Every set has its S-divisor

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Abstract:

When I study some basic geometry,sometimes double lines are at the same place,how to show it in the picture?How about a minus line?When I study a little algebraic geometry,specially the algebraic curve,I find that is just the divisor.The set is a better stage for the divisor and I call it S-divisor.

1.What is the S-divisor?

Definition 1: Suppose X is a nonempty set,we can define the S-divisor set of X as S-div(X)={ $\Sigma k_i x_i$;x_i belong to X and only finite k_i is not zero.},every element in the S-div(X) is called a S-divisor. When X is empty,we can define that S-div(X) is also empty. If there is no confusion，the S-divisor of X is also called divisor for short,denoted div(X).

Every element x in X can seem as a divisor with the coefficient 1,and the divisor –x is called the minus element of x,we also have the annihilate equation x-x= θ .For any division A belong to X,we also define its antidivisor –A by changing all the coefficients of A to their opposite.Here θ is called zero divisor,it means a divisor whose coefficients are all zero.

Remark: We can think that θ is the virtual element of empty set.The empty set is not really empty!

The coefficients of divisor means how much does a element belong to the set.For example,let a and b are two elements in the set X,then 2a-3b is a divisor of X,it means there are double a in the set X and lack triple b in the set X..

In the most situation,the coefficients are integral,but sometimes are real number and we also can extend to some algebraic object.For example,in the fuzzy maths,we often restrict the coefficient at the interval [0,1], that is just the membership functions.It also has some relations with the algebra.If the set is a group G,the group algebra ZG is a divisor with the integral coefficient.

Warn: When X has a addition,do not confuse it with the form addition in the divisor.For example,when X is a Abelian group,0+x=x is just for the group addition,but is not right for the divisor addition.

Usually,we can identify all the form addition in the set,so we have such a proposition.

Proposition 1: Let X is a set,we have div div(X)=div(X)

Theorem 1: Let X is a set,then div(X) makes a free Abelian group with its form addition and its zero element is θ .

2.Some set operations of the S-divisor

We can match the divisor with the set operations.

Proposition 2: Let X and Y are two sets,we have

1) $\mathrm{div}(X\cap Y)=\mathrm{div}(X)\cap \mathrm{div}(Y)$
2) $\mathrm{div}\{X\cup Y\}=\mathrm{div}(X)+\mathrm{div}(Y)$

where the plus symbol in 2) is divisor addition.

We can extend 1) into any sets and 2) into any finite sets,when the sets in 2) are infinite,we should change the right side into form direct sum.

We can induce the morphism of divisors from the set structure naturally,that is just linear extension.

Definition 2:Let X,Y are two sets,the map F:div $(X)\to$div(Y) is a morphism,if for any A,B in the div(X),F(A+B)=F(A)+F(B),When such F is a bijection,we call it is a isomorphism and also call the divisor div(A) is isomorphism with div(B).

Proposition 3: Let X,Y are two sets,any map f:X $\to$ Y induces a morphism F:div(X) $\to$ div(Y),any morphism F:div(X)$\to$div(Y) is also induced by a set map F:X$\to$Y.When f is injection(or surjection,or bijection),so does F.

3.The S-divisor in the algebraic geometry

Definition 3: For any divisor $A=\Sigma k_i x_i$ in div(X),we can define the degree of A as $\deg(A)=\Sigma k_i$.If deg(A)=0,A is called principal divisor.If A and B are two divisors,A-B is a principal divisor,we call that A and B are equivalence,denote by $A\approx B$.All the principal divisors make a subgroup of div(X.) and.its quotient group is called divisor class group.denoted Cl(X).

Remark: In the Proposition 1 and 2,we can change all the div into Cl.

Definition 4: For any divisor $A=\Sigma k_i x_i$,if all the $k_i\geqslant 0$，A is called a effective divisor,denoted $A\geqslant 0$. We can define the associative space of A as $L(A)=\{B\in \mathrm{div}(A);B+A\geqslant 0，\deg(B)=0\}$.

Proposition 4: If $\deg(A)<0$，then L(A)=0;If deg(A)=0,then L(A)={-A}.

Proposition 5: If $A\approx B$，we can get a divisor C,such that L(A)=L(B)+C.

Remark: These Propositions come from the algebraic geometry,but we can get it at the set level.Unfortunately,the set is not rich enough to contain something like the space of meromorphic differential forms.

4.S-divisor and Descartes product

The structure of the S-divisor is a form addition of set,how about the form multiplication?That is just the Cartesian product,maybe we can say that where is the. Descartes product,there is the S-divisor.

Now let us march the addition and the product of a set,we have the distributive law of them:for any a,b,c in the set X,（a+b）×c=a×c+b×c,here a×b is in the Cartesian product X×X.We also have a natural supplement:θ×a=θ,that means empty set by any set is also empty.

Theorem 2： Let X is a set,.then div(X) makes a form ring with the divisor addition and Cartesian product.

We can define the Cartesian product on the set(not just its elements),how about the divisor?That is the divisor on the category.For the group category,we can treat the divisor as a form direct sum.The divisor of set category is the same,but we should define the form direct sum first which is the essential part of the S-divisor.

Reference:

All the concepts in this paper are very basic.For a little algebraic geometry, the following book is enough.

Phillip A. Griffiths, Introduction to Algebraic Curves, Kuniko Weltin, trans., American Mathematical Society, Translation of Mathematical Monographs volume 70, 1985 revision