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The energy-momentum pseudo-tensor of the gravitational field is a mistake

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It is shown that the use of the standard energy-momentum pseudo-tensor of the gravitational field increases the calculated mass-energy of a closed system of gravitating bodies, whereas the energy of the gravitational field, if it exists, is negative. This discredits the pseudo-tensor because, according to the pseudo-tensor, the energy of the gravitational field is positive.
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1. If particles or bodies are attracted to each other by the forces of a field and become combined, the mass of the compound is less than the sum of the masses of its constituent bodies. This effect is called the *mass defect*. The simplest example is given by electrostatics. A proton and an electron, which are far away from each other, are attracted to each other by their own electric field, which fills the space between them. In this case, the mass-energy of the electric field is part of the masses of a proton and an electron. When a proton and an electron form a neutral hydrogen atom, these particles acquire a kinetic mass-energy due to the energy of their electric fields, which are gradually eliminated as a result of interference. Thus, we can assume that in the process of binding, particles retain their total mass-energy. I.e. some of the field energy is converted into kinetic energy of the particles. After this process, the electric field outside the atom is equal to zero, and the associated field energy, converted temporarily into kinetic energy, somehow dissipates into space. Therefore, the proton and the electron are deprived of some of their electric fields. Because of this, the mass-energy of the hydrogen atom is less than the sum of the mass-energies of a free proton and an electron by 13.6 eV.

2. A similar process under gravitational attraction proceeds somewhat differently. Consider, instead of the remote proton and electron, a spherical cloud of dust surrounded by its gravitational field. Some difference from electrostatics is immediately noticeable: the electric field, ensuring attraction of the proton to the electron, is between these particles, whereas the gravitational field, providing the attraction of dust particles to each other, is mainly outside the cloud. In the center of the cloud the gravitational field is absent at all.

Due to the mutual attraction of dust particles, the dust cloud is compressed, and dust particles acquire kinetic energy. Therefore, their mass-energy is increased, as in the case of electrostatics. However, in contrast to the electrostatics, the gravitational field of the cloud also increases! During the compression, there arises a pressure in the cloud, which eventually stops the compression. Thus, in contrast to the electrostatics, the kinetic energy of the particles of dust can turn into heat and remain in the compressed cloud, increasing its mass compared to initial value. The gravitational field of the cloud at the same time does not eliminate (like the electric field) and, on the contrary, increases, filling the area of the space vacated by the compressed cloud. Therefore, the supporters of the total energy conservation have to attribute the negative energy to the gravitational field so that the sum of the energies of the dust cloud and the gravitational field of the cloud remained unchanged during the compression. This fixed sum is mathematically equal to the Schwarzschild parameter m, which is preserved in accordance with Birkhoff's theorem during the cloud compression.

But then a problem arises. The problem is about the calculation of the negative gravitational energy, which will make the difference between m and the mass-energy of the cloud matter.

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3. To calculate the mass-energy of the matter of an unmoving dust cloud, there exists a natural formula

$$P=\int \rho dV_0=\int \rho\sqrt{-g_{rr}g_{\theta\theta}g_{\phi\phi}}drd\theta d\phi=\int \rho\sqrt{-g_{rr}}4\pi r^2 dr\ . \qquad (1)$$

Here the mass is denoted by the letter P, because the mass is the sum of moduli of 4-momenta P_i of dust particles, and $dV_0=\sqrt{-g_{rr}g_{\theta\theta}g_{\phi\phi}}drd\theta d\phi$ is an element of the 'spatial eigenvolume' in spherical coordinates. Formula (1) can be conveniently re-written in four-dimensional space-time notations. For the modulus of the 4-momentum element we have:

$$dP=\frac{dP_t}{\sqrt{g_{tt}}}=\frac{T^t_{\wedge t}}{\sqrt{g_{tt}}}dV^\wedge_t=\frac{T^t_t\sqrt{-g}_\wedge}{\sqrt{g_{tt}}}dV^\wedge_t=T^t_t(\sqrt{-g_{rr}g_{\theta\theta}g_{\phi\phi}})_\wedge dV^\wedge_t=T^t_t dV_0=\rho dV_0\,. \qquad (2)$$

Let us explain this. The 'energy-momentum tensor' is actually a tensor *density* $T^i_{\wedge k}=T^i_k\sqrt{g}_\wedge$. In writing equations for the densities we do not use Gothic letters, as is usually done. Instead, we denote the density by the sign 'wedge' $\wedge$. Such a designation was used by Kunin [1] in the Russian translation of monograph [2]. However, in contrast to [1], we use the sign $\wedge$ in the subscripts or superscripts for the densities of weight +1 or –1, respectively. For example, an element of the volume or an elementary outward-oriented area, which are the densities of weight –1, are denoted in space-time by $dV^\wedge_k$ or $da^\wedge_{ik}$, respectively, and the root of the determinant of the metric tensor is denoted by $\sqrt{g}_\wedge$.

