Planck-unit black-hole-electron Mathematical Universe Hypothesis

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In the "Trialogue on the number of fundamental physical constants", Okun, Veneziano and M. Duff debated the number, from 1 to 3, of dimensionful units required. In this article I introduce formulas for the physical constants as geometrical shapes using the frequency of 2 dimensionful units; mass k and time t, and 2 fixed dimensionless constants; the Sommerfeld fine structure constant alpha and a proposed constant denoted here as Omega. By adjusting the frequency of the units mass and time, we may solve the SI values for G, h, c, e, m_e, k_B Solutions are consistent with CODATA 2014. Baseline units (k = 1, t = 1) give mass M = 1, time T = 2π , velocity $2V = 4\pi\Omega^2$, length $2L = 4\pi^2\Omega^2$... suggesting an underlying Planck entity (geometry). From this geometry we find a ratio of units whereby they cancel to form a dimensionless unitary state ($M^9T^{11}/L^{15} = 1$). This ratio is embodied in the electron formula suggesting the electron oscillates over time between a dimensionful electric state and a dimensionless (black-hole) state. As such, wave-particle duality may be a reflection of this oscillation, the electron as a means by which a Planck size micro black hole stores information aka the dimensions of our physical universe. The base (Planck) units are geometrical (M-point, T-radians, P-line, 2V-sphere, 2L-torus) rather than physical constructs and as they may unfold from this dimensionless state we have the basis for a Planck unit Mathematical Universe.

	Calculated* (Ω, α, k, t) [4]	CODATA 2014
speed of light	$c^* = 299792458 \text{ (exact)}$	c = 299792458 (exact)
Fine structure constant	n/a	$\alpha = 137.035\ 999\ 139(31)\ [11]$
Rydberg constant	$R^*_{\infty} = 10\ 973\ 731.568\ 508$	$R_{\infty} = 10\ 973\ 731.568\ 508(65)\ [9]$
Planck constant	$h^* = .662\ 606\ 971\ 458\ e-33$	$h = .662\ 607\ 004\ 0(81)\ e-33\ [10]$
Elementary charge	$e^* = .160\ 217\ 659\ 767\ e^{-18}$	$e = .160\ 217\ 662\ 08(98)\ e-18\ [13]$
Vacuum permeability	$\mu_0^* = 4\pi/10^7$, (exact)	$\mu_0 = 4\pi/10^7$, (exact) [15]
Electron mass	$m_e^* = .910\ 938\ 274\ 226\ e-30$	$m_e = .910\ 938\ 356(11)\ e-30\ [14]$
Boltzmann's constant	$k_B^* = .137\ 951\ 019\ 400\ e-22$	$k_B = .138\ 064\ 852(79)\ e-22\ [17]$
Larmor frequency	$f_L^* = 28\ 024.953\ 740 \text{MHz}$	$f_L = 28\ 024.951\ 64(17)$ MHz [21]
Gravitation constant	$G^* = .667\ 249\ 748\ 905\ e-10$	$G = .667 \ 408(31) \ e{-10} \ [16]$
Von Klitzing constant	$R_K^* = 25\ 812.806\ 933\ 779$	$R_K = 25\ 812.807\ 455\ 5(59)\ [19]$
Bohr magneton	$\mu_B^* = .927\ 401\ 023\ 52e-23$	$\mu_B = .927\ 400\ 999\ (57)e-26\ [22]$

keywords: Planck unit theory, Mathematical Universe, fundamental physical constants, fine structure constant alpha, Omega, black hole electron, SI units, dimensions, Plato's theory of forms

1 Introduction

"...we get the possibility to establish units for length, mass, time and temperature which, being independent of specific bodies or substances, retain their meaning for all times and all cultures, even non-terrestrial and non-human ones and could therefore serve as natural units of measurements" -M Planck [3].

In this article I first construct generic geometrical formulas for the fundamental physical constants $G, h, c, e, m_e, k_B...$ using 3 dimensionful units; *mass, time, momentum,* 2 natural constants; the dimensionless Sommerfeld fine structure constant *alpha* and a proposed *Omega*, and 2 scalars; *x*, *y*.

