Derivation of the Complete Doppler effect Formula by means of a (reflecting) Newtonian telescope and some additional consequences

Albert Zotkin
[1] *tardigrados.wordpress.com,*
[2] *fragmentales.wordpress.com (Radial Serendipity),*
[3] *albertzotkin.wordpress.com,*
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Abstract: In this short paper it is given to you a derivation of the Complete Doppler effect formula in the framework of Galilean Relativity, and such a formalism is compared with that of Special Relativity (SR). Then, it is shown how useful this enhanced Galilean Relativity can be. An example of a proton-antiproton computation is provided as exercise, and finally, it is proved that time dilation isn't necessary for explaining some phenomena, as the time of light of cosmic ray muons.

I. DERIVATION OF THE COMPLETE DOPPLER FORMULA

Let us arrange a reflecting Newtonian telescope, such that the light of a distant receding star is coming along its line of sight. Light reflected off by the primary mirror has a frequency of f, which is lower than the original one f_0 emittted by the star, because of the Doppler effect. We now accelerate that primary mirror a differential speed dv towards the central diagonal secondary mirror.

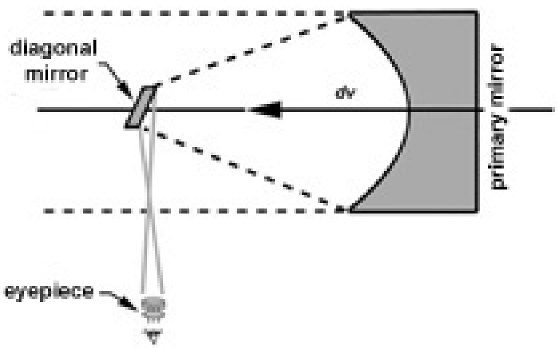

FIG. 1: A Newtonian telescope

That means the central diagonal mirror is now reflecting off light towards the objetive with a frequency slightly higher than f. That frequency will be $f + df$. Therefore, we can write the following differential equation and solve it:

$$f + df = f\left(1 + \frac{dv}{c}\right) \tag{1}$$

$$f + df = f + \frac{f\,dv}{c} \tag{2}$$

$$df = \frac{f\,dv}{c} \tag{3}$$

$$\frac{df}{f} = \frac{dv}{c} \tag{4}$$

$$\ln\left(\frac{f}{f_0}\right) = \frac{v}{c} \tag{5}$$

$$f = f_0 \exp\left(\frac{v}{c}\right) \tag{6}$$

so we have find out **the Complete Doppler effect formula**.

In that differential equation it has been used the first order approximation of the Doppler effect formula (the classical non-relativistic one). It is very important to highlight that when a diffeential of speed is integrated what we are doing is to sum over all infinitesimal quantities, it is saying, in that integration process we are uniformly accelerating the material system, and at the end of tha integration, the material system has accelerated from 0 to v.

$$\int \frac{dv}{c} = \frac{1}{c}(dv + dv + dv + ...) = \frac{v}{c} \tag{7}$$

I will mention now a person, considered himself an "expert" in the subject, that argued the use of that first order approximation was incorrect for deducing a supposed "complete formula", because from a first order it is not possible to attain any formula containing all infinite orders. Of course, that argumentation is blatantly wrong and misleading. It was Euclid who historically could show us how it is possible to approach a circle from a rectangle with an infinitesimal side. In that sense, we may see a rectangle as a first order approximation of a circle, and that supposed "expert" was absolutely mistaken. In the derivation that I have offered above, it is used something very similar to what Euclid used, and that is in the very core of the definition of integration.

Actually, the found solution is just an area. That area is $\beta = v/c$, that matches the area $\ln(f/f_0)$. Then, another supposed "expert" in the subject came to say that

I was very mistaken because that what appears in the formula as $\beta = v/c$ actually is a *rapidity*. This person pointed out to me that what I call velocity v actually is a hyperbolic velocity as defined in SR. Sure, the hyperbolic velocity as defined in SR, also called celerity, equals rapidity times c, if we replace $\beta = v/c$ for rapidity $\theta = \tanh^{-1}\beta$ we attain the famous relativistic formula for Doppler $f = f_0\sqrt{(1+v/c)/(1-v/c)}$. It was told to this second supposed "expert" in the subject that since I was not applying SR, but Galilean Relativity, there can't be any confusion at all, therefore, velocity v is postulated as a real velocity and never as a hyperbolic one. All these supposed "experts in the subject" are trying to refute the Complete Doppler formula found above, arguing that all experiments validate SR and invalidate my formula. That what these supposed "experts" claim is, of course, a great lie. They say that just because they are confident SR is well-tested and must be right, my formula must be wrong at first glance. But, if we compare both formalisms, that of SR and my formula, we get:

$$\frac{f}{f_0} = \exp\left(\frac{v}{c}\right) = 1 + \frac{v}{c} + \frac{v^2}{2c^2} + \frac{v^3}{6c^3} + \frac{v^4}{24c^4} + ... \tag{8}$$

$$\frac{f}{f_0} = \sqrt{\frac{1+v/c}{1-v/c}} = 1 + \frac{v}{c} + \frac{v^2}{2c^2} + \frac{v^3}{2c^3} + \frac{3v^4}{8c^4} + ... \tag{9}$$

This means, we would need to perform an experiment that could discriminate between both predictions with such an accuracy reaching to the third order of approximation, but that accuracy is not possible to achieve for current technology. Best precision in current experiments only reaches to the second order approximation in the $\beta = v/c$.

