A conjecture about 2-Poulet numbers and a question about primes

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Abstract. To find generic formulas for Poulet numbers (beside, of course, the formula that defines them) was for long time one of my targets; I maybe found such a formula for Poulet numbers with two prime factors, involving the multiples of the number 30, that also is rising an interesting question about primes.

Conjecture:
Any Poulet number with two prime factors can be written as \( P = (q - 30*n)*(r + 30*n) \), where \( q \) and \( r \) are primes or are equal to 1 and \( n \) is positive integer, \( n \geq 1 \).

Note: For a list of 2-Poulet numbers see the sequence A214305 that I submitted to OEIS.

Verifying the conjecture for the first few 2-Poulet numbers:

: \( P = 341 = 11*31 = (41 - 30*1)*(1 + 30*1) = (31 - 30*1)*(311 + 30*1) \);

: \( P = 1387 = 19*73 = (61 - 30*2)*(1327 + 30*2) = (79 - 30*2)*(13 + 30*2) \);

: \( P = 2047 = 23*89 = (31 - 30*1)*(2017 + 30*1) = (53 - 30*2)*(1987 + 30*2) = (83 - 30*2)*(29 + 30*2) \);

: \( P = 2701 = 37*73 = (31 - 30*1)*(2671 + 30*1) = (67 - 30*1)*(43 + 30*1) = (103 - 30*1)*(7 + 30*1) = (97 - 30*2)*(13 + 30*2) = (151 - 30*5)*(2551 + 30*5) \);


Note: It is remarkable in how many ways a 2-Poulet number can be written this way.
**Note:** The conjecture might probably be extended for all Poulet numbers not divisible by 3 or 5, not only with two prime factors.

Verifying the extended conjecture for first few Poulet numbers with more than two prime factors not divisible by 3 or 5:

: \( P = 1729 = 7 \times 13 \times 19 = (31 - 30*1)*(1699 + 30*1) = (43 - 30*1)*(103 + 30*1) ; \\

: \( P = 2821 = 7 \times 13 \times 31 = (31 - 30*1)*(2791 + 30*1) = (37 - 30*1)*(373 + 30*1) = (61 - 30*1)*(61 + 30*1) ; \\

: \( P = 6601 = 7 \times 23 \times 41 = (31 - 30*1)*(6571 + 30*1) = (53 - 30*1)*(257 + 30*1) = (71 - 30*1)*(131 + 30*1) = (191 - 30*1)*(11 + 30*1). \\

**Note:** This conjecture is rising the following question: which pairs of primes \((x,y)\), at least one of them bigger than 30, have the property that can be written as \((p - 30*n, q + 30*n)\), where \(p\) and \(q\) are primes or are equal to 1 and \(n\) is positive integer, \(n \geq 1\).