Natural constants are "natural" because the origin of their definition comes only from properties of nature and not from any human construct. The fine structure constant is presumed to be a ratio such as π and so dimensionless, as such its numerical value, as with π , is unit invariant and so it is a natural constant. Omega ($\Omega \sim 2.007135$) is not a ratio but instead is

constructed from 2 natural numbers π and log *e* and so may also be classified as a natural constant.

$$\Omega = \sqrt{\left(\frac{\pi^e}{e^{(e-1)}}\right)} \tag{1}$$

The numerical values for the fundamental constants as SI units however are a human construct, their values are set by consensus, and so to solve our formulas to reflect these values we must use 2 scalars (x, y). Likewise these scalars may be adjusted to solve any valid set of fundamental constants... were we to meet an alien civilization, we need only determine their (x, y) values in order to solve their constants.

We then note that we can solve the electron frequency formula using dimensionful units but that the ratio of these units cancel leaving the electron dimensionless. We also note that the numerical value associated with this ratio is non-trivial and denotes units of Planck time. This suggests that the electron evolves until that instant (single unit of Planck time) when these dimensionful units overlap, at that 'point' they then cancel and the electron is now dimensionless aka a micro black hole. The interval between these black-hole states is the electron frequency.

Using this ratio we can now reduce the number of units required to 2; mass and time. The SI analogs, Planck mass and Planck time are known with low precision, we can however re-write these formulas in terms of the most accurate constants; c (exact value) R (Rydberg constant, 12-13 digit precision), μ_0 (vacuum permeability, exact value) and the proposed Omega (exact value?).

2 MLTVPA formulas

Formulas for the (Planck) units in terms of $(p, k, t, \alpha, \Omega)$. The scalars (x, y) refer to the frequency of k mass and t time respectfully. The 3rd unit, the sqrt of momentum does not require an independent scalar.

M = (1)x, units = k (mass)

 $T = (2\pi)y$, units = t (time)

$$P = (\Omega) \frac{x^{4/5}}{y^{2/15}}, \text{ units} = p \text{ (sqrt of momentum)}$$
(2)

$$V = \frac{2\pi P^2}{M} = (2\pi\Omega^2) \frac{x^{3/5}}{y^{4/15}}, \text{ units} = \frac{p^2}{k} (velocity) \quad (3)$$

$$L = \frac{TV}{2} = (2\pi^2 \Omega^2) x^{3/5} y^{11/15}, \text{ units} = \frac{p^2 t}{k} (length)$$
(4)

$$A = \frac{8V^3}{\alpha P^3} = \frac{64\pi^3 \Omega^3}{\alpha} \frac{1}{x^{3/5} y^{2/5}}, \text{ units} = \frac{p^3}{k^3} (ampere)$$
(5)

3 Physical constants

The formulas for the physical constants become;

$$G^* = \frac{V^2 L}{M} = 8\pi^4 \Omega^6 x^{4/5} y^{1/5}, \ units = \frac{p^6 t}{k^4} \tag{6}$$

$$T_P^* = \frac{AV}{\pi} = \frac{128\pi^3 \Omega^5}{\alpha y^{2/3}}, \ units = \frac{p^5}{k^4}$$
(7)

$$\mu_0^* = \frac{\pi V^2 M}{\alpha L A^2} = \frac{\alpha x^{14/5}}{2048\pi^5 \Omega^4 y^{7/15}}, \text{ units} = \frac{k^6}{p^4 t}$$
(8)

$$e^* = AT = \frac{128\pi^4 \Omega^3 y^{3/5}}{\alpha x^{3/5}}, \ units = \frac{p^3 t}{k^3}$$
 (9)

$$h^* = 2\pi LVM = 8\pi^4 \Omega^4 x^{11/5} y^{7/15}, \ units = \frac{p^4 t}{k}$$
 (10)

$$k_B^* = \frac{\pi VM}{A} = \frac{\alpha x^{11/5} y^{2/15}}{32\pi\Omega}, \ units = \frac{k^3}{p}$$
 (11)

$$\epsilon_0^* = \frac{\alpha A^2 L}{\pi V^4 M} = \frac{512\pi^3 y}{\alpha x^4}, \text{ units} = \frac{t}{k^4}$$
(12)

$$f_e = T \left(\frac{M^2}{12\alpha \ 2L \ 2V \ 2P}\right)^3 = T \left(\frac{\pi^2}{3\alpha^2 AL}\right)^3, \ units = \frac{t^2 p^{15}}{k^{12}}$$
(13)

$$f_e^{-1} = 4\pi^2 (2^6 3\pi^2 \alpha \Omega^5)^3, \ (electron \ frequency^*)$$
(14)

$$m_e^* = M f_e, \ (electron \ mass^*)$$
 (15)

$$\lambda_e^* = \frac{2\pi L}{f_e}, \ (electron \ wavelength^*) \tag{16}$$

$$\alpha_G^* = \frac{Gm_e^2}{\hbar c} = f_e^2, \ (gravitation \ coupling \ constant^*)$$
(17)