Corollary I.1. *It is easy to deduce the momentum of a particle from the Doppler effect:*

$$\boldsymbol{p} = \frac{m\boldsymbol{c}}{2}\left(\mathrm{D}\left(\frac{v}{c}\right) - \mathrm{D}\left(-\frac{v}{c}\right)\right) \tag{10}$$

This generic equation (10) always holds for any Doppler factor $\mathrm{D}(v/c)$ in any theory. Where $\mathbf{c}$ is a vector in the direction of the particle movement. The above deduced complete Doppler effect factor is $\mathrm{D}(v/c) = \exp(v/c)$, therefore, the momentum that can be deduced from that factor is:

$$\mathbf{p} = m\mathbf{c}\sinh\left(\frac{v}{c}\right) \tag{11}$$

Similarly, total energy of a particle deduced from Doppler arises from that generic equation as:

$$E = \frac{mc^2}{2}\left(\mathrm{D}\left(\frac{v}{c}\right) + \mathrm{D}\left(-\frac{v}{c}\right)\right) \tag{12}$$

Therefore, we have:

$$E = mc^2\cosh\left(\frac{v}{c}\right) \tag{13}$$

It is also easy to see that for the case of SR, we would have $\mathrm{D}(v/c) = \sqrt{(1+v/c)/(1-v/c)}$. So, after some algebraic steps, we would have $E = mc^2\gamma$ and $p = mv\gamma$, where γ is Lorentz factor.

And by the way, it is also easy to see that the above generic equations hold good for the relation $E^2 - c^2p^2 = m^2c^4$, if the generic function $\mathrm{D}(v/c)$ exhibits the property $\mathrm{D}(v/c)\mathrm{D}(-v/c) = 1$, that property has to be hold by every Doppler factor pretending to be consistent with the physical effect that wants to model.

II. EXAMPLE OF COMPUTATION OF A PAIR PROTON-ANTIPROTON PRODUCTION

The following exercise was proposed by amarashiki in a discusion about the issue, because he thought that was a strong argument that would refute the model I presented.

Exercise II.1. *Compute. using YOUR definition of energy and momentum, minimal energy and minimal kinetic energy in order to produce a pair proton-antiproton in the collision of a a proton A against a proton B at rest.. Note, you must not use neither the relativistic definition of energy $E = m\gamma c^2$ nor that of momentum $p = m\gamma v$, but your own equations, namely $E = mc^2\cosh(v/c)$ and $p = mc\sinh(v/c)$. Under SR assumptions, we attain minimal energy is $7mc^2$ (where misprotonmass), and minimal kinetic energy $6mc^2$. Under "your theory" assuptions, with YOUR definicitons of energy and momentum , above written, I say that it is impossible the creation of pairs. Since creation of pairs is observed, then "your theory" is bullshit. Refute it, if you can, with equations ..*

What follows was my reply and computation: This exercise will be solved, firstly using a reference frame centered in the center of masses of both protons, therefore momentum will be null. Firstly, I am going to compute assuming that reaction will produce a pion, π^0, with all final particles at rest after the collison, (p, p, π^0). Using my model, total energy of the system is:

$$E = 2mc^2 = 2m_pc^2 + m_\pi c^2 \tag{14}$$

where

$$m = m_p\cosh(v/c) \tag{15}$$

So, for the production of that π^0, the aproaching speed for each proton towards the center of masses must be:

$$v = c\cosh^{-1}\left(1 + \frac{m_\pi}{2m_p}\right) \tag{16}$$

and since velocities in my model sum trivially and classically like vectors, we get velocicity v' of approaching of one of those protons in the reference frame where the other is at rest would be:

$$v' = v + v = 2c\cosh^{-1}\left(1 + \frac{m_\pi}{2m_p}\right) \tag{17}$$

This result belongs to reaction that produces a pion, $p + p \to p + p + \pi^0$. It is very easy to see now that the reaction that produces a pair proton-antiproton, $p+p \to p+p+p+\bar{p}$, must imply an approaching speed of a proton towards the other as:

$$v' = 2c\cosh^{-1}\left(1 + \frac{2m_p}{2m_p}\right) = 2c\cosh^{-1}(2) = 2.63392c \tag{18}$$

