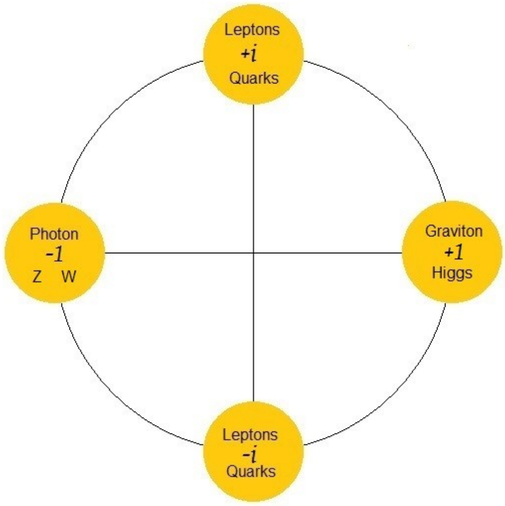
The relation of colour charge to electric charge



Dirac has shown how Einstein's expression for the relation of energy to momentum in Special Relativity can be factored into two linear parts using 4x4 Dirac matrices labelled J,K,L,M. *[Dirac, P.A.M., The Principles of Quantum Mechanics, 4th edition (Oxford University Press) ISBN 0-19-852011-5]*

$$(E/c)^2 - P^2 - Q^2 - R^2 - (mc)^2 I = (E/c + rJP + gKQ + bLR + mcsM)(E/c - rJP - gKQ - bLR - mcsM)$$

This is true for all 3! = 6 permutations of J,K,L where r,g,b and s equal +1 or -1.
The set {I, J, K, L} forms the basis of a 4-dimensional real vector space.

For leptons r,g,b all equal -1 and for quarks two of r,g,b are equal to +1 and the third equals -1. The signs are all negated for anti-particles as in the equation above.

The 3 cyclic permutations JKL = LJK = KLJ count the number of plus signs (say) for r,g,b which is 0 for leptons and 2 for quarks.

The 3 cyclic permutations LKJ = KJL = JLK count the number of minus signs (say) for r,g,b which is 3 for leptons and 1 for quarks.

For material particles r,g,b all equal -1 which is always true for leptons and true for three distinct quarks with r,g,b equal to -1 separately or a quark and an appropriate anti-quark.

Similarly, for material gauge bosons, the 3 cyclic permutations of JKL squared (which all equal -I) count the number of plus signs (say) for r,g,b which is 0 for the Z boson and the 3 cyclic permutations of LKJ squared (which all equal -I) count the number of minus signs (say) for r,g,b which is 3 for the W boson.

The photon, having zero rest mass, carries no electric charge.

The 6 permutations of JKL raised to power zero (which all equal I) present (say) a material particle which carries no electric charge.

The 6 permutations of JKL raised to power four (which all equal I) present (say) a particle with zero rest mass which carries no electric charge.

A charged particle moving in an electromagnetic field will have E, P, Q, R modified to **E**, **P**, **Q, R** by the scalar and vector potentials of the field where **E**, **P**, **Q, R** do not commute with each other. Let JKL = N, then:

$$\begin{aligned}
&(\boldsymbol{E}/c + rJ\boldsymbol{P} + gK\boldsymbol{Q} + bL\boldsymbol{R} + mcsM)(\boldsymbol{E}/c - rJ\boldsymbol{P} - gK\boldsymbol{Q} - bL\boldsymbol{R} - mcsM) \\
&= (\boldsymbol{E}/c)^2 - \boldsymbol{P}^2 - \boldsymbol{Q}^2 - \boldsymbol{R}^2 - (mc)^2 I \\
&\quad - [rJ(\boldsymbol{E}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{E}) + gK(\boldsymbol{E}\boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{E}) + bL(\boldsymbol{E}\boldsymbol{R} - \boldsymbol{R}\boldsymbol{E})]/c \\
&\quad - gbKL(\boldsymbol{Q}\boldsymbol{R} - \boldsymbol{R}\boldsymbol{Q}) - brLJ(\boldsymbol{R}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{R}) - rgJK(\boldsymbol{P}\boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{P}) \\
&= (\boldsymbol{E}/c)^2 - \boldsymbol{P}^2 - \boldsymbol{Q}^2 - \boldsymbol{R}^2 - (mc)^2 I \\
&\quad - [rJ(\boldsymbol{E}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{E}) + gK(\boldsymbol{E}\boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{E}) + bL(\boldsymbol{E}\boldsymbol{R} - \boldsymbol{R}\boldsymbol{E})]/c \\
&\quad + N[rJ(\boldsymbol{Q}\boldsymbol{R} - \boldsymbol{R}\boldsymbol{Q}) + gK(\boldsymbol{R}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{R}) + bL(\boldsymbol{P}\boldsymbol{Q} - \boldsymbol{Q}\boldsymbol{P})]
\end{aligned}$$