An algebraic journey in to geometric forest

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Abstract:

Once the famous French mathematician Lagrange remarked that as long as algebra and geometry are not inter linked one can not expect good results. Keeping this in mind, the author has attempted to establish an interesting classical Euclidean theorem by applying the algebra of matrices.

Key words: Matrix algebra, classical geometry, new finding

MSC : 0899, 51 M04

1. Results:

\[
A = \begin{bmatrix} x & y \\ a & v \end{bmatrix}
\]

(1)

\[
B = \begin{bmatrix} y & z \\ b & \end{bmatrix}
\]

(2)

\[
C = \begin{bmatrix} y & c \\ z & m \end{bmatrix}
\]

(3)

\[
D = \begin{bmatrix} x & a \\ y & v \end{bmatrix}
\]

(4)

\[
E = \begin{bmatrix} y & v \\ z & b \end{bmatrix}
\]

(5)

\[
F = \begin{bmatrix} y & e \\ c & m \end{bmatrix}
\]

(6)
\[
A + B = \begin{bmatrix}
x + y & y + z \\
\alpha + v & v + b
\end{bmatrix}
\]  
(7)

\[
C - D = \begin{bmatrix}
v - x & c - a \\
z - y & m - v
\end{bmatrix}
\]  
(8)

\[
D + E = \begin{bmatrix}
x + y & \alpha + v \\
y + z & v + b
\end{bmatrix}
\]  
(9)

\[
F - E = \begin{bmatrix}
x - v & z - v \\
\alpha - c & v - m
\end{bmatrix}
\]  
(10)

\[
A - F = \begin{bmatrix}
x - v & y - z \\
\alpha - c & b - m
\end{bmatrix}
\]  
(11)

\[
C - B = \begin{bmatrix}
v - y & c - z \\
z - v & m - b
\end{bmatrix}
\]  
(12)

Adding (7) to (12) \( 2a + 2c = \begin{bmatrix} 2x + 2v & 2y + 2c \\ 2v + 2b \end{bmatrix} \)  
(13)

But from (1) and (2), \( 2a + 2c = \begin{bmatrix} 2x + 2v & 2y + 2c \\ 2a + 2z & 2v + 2m \end{bmatrix} \)  
(14)

Comparing (13) and (14), we obtain that \( b = m \)  
(15)

2. Geometrical interpretation of Eqn. (15)

In the Euclidean figure 1, \( x, y, z \) and \( m \) respectively denote the sum of the interior angles of triangles ABD, ADE, AEF and AFC. And \( a, b \) and \( c \) denote the interior angle sum of triangles ABE, ADF and AEC respectively. \( v \) refers to 180 degrees. Adding \( x + y = v + a, y + z = v + b \) and \( m + z = v + c \). We have formulated matrices A, B, C, D, E and F by using the elements \( x, y, z, m, a, b, c \) and \( v \).
From eqn. (15) we get that the sum of the interior angles of Euclidean triangle ADF and AFC are equal.

\[
\text{Euclidean Figure 1}
\]

**Discussion:**

It is well known that the fifth Euclidean postulate holds that, If there are two triangles with equal angles \([1-7]\) So, eqn. (16) establishes the parallel postulate of Euclidean geometry.

**References**

[1] viXra:1305.0181