On Special Relativity: Root Cause of the Problems with Lorentz Transformation

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In this paper the Lorentz Transformation is shown to be merely a set of restricted equations stemmed from the Galileo transformation applied to a particular conversion reflecting the theorized principle of the speed of light invariance implemented in the direction of the relative motion between the inertial reference frames. Consequently, the Lorentz transformation is shown to be restricted to time and longitudinal space coordinate values different from zero. The deduction of the time dilation and length contraction becomes unfeasible under such restrictions. It follows that the Lorentz transformation possesses no other effects than mathematically expressing the speed of light postulate in the relative motion direction; that is, the coordinate of the tip point of a light ray traveling in the direction of the relative motion, given by $x = ct$ in the "stationary" frame, is transformed to $x' = ct'$ with respect to the "traveling" frame, with c being the light speed in empty space. Furthermore, the application of the Lorentz transformation to events having restricted coordinates is shown to result in mathematical contradictions.

Introduction

The Lorentz transformation equations constitute the backbone of the special relativity theory in which their interpretations lead to the peculiar predictions of the space- time distortion characterized by the length contraction and time dilation. The Lorentz transformation was derived by Einstein[1, 2] on the basis of the constancy of the speed of light postulate. The sought transformation, converting between the space and time coordinates of two inertial reference frames, say $K(x, y, z, t)$ and $K'(x', y', z', t')$, in relative motion at speed v, was assumed to take the following general form

$$x' = ax + bt$$

$$y' = y$$

$$z' = z$$

$$t' = kx + mt$$

where $a, b, k,$ and m are unknown real terms.

Whereas, the constancy of the speed of light postulate was expressed by the assumption that a spherical light wave front, emitted from the coinciding frame origins, would be observed as a light sphere centered at the frame origin, with its radius being expanded at the speed of light c, with respect to either frame:

$$x^2 + y^2 + z^2 = c^2t^2$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2$$

leading to

$$x^2 - x'^2 = c^2t^2 - c^2t'^2$$

In the customary derivation of the Lorentz transformation, the latter speed of light constancy equation along with the above proposed space and time transformation equations and given particular conditions would be solved for the unknown terms, yielding the following Lorentz transformation equations:

$$x' = \gamma(x - vt);$$
$$y' = y\,;$$

$$z' = z\,;$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right);$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

The above approach is rather complex, which makes inconsistent operations performed in the derivation process easily bypassed. For instance, the above constancy of the speed of light equation was obtained in the original paper on special relativity[1] through constructing it from the basic conversion expressions $x = ct$; $x' = ct'$ presenting the speed of light invariance in the relative motion direction:

$$x = ct;\; x^2 = c^2t^2;\; x^2 - c^2t^2 = 0$$
$$x' = ct';\; {x'}^2 = c^2{t'}^2;\; {x'}^2 - c^2{t'}^2 = 0$$
$$x^2 - c^2t^2 = {x'}^2 - c^2{t'}^2;$$
$$x^2 - {x'}^2 = c^2t^2 - c^2{t'}^2.$$

Obviously, the intrinsic property of the basic expressions $x = ct$; $x' = ct'$, requiring $x = 0$ when $t = 0$—thus leading to $x' = 0$ and $t' = 0$— is lost in the above constructed speed of light equation. To remedy this inconsistency, the above constructed equation should be restricted to non-zero coordinate values.

Furthermore, we can equally use the basic expressions $x = ct$; $x' = ct'$ to construct the following equation, by squaring each one and adding the resulting expressions:

$$x^2 + {x'}^2 = c^2t^2 + c^2{t'}^2,$$

which would make the y, y', z, and z' coordinates equal to zero in the above light sphere equations.

Consequently, to avoid the encountered inconsistencies in the above conventional derivation approach, a straight forward method is used in this study to derive and reveal the innate limitations of the Lorentz transformation.

The speed of light constancy principle equations, as well as the Lorentz transformation, have been the subject of analytical studies,[3-5] in which mathematical contradictory results, attributed to the Lorentz transformation and the speed of light postulate, have been unveiled. This communication provides supplementary materials to the said works, in which the attained conclusions are reconfirmed by addressing the Lorentz transformation from a different perspective using a direct derivation approach—rather than working backward through analyzing the given Lorentz transformation—leading to the same detrimental contradictions.