4 SI units

To solve as SI units we need to set values whereby M = Planck mass and T = Planck time. As a reference (section 7) we can use c (exact value), μ_0 (vacuum permeability, exact value) and the Rydberg constant (CODATA mean, 12-13 digit precision) to solve (x, y). This gives us best fit values for;

 $x = .217 \ 672 \ 822 \ 274e-7$ $y = .171 \ 585 \ 524 \ 173e-43$ $\alpha = 137.035 \ 996 \ 369$ $M = (1)x = m_P, \text{ (Planck mass)}$ $T = (2\pi)y = t_p, \text{ (Planck time)}$ V = c $L = l_p, \text{ (Planck length)}$

Results for the constants are listed on the table on page 1, (note: Planck momentum = $2\pi P^2$) [4].

5 Black hole

If we set our scalars to unitary values; x = 1, y = 1 then we find a set of baseline units as geometrical shapes that are analogous to our Planck units suggesting an underlying fundamental Planck entity.

$$M = 1$$

$$T = 2\pi$$

$$P = \Omega$$

$$V = 2\pi\Omega^{2}$$

$$L = 2\pi^{2}\Omega^{2}$$

$$A = \frac{2^{6}\pi^{3}\Omega^{3}}{\alpha}$$
(18)

6 Electron frequency

The electron frequency formula (eq.14) has no (x, y) scalars, and so it has a fixed value regardless of the units used, f_e^{-1} = .238954525 $x10^{23}$, yet it is constructed from dimensionful units (eq.13);

It should only be possible for a dimensionful constant to have a unit invariant solution if the dimensions are not independent of each other, whereby there can be such a ratio of these units that they overlap and cancel giving a fixed numerical value. The units for the electron appear to display such a ratio;

$$\frac{L^{13}}{M^9 T^{11}} = 16\pi^{19} \Omega^{30}, units = 1$$
(19)

As Ω has a fixed value, this means that if I know the values for M and T, then I can calculate L (x and y cancel);

$$\frac{L^{15}}{M^9 T^{11}} = \frac{(2\pi^2 \Omega^2)^{15}}{(1)^9 (2\pi)^{11}} \cdot \frac{(x^{3/5} y^{11/15})^{15}}{(x)^9 (y)^{11}}$$
(20)

And so mathematically we can define the unit for L (length) in terms of k (mass) and t (time) via this ratio;

$$length = k^{3/5} t^{11/15} \tag{21}$$

Likewise

$$\frac{P^{15}T^2}{M^{12}} = 4\pi^2 \Omega^{15}, units = 1$$
(22)

$$\frac{2\pi V^{12}T^2}{Q^9} = (2\pi\Omega)^{15}, units = 1$$
(23)

7 Mass and Time

Using the following format where k represents the frequency x of a 'mass unit' and t represents the frequency y of a 'time unit', our formulas become:

constant = (dimensionless component) x 'units k, t'

- k = x (dimensionless frequency) x '1 mass unit'
- t = y (dimensionless frequency) x '1 time unit' M = (1)k

 $T = (2\pi)t$

$$P = (\Omega) \ \frac{k^{4/5}}{t^{2/15}} \tag{24}$$

$$V = \frac{2\pi P^2}{M} = (2\pi\Omega^2) \frac{k^{3/5}}{t^{4/15}}$$
(25)

$$L = \frac{TV}{2} = (2\pi^2 \Omega^2) k^{3/5} t^{11/15}$$
(26)

$$A = \frac{8V^3}{\alpha P^3} = \left(\frac{64\pi^3 \Omega^3}{\alpha}\right) \frac{1}{k^{3/5} t^{2/5}}$$
(27)

$$G^* = \frac{V^2 L}{M} = (8\pi^4 \Omega^6) k^{4/5} t^{1/5}$$
(28)

