Supersymmetry in Single Crystals

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This study compares some fundamental concepts in the fields of crystallography and quantum mechanics. Edge dislocations in a single crystal behave as fermions if they repel each other and behave as bosons if they attract each other. The bosonic and fermionic characteristics of edge dislocations enable it to study some fundamental properties of soliton supersymmetry.

Introduction

Supersymmetry. According to the theory of unbroken supersymmetry, for each fermion there exists a boson superpartner and for each boson there exists a fermion superpartner with the same fundamental characteristics (including mass and charge). But, broken supersymmetry states that all superpartners have masses much greater than their associated particles, leaving no hope to observe the unbroken supersymmetry. It is believed that, observation of superpartners will assist scientists to explain the universe under a single fundamental framework, however, unlike the massive study on supersymmetry, no practical evidence on that is yet reported (1, 2).

Dislocations and Nonlinearity in Single Crystal Systems. Dislocations in single crystals are imperfections in almost perfect regions. Two main types of dislocations are known as edge and screw and third type which is the dominant one is mixed of the edge and screw. Edge dislocations are responsible for plasticity (bending) of single crystals, screw for rotating (twisting), and mixed for simultaneous bending and twisting (helical configuration). Two opposite Burger vectors $(\vec{b})$ are attributed to each type of dislocation. It totally leaves 6 Burger vectors (3–4). Also in symmetric group (S_3), the smallest non-Abelian group has 6 elements, which is the smallest perfect number, as well (5). Burger vectors as fundamental identities of dislocations explain about their directions and magnitude of distortions. Dislocations which are topological line defects have no independent identities, but they govern electrical (~ mechanical), optical, and magnetic properties of single crystals (6–8). Each dislocation with its fundamental properties including effective charge, mass, and space $(\vec{b})$ is defined as a singularity in a single crystal in which constitutive atoms with their masses and charges are designed within the crystal lattices. Therefore, each singularity finds its effective identities, introducing nonlinearity in the same identities of the constituent elements of the whole structure of a perfect single crystal (9).

Edge Dislocations and Internal Energy. Top-view schematic diagram of a dislocation-free single crystal is presented in Fig. **1A**. Figure **1B** illustrates that how due to an extra half-plane

($\perp$) there exists a compressive and tensile stress fields above and below an edge dislocation, respectively. Figure **1B** suggests that atomic mass and charge distribution around a single edge dislocation are symmetric. Bending of the single crystal due to the existence of an edge dislocation is clear in Fig. **1B**. Figure **1C** displays a single crystal with two nearly far opposite edge dislocations. Mass and charge distributions around these two edge dislocation are almost uniform, as they are considered far enough.

Dislocations as sources of excitations are increasing entropy and therefore internal energy of single crystal systems. From another point of view, edge dislocations create plasticity (bending). Plasticity is an origin of stress, which is directly proportional to the internal energy of single crystals (10, 11). A flat single crystal, suffering no plasticity (stress-free) has the lowest energy state, can be considered as the quantum-mechanical ground state (12, 13) of the system (Fig. **1A** and **C**). The quantum-mechanical ground state can be obtained only at absolute zero (14), where kinetic energy disappears and internal energy is purely potential energy. Therefore, I suggest that the ground state of a single crystal is prohibited.

A perfect amorphous model is also stress-free (no bending) (15), suggested to be the same as a perfect single crystal. Fig. **1C** could be also a representative of a perfect amorphous model including two opposite grains, separated by a dashed line. However, understanding of the duality between single crystal and amorphous systems is not yet considered in device technology, and this is the first suggestion for such systems, but many similarities between characterizations of these two systems are reported without consideration of the suggested fundamental duality (16–18). Nevertheless, many fundamental scientific studies are finished on duality of order-disorder systems (19, 20).

If a perfect single crystal (ordered system) is a system of pure potential energy, probably perfect amorphous system (disorder system) could be attributed to pure kinetic energy. Prohibition of a perfect disordered system (as a result of its duality with prohibited perfect ordered system) sounds reasonable, because its next random step will be the appearance of ordered crystalline grains violating the second law of thermodynamics (transition of higher entropy to the lower one).