Limitations of the Lorentz Transformation

Consider two inertial reference frames, $K(x, y, z, t)$ and $K'(x', y', z', t')$, in relative uniform motion along the overlapped x- and x'-axes, at a speed v. The transformation relating the space and time coordinates of the two frames is to be determined. If the time duration was considered to be unchanged from one frame to another, the coordinate conversion equation would then be governed by the Galilean transformation, namely

$$x' = x - vt \qquad (1)$$

with unchanged y and z coordinates (i.e. $y = y'; z = z'$).

It would then be inferred that the general transformation should have the following linear form;

$$x' = \gamma x + \beta t, \qquad (2)$$

where γ and β are real terms to be determined—y and z remain invariant.

For both cases described by equations (1) and (2), the origin of K' is traveling at speed v with respect to K origin. Therefore, we can conclude that the coordinate $x' = 0$ in K' would be transformed to $x = vt$ in K, by both equations. Hence, plugging the particular conversion $x' = 0; x = vt$ in the general transformation equation (2) yields the particular equation $0 = \gamma vt + \beta t$, or $\beta = -\gamma v$ (for $t \neq 0$), leading to a simplified general transformation equation

$$x' = \gamma(x - vt). \qquad (3)$$

Furthermore, under the principle of the constancy of the speed of light, another particular conversion related to the x-coordinate of the tip point of a light ray propagating in the relative motion direction is readily available, and can be expressed as $x = ct; x' = ct'$, which, when plugged in equation (3), leads to the particular equation

$$ct' = \gamma\left(ct - \frac{vx}{c}\right);$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right); \tag{4}$$

with the above restriction $t \neq 0$ being maintained, leading to the additional restriction of $x \neq 0$, since $t = x/c$ is used to get the expression vx/c^2 in equation (4).

Now, owing to the fact that the reference frame K is traveling at a speed of $-v$ with respect to K', and to the essential symmetrical property of the transformation with respect to the reference frames, the inverse of the general transformation given by equation (3) can be written as

$$x = \gamma(x' + vt'), \tag{5}$$

which must be as well restricted—by symmetry—to $t' \neq 0$.

Similarly, under the principle of the constancy of the speed of light, plugging the particular conversion of the tip point x'-coordinate of a light ray propagating in the relative motion direction, expressed as $x' = ct'; x = ct$, in the general transformation equation (5) leads to the particular equation

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right), \tag{6}$$

equally maintaining the above restriction $t' \neq 0$, leading to $x' \neq 0$.

Substituting equations (3) and (4) in equation (6), leads after simplification to

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{7}$$

It follows that equations (3–7) constitute the Lorentz transformation—and its inverse—although equations (4) and (6) are shown to be merely particular equations obtained from the special conversion $x = ct; x' = ct'$, expressing the constancy of the speed of light principle in the direction of the relative motion, when plugged in the general transformation equations (3) and (5). In addition, as demonstrated above, the Lorentz transformation equations (3–7) are restricted to values of x, t, x', and t' different from zero.

Furthermore, since γ was determined with the use of the particular equations (4) and (6), equations (3) and (5) would bear the same limitations as equations (4) and (6). It follows that all Lorentz transformation equations are limited to the particular conversion $x = ct; x' = ct'$, and to coordinate values not equal to zero. These results have been confirmed in an earlier study through mathematical analyses of the Lorentz transformation.[3-5]

Conflicting Findings

The invalid generalization of the particular equations (4) and (6) would result in mathematical conflicts. Indeed, substituting equation (4) into equation (6), returns

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right),$$

which can be simplified to

$$t(\gamma^2 - 1) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right). \tag{8}$$

