.14), we focused full attention on developing the particle (positive energy, positive vacuum) Lagrangian, and did no further development for the antiparticle Lagrangian. That is clearly a "to do" item for which the development path is well-laid out by the particle development. Whether this broken symmetry between particles and antiparticles translates into something that can be or has been observed is unclear at the moment, but is worth further exploration.

In section 15, we laid out how the orbital angular momentum is to be developed, but stopped short adding the several additional sections akin to sections 9 through 11 to do this complete development. This is a worthwhile pursuit, and could lead to further validation opportunities by finding factors for orbital angular momentum of an analogous nature to the spin g-factors developed here.

Another important topic for further development is renormalization, which has bedeviled physicists for years. In many ways, doing renormalization is like trying to walk on air, because everything becomes scale dependent and as soon as one changes the scale, everything else changes. This is especially so the moment that rest mass cannot even be regarded as an invariant. In the 20th century, physicists learned the value of finding *invariant* quantities in nature, and of understanding the symmetries and conservation laws behind those invariants. Renormalization poses a severe challenge that notion, because even the supposedly reliable ground of an invariant rest mass apparently is no more. But the constant-mass renormalization of section 14 appears to put an end to that quandary: mass is a constant, even as one shifts the renormalization scale, *because a variation in the mass along the renormalization scale can always be gauged out*. So equation (14.21), which is the Ward identity, should be fully developed by developing the vertex of (14.20) and then writing out in full, the first-order differential equation (14.21) in $\partial / \partial p_{\mu}$ as a function of e, ϕ, **A,** *m* and the vacuum $v_+{}^3$. What (14.14) and (14.15) tell us, is that once we do extract the full differential equation which is the Ward identity (14.21), and then work to solve that equation as a function of p_{μ} or really the magnitude of $\mu = \sqrt{p_{\mu} p^{\mu}}$, we can treat the mass *m* as well as the vev v_+ as constant numbers, so that the only variables in the differential equation are the gauge fields A^{μ} and the running charge $e = \sqrt{4\pi\alpha}$. Being able to treat the mass as a constant will make mathematical life much simpler when solving (14.21), and the deduced running of the A^{μ} will then be a measure the gauge transformations which were applied to keep the mass constant. Additionally, (14.20) should provide a complete, closed expression for $\Gamma^{\mu}\left(p,0,p\right)$, so (14.21) should provide a fully closed form for the differential equation to be solved.

Another avenue for development that appeared along the way which we bypassed in the present development, was non-Abelian, Yang-Mills gauge theory. In (9.9) and its later counterparts (10.6) and (11.8), we ran into commutator terms $\left[A^{\mu}, A^{\nu}\right]$. We were not seeking these, but they appeared anyway. To simplify development and restrict ourselves to electrodynamics for the time being, we set $\left[A^{\mu}, A^{\nu}\right]=0$. But Yang-Mills theory is distinguished from Abelian gauge theory solely and exclusively by the fact that its gauge fields do not commute, $\left[A^{\mu}, A^{\nu}\right]\neq 0$, because they are now defined as the *matrices* $A^{\mu}\equiv\lambda^{i}A^{i\mu}$ where λ^{i} are the group structure constants for a gauge group for SU(N) . Were we to forego setting $\left[A^{\mu}, A^{\nu}\right]=0$, we would have additional terms in results such as (12.5) and (12.11) which would represent non-Abelian theory, and then the running of renormalization would invert the running of coupling and charges and produce asymptotic freedom, etc., and all of this development could then be applied, most notably, to weak and strong interactions. Clearly, this too is an avenue that ought to be pursued.

The one final aspect of this development which is nothing if not stunning, exemplified by (12.5) and (12.11), is that all of the time and space dependencies of Maxwell's equations are "revealed" from gauge fields which start out as Heisenberg operators without any explicit space or time dependency. This time and space dependency is revealed because Dirac's equation and its spinor solutions created commutation relationships with the canonical momentum that placed a time or space dependency back into the Dirac equation for all of the gauge fields A^{μ} to reveal the electric and magnetic fields **E** and **B**, and then went even further to commute the electric and magnetic fields with the canonical momentum to put $\boldsymbol{\sigma}\cdot\mathbf{B}$, $\nabla\times\mathbf{E}$, $\nabla\cdot\mathbf{E}$ and $\nabla\times\mathbf{B}$ terms right into the middle of the Dirac Lagrangian. And the only reason we did not have a $\nabla\cdot\mathbf{B}$ magnetic monopole term is because we turned off any non-Abelian interactions. It is a testament to the economy of nature that Dirac, Heisenberg and Maxwell all converge in this way. But what is tremendously profound is the realization that one can start with an equation which has no space or time dependencies, and then by the simple application of Heisenberg commutation to various commutator relationships that emerge, end up with a full space and time dependency for all of the electric and magnetic fields akin to those of Maxwell's equations. If Higgs theory is about "revealing" fields and masses which are not in our original Lagrangian, then what is demonstrated here is that that Dirac and Heisenberg theory reveal *space and time dependencies* even when they are not in the original Lagrangian. If the entirety of our experience in the physical universe is about what transpires in space over time, then the fact that space and time dependencies can emerge from a Lagrangian without any *ab initio* space and time dependency – what Wheeler in a Geometrodynamic mind might call "spacetime without spacetime" – is a deeply penetrating insight into the nature of our world.

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