So, that means minimal kinetic energy would be:

$$E_k = m_p c^2(\cosh(2.63392) - 1) = 6m_p c^2 \tag{19}$$

And total minimal energy would be:

$$E = m_p c^2 \cosh(2.63392) = 7m_p c^2 \tag{20}$$

This translated to the SR formalisms, where constant c is assumed to be a limit speed that cannot be exceeded by anything, we will get a velocity:

$$v'' = \tanh(2.63392)c = 0.989743c \tag{21}$$

That is the trick that SR succeeded in fooling all theoretical physics since more than a century now. They (mainstream physicists) believe that particles cannot exceed c, but the truth is that speed is routinely being exceeded in any particle accelerator, even in muons produced by cosmic rays in upper layers of the Earth's atmosphere. In order to perpetrate that deception, SR proponents devised fictitious effects like time dilation and length contraction, or even a more absurd one, concerning relativity of simultaneity of events, and theoretical tricks as Einstein convention for synchronisation of distant clocks at rest.

III. TIME DILATION IS A FALLACY

Let's see now how so called time dilation, claimed to have been tested with success in cosmic ray muons, actually is a big fallacy. Muons exhibit a mean life of 2.19703(4) 10^{-6} s. But then, a muon, produced in upper layers of Earth atmosphere, would have lack of time for arriving to the ground even if could travel at speed of light c., or at best only a smalll quantity would be detected , but that is not observed. Mainstream reasoning is that muons have to move at high speeds, not superluminal ones but relativistic. Those relativistic muons would move at speeds about $0.999c$ or higher. Under SR assumptions, motion at such high speeds would produce a meaningful dilation of tke proper time of the muon, threrefore its mean life would be longer. We can prove that reasoning is a fallacy. What actually is happening is those muons preserve unchanged thieir mean life,2.19703(4) 10^{-6} s, but their speeds are higher than c. Let us see with numbers why that mainstream reasoning is a fallacy. Let us assume a muon, when created in upper layers of the Earth atmosphere, exhibit a total energy of $E = 20$ GeV. Then, from that energy it is easy to compute its speed with respect to te ground detector, since:

$$E = mc^2 \cosh\left(\frac{v}{c}\right) \tag{22}$$

$$v = c\cosh^{-1}\left(\frac{E}{mc^2}\right) \tag{23}$$

and since muon rest energy is $E_0 = mc^2 = 105.658367(4)$ MeV, we get:

$$v = c\cosh^{-1}\left(\frac{20 \times 10^9}{105.6 \times 10^6}\right) = 5.93697c \approx 6c \tag{24}$$

In other words, muons with energy 20 GeV, produced in upper layers of the Earth atmosphere, arrive to the ground detector on time because their speed is about six times the speed of light. This also show in a compelling way that muon-neutrinos, debris of muon decay, measured in OPERA experiment, actually travelled at superluminal speeds, although as it has been reliably shown, it is more than evident SR formalisms mask that reality.

Corollary III.1. *We can see the differential equation from whcih we could integrate the Doppler effect under SR assumptions*

$$\frac{df}{f} = \frac{dv}{c(1 - \frac{v^2}{c^2})} \tag{25}$$

and integrating

$$\ln\left(\frac{f}{f_0}\right) = \tanh^{-1}\left(\frac{v}{c}\right) \tag{26}$$

$$\ln\left(\frac{f}{f_0}\right) = \frac{1}{2}\ln\left\{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right\} \tag{27}$$

$$f = f_0\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \tag{28}$$

The problem with that SR equation (25) is its unclear physical origin. We can't see what physical effect that differential equation pretends to model. In contrast, equaton (1) clearly is modelling a first order approximation (classical) Doppler effect Anyway, let's try to analyze a little further that SR relation . We can see that when integrating it we get $\ln(\frac{f}{f_0}) = \tanh^{-1}(\frac{v}{c})$, and notice that hyperbolic arctangent actually is a rapidity:

$$\theta = \tanh^{-1}\left(\frac{v}{c}\right) \tag{29}$$

and that means $d\theta$, so when we integrate it:

$$d\theta = \frac{dv}{c(1 - \frac{v^2}{c^2})} \tag{30}$$

we get rapidity θ. So, this corollary proves that in the Complete Doppler formula I derived above you can't be mystified regarding any speed as a hyperbolic speed, neither a β as a rapidity θ, because we can clearly see this latter has its own differential equation.

[1] "The Complete Doppler Formula: Return to the Origin".Albert Zotkin. viXra:1209.0003

[2] "Cosmic ray muons and relativistic time dilation". Preston College Cosmic Ray Group.

[3] "Experimental Tests of General Relativity". Mester, John (2006).

[4] "An experimental study of the rate of a moving clock". Ives, H. E.; Stilwell, G. R. (1938).