$$T_P^* = \frac{AV}{\pi} = \left(\frac{128\pi^3 \Omega^5}{\alpha}\right) \frac{1}{t^{2/3}}$$
(29)

$$\mu_0^* = \frac{\pi V^2 M}{\alpha L A^2} = \left(\frac{\alpha}{2048\pi^5 \Omega^4}\right) \frac{k^{14/5}}{t^{7/15}}$$
(30)

$$e^* = AT = \left(\frac{128\pi^4\Omega^3}{\alpha}\right) \frac{t^{3/5}}{k^{3/5}}$$
(31)

$$h^* = 2\pi LVM = (8\pi^4 \Omega^4) k^{11/5} t^{7/15}$$
(32)

$$k_B^* = \frac{\pi V M}{A} = \left(\frac{\alpha}{32\pi\Omega}\right) k^{11/5} t^{2/15}$$
 (33)

$$\epsilon_0^* = \frac{\alpha A^2 L}{\pi V^4 M} = \left(\frac{2^9 \pi^3}{\alpha}\right) \frac{t}{k^4}$$
(34)

$$f_e^* = (2^{20} 3^3 \pi^8 \alpha^3 \Omega^{15}) \tag{35}$$

$$m_e^* = M(f_e) \tag{36}$$

$$\lambda_e^* = \left(\frac{2\pi}{f_e}\right)L\tag{37}$$

$$\alpha_G^* = f_e^2 \tag{38}$$

$$\mu_B^* = \frac{eh}{4\pi M(f_e)} = \left(\frac{2^8 \pi^7 \Omega^7}{\alpha f_e}\right) k^{3/5} t^{16/15}$$
(39)

$$f_L^* = (2^{26} \pi^{11} 3^3 \gamma \Omega^{18} \alpha^2) \frac{t^{3/5}}{k^{8/5}}$$
(40)

$$(R^{-1})^* = \frac{4\pi l_p \alpha^2 m_P}{m_e} = (2^{23} 3^3 \pi^{11} \alpha^5 \Omega^{17}) k^{3/5} t^{11/15}$$
(41)

We note that non-integer units of k and t can be converted to integer units but in the process relevant constants, being those constants that include Ω , become multiples of Ω^{15} ; $G^5 - > \Omega^{30}$, $e^5 - > \Omega^{15}$, $h^{15} - > \Omega^{60}$, such that the charge constants give Ω^{15} , the mass constants Ω^{30} , these are reflected in $f_e - > \Omega^{15}$ and the gravitational coupling constant $\alpha_G = f_e^2 - > \Omega^{30}$.

I showed in a previous article [4] that we may solve the fundamental constants in terms of; c and μ_0 (exact), R (12-13 digit precision) and α . By replacing α with Ω (exact) we can formulate and solve the following.

$$\begin{split} c &= (m \ s^{-1}) = k^{(3/5)} t^{(-4/15)} \\ \mu_0 &= (kg \ m \ s^{-2} \ A^{-2}) = k^{(14/5)} t^{(-7/15)} \\ R^{-1} &= (m) = k^{(3/5)} t^{(11/15)} \\ z &= (m^{15} kg^{-9} s^{-11}) = k^0 t^0 = 1 \end{split}$$

In the previous paper [4] I proposed the sqrt of Planck momentum as the link between charge and mass giving the formula and units for the ampere as;

$$A_Q = \frac{8c^3}{\alpha Q^3}, \ units = A = \frac{m^2}{kgs^2\sqrt{kg.m/s}}$$
(42)

$$A_Q^2$$
, units = $A^2 = \frac{m^3}{kg^3s^3}$ (43)

Alpha becomes

)

$$\alpha^{26} = \frac{c^{53}}{2^{295} 3^{21} \pi^{157} \mu 0^9 \Omega^{225} R^7} \tag{44}$$

$$\alpha^{26}$$
; $units = \frac{A^{18}m^{33}}{kg^9s^{17}} = z^4$, $units = (k^0t^0)^4 = 1$ (45)

