Role of ‘number of moles’ in understanding the Planck’s quantum hypothesis

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Abstract: If one is willing to express the Wien’s displacement constant b in terms of the electric charge e and the Boltzmann constant k_B, then the Planck’s quantum and discrete nature of energy can be understood. By considering the universal gas constant R, 110 years of a great historical puzzle can be addressed and understood in terms of ‘ n moles’ concept where $n = 1, 2, 3, ..$

Key words: Hubble length, Hubble volume, Cosmic critical density, Hubble mass, CMB radiation, Wien’s displacement constant, Planck’s law, Planck’s constant, Boltzmann constant, Universal gas constant and Avogadro number.

1. Introduction

Once Planck had discovered the empirically fitting function, he constructed a physical derivation of this law [1]. His thinking revolved around entropy rather than being directly about temperature. Planck considered a cavity with perfectly reflective walls; the cavity contained finitely many hypothetical well separated and recognizable but identically constituted, of definite magnitude, resonant oscillatory bodies, several such oscillators at each of finitely many characteristic frequencies. The hypothetical oscillators were for Planck purely imaginary theoretical investigative probes, and he said of them that such oscillators do not need to “really exist somewhere in nature, provided their existence and their properties are consistent with the laws of thermodynamics and electrodynamics”. Planck did not attribute any definite physical significance to his hypothesis of resonant oscillators, but rather proposed it as a mathematical device that enabled him to derive a single expression for the black body spectrum that matched the empirical data at all wavelengths. He tentatively mentioned the possible connection of such oscillators with atoms. In this paper an attempt is made to understand the quantum nature of energy with “ n moles concept” where $n = 1, 2, 3, ..$

2. Planck’s quantum hypothesis

Partly following a heuristic method of calculation pioneered by Boltzmann for gas molecules, Planck [2] considered the possible ways of distributing electromagnetic energy over the different modes of his hypothetical charged material oscillators. This acceptance of the probabilistic approach, following Boltzmann, for Planck was a radical change from his former position, which till then had deliberately opposed such thinking proposed by Boltzmann. Heuristically, Boltzmann had distributed the energy in arbitrary merely mathematical quanta E, which he had proceeded to make tend to zero in magnitude, because the finite magnitude E had served only to allow definite counting for the sake of mathematical calculation of probabilities, and had no physical significance. Referring to a new universal constant of nature, h Planck supposed that, in the several oscillators of each of the finitely many characteristic frequencies, the total energy was distributed to each in an integer multiple of a definite physical unit of energy, E, not arbitrary as in Boltzmann’s method, but now for Planck, in a new departure, characteristic of the respective characteristic frequency. His new universal constant of nature, h, is now known as Planck’s constant. Planck explained further that the respective definite unit, E, of energy should be proportional to the respective characteristic oscillation frequency ν or inversely proportional to the respective characteristic oscillation wavelength λ of the hypothetical oscillator, and in 1901 he expressed this with the constant of proportionality h:

$$\mathrm{E} = nh\nu \cong \frac{nhc}{\lambda} \tag{1}$$

where $n = 1, 2, 3, ..$ This is known as Planck’s relation. Planck did not propose that light propagating in free space is quantized. The idea of quantization of the free electromagnetic field was developed later, and eventually incorporated into what we now know as quantum field theory. In 1906 Planck acknowledged that his imaginary resonators, having linear dynamics, did not provide a physical explanation for energy transduction between frequencies. Present-day physics explains the transduction between frequencies in the presence of atoms by their quantum excitability, following Einstein. Planck believed that in a cavity with perfectly reflecting walls and with no matter present, the electromagnetic field cannot exchange energy between frequency components. This is because of the linearity of Maxwell's equations. Present-day quantum field theory predicts that, in the absence of matter, the electromagnetic field obeys nonlinear equations and in that sense does self-interact. Such interaction in the absence of matter has not yet been directly measured because it would require very high intensities and very sensitive and low-noise detectors, which are still in the process of being constructed. Planck believed that a field with no interactions neither obeys nor violates the classical principle of equiparition of energy and instead remains exactly as it was when introduced, rather than evolving into a black body field. Thus, the linearity of his mechanical assumptions precluded Planck from having a mechanical explanation of the maximization of the entropy of the thermodynamic equilibrium thermal radiation field. This is why he had to resort to Boltzmann's probabilistic arguments. Later, in 1924, Satyendra Nath Bose [3] developed the theory of the statistical mechanics of photons, which allowed a theoretical derivation of Planck's law. Ultimately, Planck's law of black-body radiation contributed to Einstein's concept of quanta of light carrying linear momentum, which became the fundamental basis for the development of quantum mechanics.