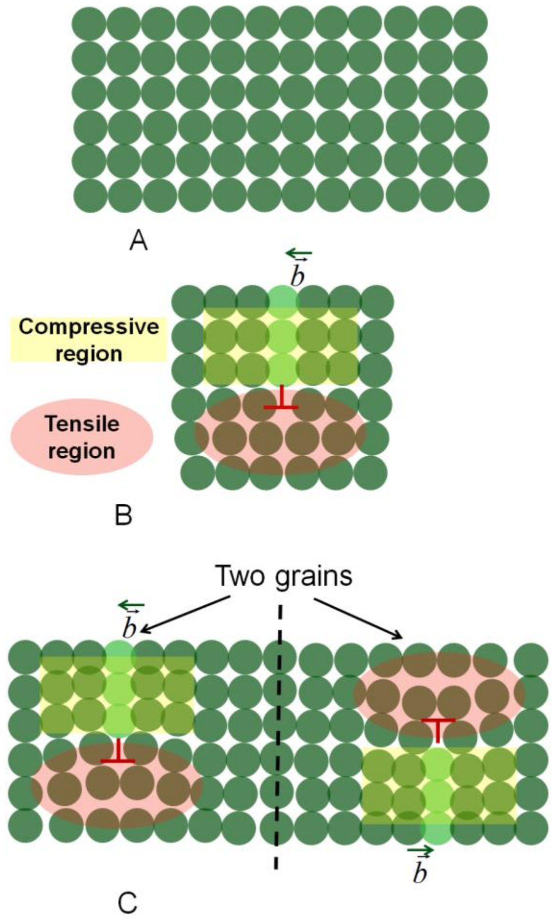

Fig. 1. Top-view schematic diagram of single-crystals. (A) Perfect (dislocation-free) single-crystal. (B) Single-crystal with one edge dislocation[†]. (C) Two opposite edge dislocations creating no bending which is same as a perfect single-crystal. A perfect single-crystal can be a representative of an amorphous model with two equal opposite grains, separated by a dashed line.

Edge Dislocations as Representatives of Fermions in Single Crystal Systems. Because two opposite Burger vector edge dislocations are making the ground state of a single crystal, therefore, I refer them as fermions. In this investigation, a single crystal system is also referred as a "fermionic system". Burger vector $(\vec{b})$ is the effective space of a dislocation (fermion) in a single crystal. In this work, I considered only a dislocation-free single crystal system and a single crystal system with a pair of opposite edge dislocations, as the ground states. However in reality, there could be degeneracy in the ground state of single crystals with 2n dislocations where half of them have Burger vectors with opposite sign of the other half. This condition may follow the model of fermions (edge dislocations) bound to vortices cores (21). Later in this study, I will show how dislocations rotate around their own central cores.

Duality of Fermions and Bosons as the Key Point of Supersymmetry. Fig. **2A** displays two very close edge dislocations. The asymmetry in mass and charge distribution around these two fermions are clear in Fig. **2A**. As much as these fermions become closer, the mass and charge density between them become higher, applying a pressure on both of them. It is worthy to mention that the total mass and charge of each fermion remain the same. However they undergo nonlinear distributions.

It is known that, dislocations during the growth procedure of a single crystal interact with each other and sometimes they merge to loops (4, 22). However unlike some of the traditional believes, it is not acceptable that they annihilated to a perfect region or a better region of crystallinity (23, 24). An old belief states that two opposite Burger vector edge dislocations merge to a perfect single crystal as shown in Fig. **2B**. Merging of dislocations to nothing is explained based on attractions between tensile-compressive regions, as already I indicated them in Fig. **1C**. However, attraction is a spontaneous phenomena and how it is possible that a spontaneous phenomena leads a system to zero entropy? Therefore, Fig. **2B** is prohibited which is in agreement with my previous statement about the prohibition of existence of perfect single crystals as ordered systems. This prohibition will be visualized at the end of this study, where dislocations merge to a single dislocation and not to a perfect region.

Fig. **2B** indicates if such merging happens, then there will be a pure attractive force between these two dislocations. Moreover, these half integer extra planes convert to a full integer plane. A single crystal can accept infinite number of these full integer planes to build up its ground state. Therefore, I call the attractive pairs of dislocations as bosons. Mass and charge distribution around these bosons are linear and uniform. Earlier, I claimed that edge dislocations (in the role of fermions) find their fundamental identities by introducing nonlinearity in the whole system. Now, one may claim that as there is no disturbance in the whole crystal system induced by these bosons, therefore they have zero mass and change. From another point of view another one may claim that they have full mass and charge of the crystal. This could be interpreted as the duality of nothing and everything.