The constants then become;

$$h^{3} = \frac{2\pi^{10}\mu_{0}^{3}}{3^{6}c^{5}\alpha^{13}R^{2}}, units = \frac{kg^{3}}{A^{6}s}z = \frac{kg^{3}m^{6}}{s^{3}}$$
(46)

$$G^{5} = \frac{\pi^{3}\mu 0}{2^{20}3^{6}\alpha^{11}R^{2}}, units = \frac{kgm^{3}}{A^{2}s^{2}} = zkg^{4}s = \frac{m^{15}}{kg^{5}s^{10}}$$
(47)

$$m_e^3 = \frac{16\pi^{10}R\mu0^3}{3^6c^8\alpha^7}, units = \frac{kg^3s^2}{A^6m^6}z = \frac{kg^{12}s^{11}}{m^{15}}z = kg^3 \quad (48)$$

$$m_P^{15} = \frac{2^{25}\pi^{13}\mu_0^6}{3^6c^5\alpha^{16}R^2} = \frac{kg^6m^3}{A^{12}s^7} = \frac{kg^{24}s^{11}}{m^{15}}z = kg^{15}$$
(49)

$$l_p^{15} = \frac{\pi^{22} \mu_0^9}{2^{35} 3^{24} c^{35} \alpha^{49} R^8} = \frac{k g^9 s^{17}}{A^{18} m^{18}} = \frac{k g^{36} s^{44}}{m^{45}} z^4 = m^{15}$$
(50)

$$t_p^{15} = \frac{\pi^{22} \mu_0^9}{2^{20} 3^{24} c^{50} \alpha^{49} R^8} = \frac{kg^9 s^{32}}{A^{18} m^{33}} = \frac{kg^{36} s^{59}}{m^{60}} z^4 = s^{15}$$
(51)

$$e^{3} = \frac{4\pi^{5}}{3^{3}c^{4}\alpha^{8}R}, units = \frac{s^{4}\sqrt{z}}{m^{3}} = (As)^{3}$$
(52)

Using the SI values for (c, μ_0, R) we can solve (m_P, t_p, α) from which in turn we can derive the frequencies (numerical values x, y) associated with (k, t). We can then numerically solve all formulas with units by using (k, t) instead of the SI units (kg, m, s, A, k) via $c - > c^*, \mu_0 - > \mu_0^*, R - > R^*$;

$$k_B^3 = \frac{\pi^5 \mu_0^3}{3^3 2 c^4 \alpha^5 R}, \ units = (k^{11/5} t^{2/15})^3$$
 (53)

$$\frac{\mu_0^3}{t^{7/15}}(units) = \left(\frac{k^{14/5}}{t^{7/15}}\right)^3 \left(k^{3/5}t^{11/15}\right) / \left(\frac{k^{3/5}}{t^{4/15}}\right)^4 = \left(k^{33/5}t^{2/5}\right)$$
(54)

$$T_p^5 = \frac{2^{10} 3^3 c^{15} \alpha^3 R}{\pi^4 \mu_0^3}, \ units = (\frac{1}{t^{2/3}})^5$$
(55)

$$\mu_B^3 = \frac{\pi^2}{2^7 3^3 c \alpha^{14} R^4}, \ units = (k^{3/5} t^{16/15})^3 \tag{56}$$

$$f_L^3 = \frac{3^3 \gamma^3 c^4}{2^5 \pi^8 \mu 0^3 \alpha R^2}, \ units = (t^{3/5} k^{8/5})^3 \tag{57}$$

$$A_{Q}^{5} = \frac{2^{10}\pi 3^{3}c^{10}\alpha^{3}R}{\mu_{0}^{3}}, \ units = (\frac{1}{k^{3/5}t^{2/5}})^{5}$$
(58)

8 Geometry

As noted, the black-hole units suggest geometrical shapes. For example, we may interpret the geometry of 2V (velocity) as the surface area of a sphere $4\pi r^2$, 2L (length) as the surface area of a torus $4\pi^2 r^2$, electron frequency as volume of a torus $4\pi^2 r^3$, Mass (M) as an integer in terms of units of Planck mass and Time (T) in radians. If we apply this to the Bohr radius *r* by replacing L = Planck length l_p with L = $2\pi^2\Omega^2$, we may incorporate *n* in the radial axis whereby length in our 3-D sense reduces to a geometrical shape - this torus. It may be that there is no 'empty' space within the atom, rather the Bohr radius itself is a geometrical construct.

$$r_{Bohr} = \alpha n^2 \lambda_e = \frac{\alpha n^2 l_p}{f_e} = \frac{\alpha n^2 2\pi^2 \Omega^2}{f_e} = \frac{\alpha}{f_e} \cdot 2\pi^2 (n\Omega)^2 \quad (59)$$

9 Discussion

I have described a solution to the physical constants G, h, e, c, k_B , μ_0 , m_e ... in terms of the frequency of 2 dimensionful units; mass and time, and 2 dimensionless constants; alpha and Omega.