4. To calculate the total 4-momentum of gravitational field plus matter a *nontensor* density was proposed. It contains derivatives of the first and second orders of the metric tensor of the coordinate system used. We denote it by $H^i_{\wedge k}$. In [3 (89.3)], it is compactly written as a private divergence. This density is called the pseudo-tensor of gravitational field plus matter. It is the sum of the tensor of the matter and pseudo-tensor of the gravitational field:

$$H^i_{\wedge k}=T^i_{\wedge k}+t^i_{\wedge k}=\partial_l[g^{im}_\wedge(\Gamma^l_{km}-\delta^l_{(k}\Gamma_{m)})-\frac{1}{2}\delta^i_k g^{mn}_\wedge(\Gamma^l_{mn}-\delta^l_m\Gamma_n)]/8\pi,\quad \Gamma_m=\Gamma^n_{mn}\,. \qquad (3)$$

Pseudo-tensor (3) is used in the formula [3, (88.4)]

$$J_k=\int_V H^i_{\wedge k}dV^\wedge_i=\int_V(T^i_{\wedge k}+t^i_{\wedge k})dV^\wedge_i \qquad (4)$$

to calculate what is called the total momentum of the gravitational field plus the matter at $k=1,2,3$ and the total energy of the gravitational field plus the matter at $k=t$:

$$J_t=\int_V H^i_{\wedge t}dV^\wedge_i=\int_V(T^i_{\wedge t}+t^i_{\wedge t})dV^\wedge_i \qquad (5)$$

From these formulas it follows that the proposed pseudo-tensor of the gravitational field $t^i_{\wedge k}$ must provide a negative contribution to the total energy (5).

The value of J_t was calculated for an isolated material system [3, § 91] by integration over the hypersurface $t=\mathrm{const}$ with the use of space-time, which took an approximately Schwarzschild form at infinity:

$$ds^2=(1-2m/r)dt^2-(1+2m/r)(dx^2+dy^2+dz^2)\,. \qquad (6)$$

Then the expected result was obtained

$$J_t=m\,. \qquad (7)$$

The same result was obtained again in [4, §92] and [4, §97], but here we deal with the explicitly represented pseudo-tensor component $t^t_{\wedge t}$, which was equal to the sum of the pressures in the case under study and was positive (!):

$$J_t=\int(T^t_{\wedge t}+t^t_{\wedge t})dV^\wedge_t=\int(T^t_t-T^1_1-T^2_2-T^3_3)\sqrt{-g}_\wedge(dxdydz)^\wedge=\int(\rho+3p)\sqrt{g_{tt}}dV_0=m\,. \qquad (8)$$

Here we feel bewildered. How can a positive addition to the matter tensor component, $T^t_{\wedge t}$, in the form of the pseudo-tensor component of the gravitational field,

$$t^t_{\wedge t} = (-T^1_1 - T^2_2 - T^3_3)\sqrt{-g}_{\wedge} > 0\,, \qquad (9)$$

describe the negative gravitational energy? Why does integral (8) with this positive addition have the value m, which is less than the value of integral (1) equal to P?

The answer is simple. Integrals (5) or (8) do not yield the energy J corresponding pseudo-tensor (3). In formulas (5) and (8), the element of the component $dJ_t = H^t_{\wedge t} dV^{\wedge}_t$ of the 4-momentum J_k is integrated. Therefore, the value of this integral is significantly less than the energy J. This is evident because it contains $\sqrt{g_{tt}} < 1$. It is important, however, that integral (4), in principle, does not make sense. The fact is that it is impossible to integrate the tensor values that are located at different points in space. In order to integrate them, one first needs to transfer them at some common point. If this is not done, then in using the curvilinear coordinates the components of the integral do not form a geometric quantity (vector) because formulas (4), (5) and (8) assume the arithmetic addition of the components of the vectors dJ_k belonging to different points of space in which the coordinate frame of reference can be differ from each other. Therefore there is no frame of reference to which the components of the integral could belong. And without a frame of reference components are always meaningless.

5. To correctly obtain the mass-energy J in the case of a spherical unmoving body, advantage can be taken of the fact that in this case the infinitesimal vectors dJ_k are all parallel to each other, and hence we can add up their moduli that do not change when they are transferred to a single point for summation. Therefore, we can easily integrate the infinitesimal moduli $dJ = dJ_t / \sqrt{g_{tt}}$. Then, instead of (8), we obtain for 'the total energy of the gravitational field plus the matter,' according to the proposed pseudo-tensor (3), the integral

$$J = \int (\rho + 3p) dV_0 = P + \int 3p dV_0\,, \qquad (10)$$

that completely discredits this pseudo-tensor.

Conclusions

The standard pseudo-tensor (3) does not yield the 'total 4-momentum of the gravitational field plus the matter', m, and physically meaningless. For details, see [4].

References

1. Schouten J.A. Tenzornyi analiz dlya fizikov (Tensor Analysis for Physicists) (Moscow: Nauka, 1965) [in Russian].
2. Schouten J. A. Tensor Analysis for Physicists (Oxford: Clarendon, 1951).
3. Tolman R.C. Relativity, Thermodynamics and Cosmology (Oxford: Clarendon Press, 1934).
4. Khrapko R.I. Myth of the Gravitational Field Energy http://khrapkori.wmsite.ru/ftpgetfile.php?id=112&module=files [in Russian].

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