Since equations (4) and (6) reflect a special case of the general transformation satisfying the particular conversion $x = ct; x' = ct'$, then equation (8) can be written as

$$t(\gamma^2 - 1) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \tag{9}$$

If equations (4), (6) and (9) were generalized (i.e. applied to conversions other than $x = ct; x' = ct'$, or $t = x/c\,; t' = x'/c$), and particularly applied to an event with the restricted time $t' = 0$, then according to equation (4), the transformed t-coordinate with respect to K would be $t = vx/c^2$. Consequently, for $t \neq 0$, equation (9) would reduce to

$$t(\gamma^2 - 1) = t\gamma^2, \tag{10}$$

yielding the contradiction,

$$\gamma^2 - 1 = \gamma^2\,, \text{ or } \quad 0 = 1\,.$$

It follows that the conversion of the restricted time coordinate $t' = 0$ to $t = vx/c^2$, for $x \neq 0$, by Lorentz transformation equation (4), is proved to be invalid, since it leads to a contradiction when used in equation (9), resulting from the Lorentz transformation equations for $t \neq 0$ (i.e. beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$)—Letting $t = 0$ would satisfy equation (10), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $t' = 0$ would be $t = vx/c^2 = 0$, yielding $v = 0$, as we're addressing the conversion of $t' = 0$ to $t = vx/c^2$ for $x \neq 0$.

A similar contradiction is obtained by substituting equation (6) into equation (4), and applying equation (6) for the conversion $t = 0$; $t' = -vx'/c^2$ of the restricted time coordinate $t = 0$.

Furthermore, substituting equation (3) into equation (5), yields

$$x = \gamma(\gamma(x - vt) + vt')\,;$$
$$x(\gamma^2 - 1) = \gamma v(\gamma t - t')\,;$$
$$x(\gamma^2 - 1) = \gamma vt\left(\gamma - \frac{t'}{t}\right). \qquad (11)$$

Since equations (3) and (5), along with equations (4) and (6), satisfy the particular conversion $x = ct; x' = ct'$, equation (11) can be written as

$$x(\gamma^2 - 1) = \gamma vt\left(\gamma - \frac{x'}{x}\right). \qquad (12)$$

If equations (4), (6) and (12) were generalized (i.e. applied to conversions other than $x = ct; x' = ct'$), and particularly applied to an event with the restricted coordinate $x' = 0$, then according to equation (3), the transformed x-coordinate with respect to K would be $x = vt$. Consequently, for $x \neq 0$, equation (12) would reduce to

$$x(\gamma^2 - 1) = x\gamma^2, \qquad (13)$$

$$\gamma^2 - 1 = \gamma^2, \text{ or } \quad 0 = 1.$$

It follows that the conversion of the of the restricted space coordinate $x' = 0$ of K' origin to $x = vt$, at time $t > 0$, with respect to K by Lorentz transformation equation (3), is invalid, since it leads to a contradiction when used in equation (12), resulting from Lorentz transformation equations, for $x \neq 0$ (i.e. beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$)—Letting $x = 0$ would satisfy equation (13), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $x' = 0$ would be $x = vt = 0$, yielding $v = 0$, as we're addressing the conversion of $x' = 0$ to $x = vt$ for $t > 0$.

A similar contradiction would follow upon substituting equation (5) into equation (3), and applying equation (5) for the conversion $x = 0$; $x' = -vt'$ of the restricted time coordinate $x = 0$.

It is worth mentioning that another conflict would emerge upon letting $t' = 0$ in the conversion $x' = ct'; x = ct$, since this results in $x' = 0$ (a restricted coordinate for Lorentz transformation) which is converted into $x = vt$ by equation (3), leading to the conflicting equality $vt = ct$, or $v = c$ — Alternatively, $t' = 0$ is restrictively converted into $t = vx/c^2$ by equation (4), leading to $x = vx/c$, or $v = c$.

It follows that, the application of the Lorentz transformation to events having any of the determined restricted coordinates (i.e. x, x', t, or t' is equal to zero) is unfeasible, as it leads to contradictions. Consequently, the interpretation of the time dilation and length contraction would not be possible, since the former requires co-local events (i.e. $x' = 0$) and the latter simultaneous events (i.e. $t = 0$).

Conclusion

The Lorentz transformation is demonstrated to be restricted to events having non-zero time coordinates and non-zero space coordinates along the reference frames axes parallel to the relative motion direction. With such imposed coordinate restrictions, the effects of the time dilation and length contraction become unfeasible. Furthermore, the Lorentz transformation is shown to be limited to merely expressing mathematically the speed of light postulate in the relative motion direction, with no practical results or predictions being obtained from its application.

References

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