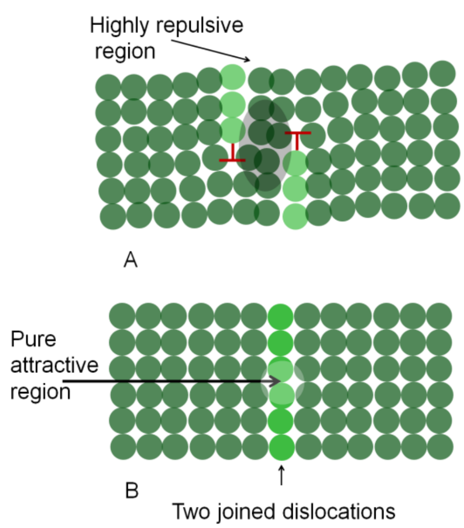

Fig. 2. Top-view schematic diagram of single crystals. (A) Two very close edge dislocations (two fermions) in a single crystal. (B) A Perfect single crystal, where two half extra planes, called as edge dislocations, are merged to a perfect plane (two bosons).

Broken Supersymmetry. For more simplicity in this study, edge dislocations in Fig. **3A–D** are considered based on their compressive and repulsive regions with more emphasis on repulsive-attractive forces between them. Fig. **3A** displays a schematic diagram of an edge dislocation (a single dislocation is not fermion or boson). Fig. **3B** and **3C** show pairs of perfect fermions and bosons (with exaggeration on their comparative directions) which are prohibited because of the presence of pure strong internal repulsion and attraction, respectively. It means that for each prohibited perfect fermion pair there exists a prohibited perfect boson pair, with the same shape, mass, and charge, conforming the prohibition of unbroken supersymmetry. Now, the arising question is about the interactions of dislocations.

Because, edge dislocation pairs cannot be parallel or opposite to each other, therefore, they must rotate compared to each other. Fig. **3D** represents how two rotated edge dislocations are merging together. During the merging, compressive regions attract the repulsive regions and vice versa. Therefore, dislocations are partially fermions and partially bosons. The presence of both repulsive and attractive forces at the same time between two fermions-bosons is clear in Fig. **3D**.

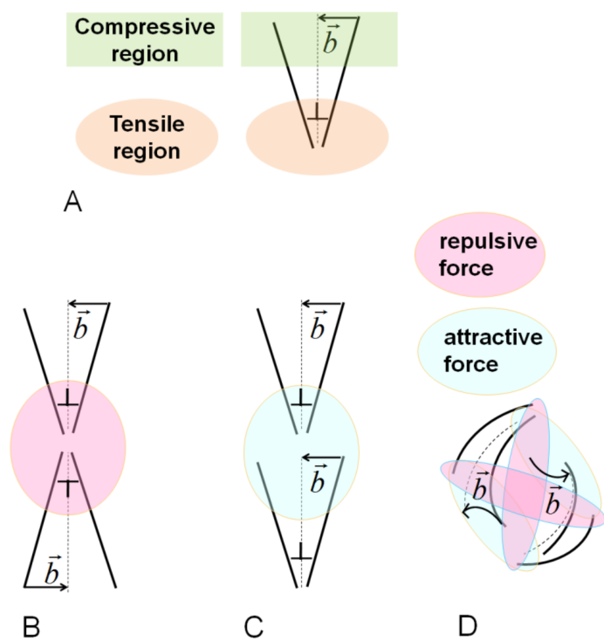

Fig. 3. Schematic diagrams of edge dislocations based on attractive-repulsive forces. (A) A single edge dislocation. (B) A pair of perfect fermions with pure repulsive force. (C) A pair of perfect bosons with pure attractive force. (D) A pair of fermions-bosons with both attractive and repulsive forces, which appears in the real single crystals‡.

Experiment

The experiment to verify all the above statements is already reported in reference (4). It enables it to track dislocations at different stages (depths) of growth. For the visualization of edge dislocations here, wet chemical etching was carried out by immersing GaN single crystals in hot acid (130 °C-H_3PO_4) for 180 seconds. Edge dislocations appeared in the form of 12-sided hexagonal pyramids.

Interactions of Dislocations during the Merging. A typical image (taken by a field-emission scanning electron microscope (FE-SEM)) of merging pyramids is presented in Fig. **4**. Far and separated dislocations (4) behaved nearly like independent particles (quasiparticles) which almost kept their full pyramid shapes, unchanged, implying weak interactions between these nearly repelled fermions in the fermionic system (single crystal). Close pyramids behave nearly like waves and interact with each other very strongly as nearly bosons (collective excitations) in a fermionic system (25).