3. CMB radiation and to fit the Wien's displacement constant

At any given cosmic time, the product of 'critical density' and 'Hubble volume' gives a characteristic cosmic mass and it can be called as the 'Hubble mass'. Interesting thing is that, Schwarzschild radius of the 'Hubble mass' again matches with the 'Hubble length'. Most of the cosmologists believe that this is merely a coincidence. At any given cosmic time, 'Hubble length' can be considered as the gravitational or electromagnetic interaction range. If one is willing to think in this direction, by increasing the number of applications of Hubble mass and Hubble volume in other areas of fundamental physics like quantum physics, nuclear physics, atomic physics and particle physics [7-14] - slowly and gradually - in a progressive way, concepts of 'Black hole Cosmology' can be strengthened and can also be confirmed.

Authors noticed two approximate methods for estimating the CMB radiation. Geometric mean of the two methods is fitting with the observational CMB wavelength accurately [4-6]. With reference to the Wien's displacement law and the current CMB radiation temperature and wavelength,

1. Considering the Coulomb scale (a basic scale similar to the Planck scale), let $\left(M_C\right)^{\pm} \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$ represents a characteristic unified charged mass unit at the initial conditions of the Hubble volume,
2. Let $M_t \cong \frac{c^3}{2GH_t}$ is the characteristic mass of the Hubble volume at any cosmic time and
3. Let $M_0 \cong \frac{c^3}{2GH_0}$ is the characteristic mass of the present Hubble volume.
4. Let $H_C \cong \frac{c^3}{2GM_C}$ is the characteristic Hubble constant of the initial Hubble volume.
5. Let $\left(\lambda_m\right)_C$ and T_C represent the characteristic strongly emitted CMB wavelength and temperature of the initial Hubble volume respectively.

Method-1: In the published papers the authors proposed that [7-14], at any given cosmic time,

$$\left[1+\ln\left(\frac{M_t}{M_C}\right)\right] \cong \left(\frac{\rho_c}{\rho_m}\right)_t \cong \left(\frac{\text{critical density}}{\text{matter density}}\right)_t \tag{2}$$

$$\left[1+\ln\left(\frac{M_t}{M_C}\right)\right]^2 \cong \left(\frac{\rho_c c^2}{aT_t^4}\right) \cong \left(\frac{\text{critical energy density}}{\text{thermal energy density}}\right)_t \tag{3}$$

With this idea, semi empirically wavelength of the most strongly emitted CMB radiation can be expressed as

$$\left(\lambda_m\right)_t \cong \left[1+\ln\left(\frac{M_t}{M_C}\right)\right]\frac{G\sqrt{M_t M_C}}{c^2} \tag{4}$$

Note that this expression is free from the 'radiation constants'. If H_0 is close to 70 km/sec/Mpc, obtained (most strongly emitted) wavelength of the CMB radiation is 1.37 mm.

Method-2: This method is based on the pair annihilation of $\left(M_C\right)^{\pm}$. Thermal energy can be expressed as

$$k_B T_t \cong \sqrt{\frac{M_C}{M_t}} \cdot \left[\left(M_C\right)^{+} + \left(M_C\right)^{-}\right]c^2 \cong \sqrt{\frac{M_C}{M_t}} \cdot 2M_C c^2 \tag{5}$$

Based on the Wien's displacement law,

$$\left(\lambda_m\right)_t \cong \frac{b}{T_t} \cong \sqrt{\frac{M_t}{M_C}} \cdot \frac{bk_B}{2M_C c^2} \tag{6}$$

If H_0 is close to 70 km/sec/Mpc, obtained (most strongly emitted) wavelength of the CMB radiation is 0.822 mm.

Method-3: Considering the geometric mean wavelength of wavelengths obtained from methods-1 and 2, wavelength of the most strongly emitted CMB radiation can be expressed as

$$\left(\lambda_m^2\right)_t \cong \left[1+\ln\left(\frac{M_t}{M_C}\right)\right] \cdot \left(\frac{M_t}{M_C}\right) \cdot \left(\frac{bk_B G}{2c^4}\right) \tag{7}$$

$$\left(\lambda_m\right)_t \cong \sqrt{\left[1+\ln\left(\frac{M_t}{M_C}\right)\right] \cdot \left(\frac{M_t}{M_C}\right) \cdot \left(\frac{bk_B G}{2c^4}\right)} \tag{8}$$

