The relation of colour charge to electric charge

Dirac has shown how the Klein-Gordon equation can be factored into two linear parts using 4x4 Dirac gamma matrices.

[Dirac, P.A.M., The Principles of Quantum Mechanics, 4th edition (Oxford University Press) ISBN 0-19-852011-5]

$$(\partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 + m^2)\, I = (-i[s\gamma^0\partial_t + r\gamma^1\partial_x + g\gamma^2\partial_y + b\gamma^3\partial_z] - mI)\,(i[s\gamma^0\partial_t + r\gamma^1\partial_x + g\gamma^2\partial_y + b\gamma^3\partial_z] - mI)$$

where r,g,b and s equal +1 or -1.

For leptons r,g,b all equal -1 and for quarks two of r,g,b are equal to +1 and the third equals -1. The signs are all negated for anti-particles as in the equation above.

When s = +1, count the number of plus signs (say) for r,g,b which is 0 for leptons and 2 for quarks.

When s = -1, count the number of minus signs (say) for r,g,b which is 3 for leptons and 1 for quarks.

For material particles r,g,b all equal -1 which is always true for leptons and true for three distinct quarks with r,g,b equal to -1 separately or a quark and an appropriate anti-quark.

A charged particle moving in an electromagnetic field will have ∂_t , ∂_x , ∂_y , ∂_z modified to ${}^*\partial_t$, ${}^*\partial_x$, ${}^*\partial_y$, ${}^*\partial_z$ by the scalar and vector potentials of the field, where ${}^*\partial_t$, ${}^*\partial_x$, ${}^*\partial_y$, ${}^*\partial_z$ do not commute with each other. Thus:

$$(-i[s\gamma^0\,{}^*\partial_t + r\gamma^1\,{}^*\partial_x + g\gamma^2\,{}^*\partial_y + b\gamma^3\,{}^*\partial_z] - mI)\,(i[s\gamma^0\,{}^*\partial_t + r\gamma^1\,{}^*\partial_x + g\gamma^2\,{}^*\partial_y + b\gamma^3\,{}^*\partial_z] - mI)$$

$$= ({}^*\partial_t^2 - {}^*\partial_x^2 - {}^*\partial_y^2 - {}^*\partial_z^2 + m^2)\, I$$

$$+ s\gamma^0[r\gamma^1({}^*\partial_t{}^*\partial_x - {}^*\partial_x{}^*\partial_t) + g\gamma^2({}^*\partial_t{}^*\partial_y - {}^*\partial_y{}^*\partial_t) + b\gamma^3({}^*\partial_t{}^*\partial_z - {}^*\partial_z{}^*\partial_t)]$$

$$+ gb\gamma^2\gamma^3({}^*\partial_y{}^*\partial_z - {}^*\partial_z{}^*\partial_y) + br\gamma^3\gamma^1({}^*\partial_z{}^*\partial_x - {}^*\partial_x{}^*\partial_z) + rg\gamma^1\gamma^2({}^*\partial_x{}^*\partial_y - {}^*\partial_y{}^*\partial_x)$$

$$= ({}^*\partial_t^2 - {}^*\partial_x^2 - {}^*\partial_y^2 - {}^*\partial_z^2 + m^2)\, I$$

$$+ s\gamma^0[r\gamma^1({}^*\partial_t{}^*\partial_x - {}^*\partial_x{}^*\partial_t) + g\gamma^2({}^*\partial_t{}^*\partial_y - {}^*\partial_y{}^*\partial_t) + b\gamma^3({}^*\partial_t{}^*\partial_z - {}^*\partial_z{}^*\partial_t)]$$

$$+ \gamma^1\gamma^2\gamma^3[r\gamma^1({}^*\partial_y{}^*\partial_z - {}^*\partial_z{}^*\partial_y) + g\gamma^2({}^*\partial_z{}^*\partial_x - {}^*\partial_x{}^*\partial_z) + b\gamma^3({}^*\partial_x{}^*\partial_y - {}^*\partial_y{}^*\partial_x)]$$