For a better understanding, 4 typical pairs of merging pyramids are selected and marked by arrows. However these pairs are not isolated from other pairs, but still they are very useful for understanding the mechanism of merging (annihilation) of dislocations. Distance between pyramids decreases from the 1^{st} to 4^{th} pair, implying 4 typical successive steps of merging (annihilation) mechanism. In 4^{th} pair, two merging pyramids are approaching to create a single full pyramid (Fig. **4** inset) as the output, same as two merging stable solitons (self-localized wave packets exhibiting particle properties) leaving a single soliton as the output (26). Merging pyramids in each pair are rotated through the highest symmetry with each other to leave a single 12-sided hexagonal pyramid of the same characteristics as the output. These rotations while formation of loops might be attributed to conservation of angular momentum in perfect inelastic collisions (merging) of solitons (27). It is clear in the picture that even the merging pyramids are under influence of other pyramids in their vicinities, which means practically there is no perfect fermionic or bosonic states for these solitons. Observation of more than two pyramids in dislocation loops, shown by rectangles, is a strong proof to oppose the theory stating dislocations with opposite Burger vector attract each other which implies the force on the third dislocation must be pure repulsive.

I believe that, parallel directions of merging pairs are governed by edge dislocations (plasticity, linear momentum), specified by directions of arrows. Symmetric orientations of pyramids are also suggested to be governed by screw dislocations (rotation, angular momentum) (4). This study does not explore much about screw dislocations, however I will explain the role of them in more details in the near future. Symmetric orientations and parallel directions of pyramids take nonlinear discrete values attributed to nonlinearity in discreteness of atomic arrangement in the single crystal. Therefore, linear and angular motions of dislocations smoothly changes in nonlinear discrete values, but at the present work there is limitation of such visualizations.

Peierl-Nabarro Stress during the Merging (Annihilations). The etching rate in a single crystal is a function of many factors such as the etching agent and its concentration, temperature, and chemical atomic bonding of the crystal. H_3PO_4 solution is very sensitive to shear stresses of different chemical atomic bonding around each edge dislocation. For nearly a full pyramid (Fig. **4** Inset) a symmetric etching has resulted to a symmetric topology as a result of symmetric disturbance of atomic bonding around its dislocation line. Asymmetric topologies of merging pyramids are attributed to asymmetric rearrangements of chemical atomic bonding around their dislocation lines (Fig. **4**) (4, 28).

Slower etching rate among merging pyramids is explained by high stress fields within them, already known as Peierls stress calculated for a Peierls-Nabarro model of discrete lattices in crystals (29, 30). The Peierls stress is a barrier through merging of dislocations which reduces the speed of their interactions. This speed reduction of pyramids also confirms the conservation of linear momentum for perfectly inelastic collisions of solitons. The velocity of dislocations is bounded between zero and speed of sound in the material (here single crystal GaN), which also confirms the solitonic nature of dislocations (31, 32). Critical growth temperature (here ~1150° C for GaN) provides enough energy for solitons (edge dislocations) to tunnel the barrier height during the merging (annihilation) (33). Clear boundaries between these merging edge dislocations in Fig. **4** confirm their dual particle-wave characteristics.

Shiny tips of pyramids are attributed to central effective charge of dislocation cores (4). The shiny tips are almost symmetric in scattered shape and equal in scattered size for separated far pyramids. However, they become smaller and asymmetric for merging pyramids. Merging of dislocations with same tips to a single one with one original tip, leaves immediate extra charges of vanishing tips which might be recombined with external impurities, otherwise remain in the crystal and introduce some other kinds of point defects, according to the law of conservation of charge. Dust particles adsorbed at the tips of some pyramids (Fig. **4**) supports the electrical activity of dislocation cores. It seems that merging of dislocations in single crystals without presence of impurities or/and introducing electron/hole vacancies (self-interstitials) is impossible. Fig. **4** clearly shows that for each merging pyramid the size and shape of central effective charge decrease proportionally with its body mass and Burger vector (size). Even the shape of charge scattering distribution for each pyramid shows a direct relation with the shape of pyramid and therefore its mass scattering distribution. The mass-to-charge ratio (m/Q) is a physical quantity which appears in many scientific fields (34, 35), that is visualized here. Moreover, this experiment visualized a nonlinear and asymmetric distribution of m/Q. Additionally, as the effective electrical charges of merging cores repel each other, one can consider the dislocation bodies (including mass) also repel each other that I already described it as the stress between merging dislocations.