At present, the measured CMBR wavelength can be expressed as

$$\left(\lambda_m\right)_0 \cong \sqrt{\left[1+\ln\left(\frac{M_0}{M_C}\right)\right] \cdot \left(\frac{M_0}{M_C}\right) \cdot \left(\frac{bk_B G}{2c^4}\right)} \cong 1.064 \text{ mm} \tag{9}$$

where H_0 is close to 70 km/sec/Mpc and $M_0 \cong c^3/2GH_0$. This is a very accurate fit and needs a special analysis. The most important point is that, as the Hubble volume is expanding, its expansion rate can be checked with $\frac{d}{dt}\left(\lambda_m\right)_t$. Present observations indicate that, CMB radiation is smooth and uniform. Thus it can be suggested that, at present there is no detectable cosmic expansion or cosmic acceleration. Drop in 'cosmic temperature' can be considered as a measure of cosmic expansion and 'rate of decrease in cosmic temperature' can be considered as a measure of cosmic 'rate of expansion'. But if rate of decrease in temperature is very small and is beyond the scope of current experimental verification, then the two possible states are: a) cosmic temperature is decreasing at a very slow rate and universe is expanding at a very slow rate and b) there is no 'observable' thermal expansion and there is no 'observable' cosmic expansion. Thus in a semi empirical approach, it can be suggested that, the wavelength of the CMB radiation follows the following three conditions.

$$\left(\lambda_m\right)_t \propto \sqrt{1+\ln\left(\frac{M_t}{M_C}\right)} \tag{10}$$

$$\left(\lambda_m\right)_t \propto \sqrt{\frac{M_t}{M_C}} \tag{11}$$

$$\left(\lambda_m\right)_t \propto \sqrt{\frac{bk_B G}{2c^4}} \tag{12}$$

$\sqrt{\frac{bk_B G}{2c^4}} \cong 1.2855 \times 10^{-35}$ m seems to be a constant and can be considered as the characteristic classical thermal wave length. With reference to the assumed initial conditions, i.e if $M_t \to M_C$,

$$\left(\lambda_m\right)_C \cong \sqrt{\left(\frac{bk_B G}{2c^4}\right)} \cong 1.2855 \times 10^{-35} \text{ m} \tag{13}$$

At beginning, if $\left(\lambda_m\right)_C T_C \cong b$ and $aT_C^4 \cong \frac{3H_C^2 c^2}{8\pi G}$, it is also noticed that,

$$\left(\lambda_m\right)_C \cong b\left(\frac{3H_C^2 c^2}{8\pi Ga}\right)^{-\frac{1}{4}} \cong 1.295 \times 10^{-35} \text{ m} \tag{14}$$

From this strange coincidence it can be suggested that,

$$\sqrt{\frac{bk_B G}{2c^4}} \cong b\left(\frac{8\pi Ga}{3H_C^2 c^2}\right)^{\frac{1}{4}} \tag{15}$$

where $a \cong \frac{4}{3} \cdot \frac{k_B}{b^3}$, $H_C \cong \frac{c^3}{2GM_C}$ and $M_C \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$. It needs a very critical analysis. From this relation, b can be expressed as

$$b \cong \frac{512\pi}{9} \frac{e^2}{4\pi\varepsilon_0 k_B} \cong 2.97385 \times 10^{-3} \ {}^0\text{K.m} \tag{16}$$

Here error is 3% and can be accounted for the emission efficiency of a black body $(\approx 97\%)$.

4. To understand the Planck's quantum nature of energy

If one is willing to express the Wien's displacement constant b in terms of electric charge e and thermal energy constant k_B, then the Planck's quantum nature of energy can also be understood. For this purpose one can proceed in the following way. At any given cosmic time, if a is the radiation energy constant and b is the Wien's displacement constant, a can be expressed as

$$a \cong \frac{8\pi^5}{15} \frac{k_B^4}{h^3 c^3} \cong \frac{8\pi^5}{15}\left(\frac{k_B^3 b^3}{h^3 c^3}\right)\frac{k_B}{b^3} \cong \frac{8\pi^5}{15}\left(\frac{k_B b}{hc}\right)^3 \frac{k_B}{b^3} \tag{17}$$

It is noticed that, $\frac{8\pi^5}{15}\left(\frac{k_B^3 b^3}{h^3 c^3}\right) \cong 1.3333991714 \cong \frac{4}{3}.$

