A conjecture about primes based on heuristic arguments involving Carmichael numbers

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. The number 30 is important to me because I always believed in the utility of classification of primes in primes of the form 30k + 1, 30k + 7, 30k + 11, 30k + 13, 30k + 17, 30k + 19, 30k + 23 and 30k + 29 (which may be interpreted as well as primes of the form 30h – 29, 30h – 23, 30h – 19, 30h – 17, 30h – 13, 30h – 11, 30h – 7 and 30h – 1). The following conjecture involves the multiples of the number 30 and is based on the study of Carmichael numbers.

Conjecture: For any three distinct primes p, q, r there exist a positive integer n so that the numbers x = 30*n – p, y = 30*n – q and z = 30*n – r are all three primes.

Comments

I already showed in the article "A list of 13 sequences of Carmichael numbers based on the multiples of the number 30", posted on VIXRA, the importance of the multiples of 30 in the study of Carmichael numbers.

I shall list randomly a number of ways in which a Carmichael number with three prime factors can be written in function of the multiples of the number 30 (we note with C a Carmichael number):

C = (30*n – p)*(60*n – q)*(90*n – r), where n is a positive integer and p, q, r are primes. Examples:

C = 8911 = 7*19*67 = (30 – 23)*(60 – 41)*(90 – 23);
C = 15841 = 7*31*73 = (30 – 23)*(60 – 29)*(90 – 17);
C = 29341 = 13*37*61 = (30 – 17)*(60 – 23)*(90 – 29).

C = (30*n – p)*(90*n – q)*(120*n – r), where n is a positive integer and p, q, r are primes. Example:

C = 52633 = 7*73*103 = (30 – 23)*(90 – 17)*(120 – 17).

But the most appealing form is the following one: C = (30*n – p)*(30*n – q)*(30*n – r), where n is a positive integer and p, q, r are primes.

Examples:

C = 1729 = 7*13*19 = (30*1 - 23)(30*1 - 17)(30*1 - 11);
C = 1729 = 7*13*19 = (30*9 - 263)(30*9 - 257)(30*9 - 251);
C = 2821 = 7*13*31 = (30*9 - 263)(30*9 - 257)(30*9 - 239);
C = 6601 = 7*23*41 = (30*6 - 173)(30*6 - 157)(30*6 - 139);
C = 8911 = 7*19*67 = (30*3 - 83)(30*3 - 71)(30*3 - 23);
C = 15841 = 7*31*73 = (30*3 - 83)(30*3 - 59)(30*3 - 17).

In fact, ny initial intention was to conjecture that any Carmichael number can be written in this form, in other words that for any three prime factors p, q, r of a 3-Carmichael number there exist a positive integer n so that the numbers x = 30*n – p, y = 30*n – q and z = 30*n – r are all three primes.

Note: The reason for which I chose 3 primes for the conjecture instead of 2 or 4 is that 3 is the minimum number of prime factors of a Carmichael number but also because I would relate this conjecture with the study of Fermat’s last theorem.

Note: The conjecture implies of course that for any pair of twin primes (p,q) there exist a pair of primes (30*n - p,30*n - q) so that there are infinitely many pairs of primes.