A comparison between 4 dislocation pairs in Fig. **4** and Fig. **3D**, reveals that a single merging pyramid in each pair, has 2 parts. One part behaves as a fermion and the other part as a boson. ***Each merging dislocation in a pair behaves as a fermion from its free side, while as a boson from its merging side***[§]. Even the shape of each pyramid is the same as a particle (quasiparticle) from its free side while as a wave (collective excitations) from its merging side. From another point of view, repulsive force between two merging pyramids repels fermions and makes them far from each other. Attractive force between bosons also let them attract and merge to each other. The Peierls stress could be related to the repulsion between fermionic sides of dislocations while bosonic sides are trying to merge together. Peierls stress lets the second law of thermodynamics to be conserved. For each dislocation in a pair the Burger vector, charge, and mass of its bosonic sides is less than its fermionic side. The reduction in fundamental properties of each bosonic side gets placed very smoothly, roughly (nonlinearly and asymmetrically) of the order of Gallium and Nitrogen atoms masses and charges and their chemical bonding distances. Finally, the bosonic sides totally vanish, while fermionic sides remain unchanged. It helps to understand the broken supersymmetry. The final single dislocation will find its bosonic either fermionic state while interacting with the surrounding dislocations.

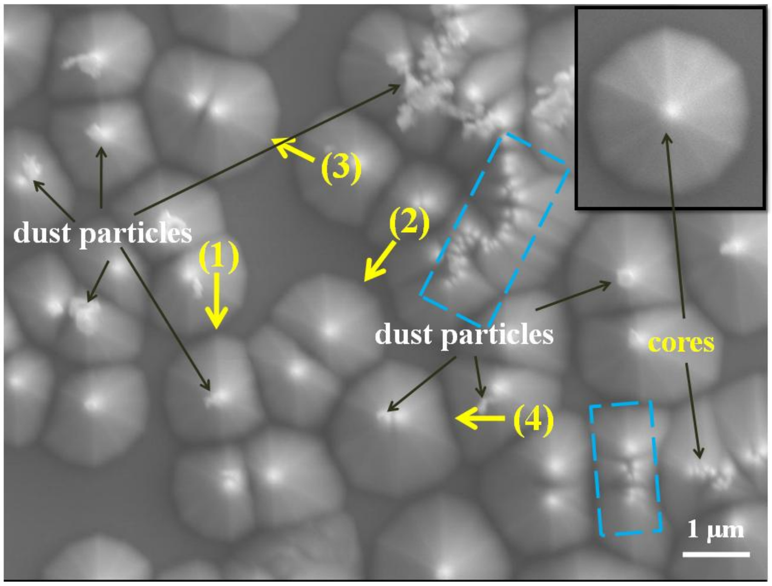

Fig. 4. FE-SEM image of merging pyramids. The 4 arrows show 4 typical merging pairs. Direction of each arrow indicates the direction of merging. Merging more than 2 typical pyramids are distinguished by rectangles. The inset shows an almost non-merging pyramid.

conclusion

In conclusion, edge dislocation pairs in single crystals are fermions if they are repelling each other and bosons if attracting each other. However in reality, edge dislocations cannot be perfect fermions or bosons. The definition of these prohibited fermions and bosons satisfies the prohibition of unbroken supersymmetry. Edge dislocations (solitons) merge together under certain conditions during the crystal growth. Merging edge dislocations are behaving partially bosons and partially fermions. Fermionic sides keep their fundamental properties unchanged during the merging of edge dislocations. Only bosonic sides of dislocations merge together. This merging which is along with loosing effective space (Burger vector), mass, and charge of bosonic sides, may help a better understanding of broken supersymmetry. This study clearly shows the conservations of charge, linear momentum, and angular momentum of dislocations. Furthermore, visualization of Peierls stress between merging solitons is another advantage of this study.

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Footnotes

[†]A single dislocation in a single crystal cannot be a boson or fermion, but the presence of second dislocation gives all of them the identity of being fermions or bosons. I will explain more about it in the near future.

[‡]Rotation operators in Fig. 3D are screw dislocations that I already pointed them out in reference (4). Later I will explain them in more details.

[§]It is a fallacy, as even roughly fermionic and bosonic sides are under influence of other dislocations. Therefore, even the half part of a dislocation cannot be a perfect boson either fermion.