Like photon's frequency-wavelength relation, $\nu\lambda = c$, in a classical approach, independent of the Planck's constant, at any given cosmic time, radiation constant a can be expressed as

$$a \cong \frac{4}{3}\cdot\frac{k_B}{b^3} \tag{18}$$

This is a very sensitive point. Please note that Einstein used Wien's displacement law and Bohr's correspondence principle for deriving the Planck's law [2,3]. From the proposed idea if $a \cong \frac{4}{3}\frac{k_B}{b^3}$ and from Planck's quantum theory if $a \cong \frac{8\pi^5}{15}\cdot\frac{k_B^4}{h^3 c^3}$, hc can be expressed as

$$hc \cong \left(\frac{2\pi^5}{5}\right) b k_B \cong 4.9652 b k_B \tag{19}$$

Please note that from Planck's law of radiation [1], the number 4.9652 can be estimated with the expression

$$x = \ln 5 - \ln(5 - x) \cong 4.96511423. \tag{20}$$

From relation (16) and considering the universal gas constant [15,16], b can be expressed as

$$b \cong \frac{512\pi}{9}\cdot\frac{e^2}{4\pi\varepsilon_0 k_B} \cong \frac{512\pi}{9}\cdot\frac{e^2}{4\pi\varepsilon_0 (R/N_A)} \tag{21}$$

where R is the universal gas constant and N_A is the Avogadro number [17-26] and can be considered as an index for one mole interacting oscillators. For N_A oscillators i.e for one mole number of oscillators

$$b \cong \frac{512\pi}{9}\cdot\frac{(N_A)e^2}{4\pi\varepsilon_0 R} \tag{22}$$

It can be suggested that,

$$b \propto N_A \tag{23}$$

$$b \propto \frac{e^2}{4\pi\varepsilon_0 R} \tag{24}$$

For $n = 1,2,3,..$ mole interacting oscillators

$$b \cong \frac{512\pi}{9}\cdot\frac{(n.N_A)e^2}{4\pi\varepsilon_0 R} \tag{25}$$

As the ratio $(N_A k_B / R) \cong 1$, now it can suggested that, for one mole interacting oscillators

$$hc \cong \left(\frac{2\pi^5}{5}\right)^{\frac{1}{3}} \cdot b k_B \cong \left(\frac{2\pi^5}{5}\right)^{\frac{1}{3}} \cdot \frac{512\pi}{9}\cdot\frac{e^2}{4\pi\varepsilon_0} \tag{26}$$

$$\cong 887.39 \frac{e^2}{4\pi\varepsilon_0}$$

and as the ratio $(nN_A k_B / R) \cong n$, for $n = 1,2,3,..$ mole interacting oscillators

$$n(hc) \cong n.\left(\frac{2\pi^5}{5}\right)^{\frac{1}{3}} \cdot \frac{512\pi}{9}\cdot\frac{e^2}{4\pi\varepsilon_0} \tag{27}$$

$$\cong \left(\frac{2\pi^5}{5}\right)^{\frac{1}{3}} \cdot \frac{512\pi}{9}\cdot\frac{ne^2}{4\pi\varepsilon_0}$$

Now the famous Planck's law for $n = 1,2,3,..$ mole interacting oscillators can be expressed as

$$n\left(\frac{hc}{\lambda}\right) \cong \left(\frac{2\pi^5}{5}\right)^{\frac{1}{3}} \cdot \frac{512\pi}{9}\cdot\frac{ne^2}{4\pi\varepsilon_0 \lambda} \cong 887.39 \frac{ne^2}{4\pi\varepsilon_0 \lambda} \tag{28}$$

In this way the concept of 'discrete energy' or 'quantum of energy' can be expressed. Not only that, 110 years of a historical puzzle can be expressed in terms of 'mole concept'.

5. Conclusions

5.1 About the Hubble volume and the Hubble mass

Please note that even though it was having strong footing, Mach's principle was not implemented successfully in modern physics and modern cosmology. One of the main motivations behind formulating the general theory of relativity was to provide a mathematical description to the Mach's principle. However, soon after its formulation, it was realized that the theory does not follow Mach's principle. As the theoretical predictions were matching with the observations, Einstein believed that the theory was correct and did not make any farther attempt to reformulate the theory to explain Mach's principle. Later on, several attempts were made by different researchers to formulate the theory of gravity based on Mach's principle. However most of these theories remain unsuccessful to explain different physical phenomena. Whether universe is a black hole or something else, one can find many interesting applications of Hubble volume and its corresponding Hubble mass in the current and past aspects of the universe. Hence magnitudes of Hubble length, Hubble volume and Hubble mass can be considered as characteristic back ground conditions for the observed atomic and cosmological physical phenomena.

5.2 Planck's quantum and discrete energy hypothesis and the Mole concept

In this paper qualitatively and quantitatively authors made a simple attempt to understand the Planck's quantum nature of energy. If fact - the integral nature of quantum of energy seems to be different from the integral nature of electric charge. 'Mole concept' and the role of Avogadro number as a fundamental physical number must be reviewed at fundamental level in all respects [27, 28].

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