Setting the scalars to unitary values gives a baseline set of geometries, as these reflect the Planck units they could be aspects of an underlying Planck entity, a possible candidate being a Planck micro black hole.

The model suggests that there is ratio of units that is also dimensionless in that the units cancel $(kg^9s^{11}/m^{15} = 1)$. The electron frequency formula f_e reflects this ratio, as such f_e is both dimensioned and dimensionless. The electron frequency may also therefore be considered as a natural (unit invariant) constant.

This dimensionless ratio of units has a numerical solution which is also reflected in the electron frequency formula f_e . If this electron frequency refers to a number of Planck time units, then when the electron reaches this sum, the units converge and cancel, the electron is a now a micro black hole. At the next unit of Planck time the units begin to unfold again, the electron undergoing this wave-state to black-hole state oscillation over time. This may hold implications for wave-particle duality [5].

I proposed a dimensionful constant which I have denoted Omega. The solution for Ω was chosen for its symmetrical elegance, for it resembles Euler's identity ($e^{i\pi} + 1 = 0$), perhaps the most famous formula in all mathematics. As the range of possible values for Ω is extremely limited, the value of Ω is fixed by the value of α (and vice-versa), this formula $\pi^e/e^{(e-1)}$ is quite likely the only non-complex solution within this range. As π and e can be constructed from integers in series, constants based upon π and e could be natural occurring, the precision of π and e and so the precision of the physical constants increasing as the universe grows [7].

Omega gives a value for alpha slightly lower than the CO-DATA value. Clearly a precise match is desirable although likewise a divergence is revealing. However the importance maybe overstated at this level for as L. Smolin noted, 'most mathematical laws used in physics do not uniquely model the phenomena they describe. In most cases the equation describing the law could be complicated by the addition of extra terms, consistent with the symmetries and principles expressed whose effects are merely too small too measure given state of the art technology. These "correction terms" may be ignored because they don't measurably affect the predictions, but only complicate the analysis'. [8].

The geometry for mass M = 1 suggests integer (Planck mass) events and $T = 2\pi$ (radians) as a unit of (Planck) time being a measure in radians of rotation. The other Planck units have spherical geometries and so we might conclude that rotation or spin is an inherent property. The black-hole electron is dimensionless but has the dimensioned units 'folded' within. In this context our 'physical' universe reduces to a projection of this underlying universe of geometrical forms as proposed by the ancient Greek philosophers [5] and from this lies the basis for a Planck unit Mathematical Universe Hypothesis.

J. Barrow and J Webb in their 2005 Scientific American article on the fundamental constants wrote;

'Some things never change. Physicists call them the con-

stants of nature. Such quantities as the velocity of light, c, Newton's constant of gravitation, G, and the mass of the electron, m_e , are assumed to be the same at all places and times in the universe. They form the scaffolding around which the theories of physics are erected, and they define the fabric of our universe. Physics has progressed by making ever more accurate measurements of their values. And yet, remarkably, no one has ever successfully predicted or explained any of the constants. Physicists have no idea why they take the special numerical values that they do. In SI units, c is 299,792,458; G is 6.673e-11; and m_e is 9.10938188e-31 -numbers that follow no discernible pattern. The only thread running through the values is that if many of them were even slightly different, complex atomic structures such as living beings would not be possible.

The desire to explain the constants has been one of the driving forces behind efforts to develop a complete unified description of nature, or "theory of everything". Physicists have hoped that such a theory would show that each of the constants of nature could have only one logically possible value. It would reveal an underlying order to the seeming arbitrariness of nature.' [2]

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- 14. Electron charge http://physics.nist.gov/cgi-bin/cuu/Value?me
- 15. Vacuum of permeability http://physics.nist.gov/cgi-bin/cuu/Value?mu0
- 16. Gravitation constant http://physics.nist.gov/cgi-bin/cuu/Value?bg
- 17. Boltzmann constant http://physics.nist.gov/cgi-bin/cuu/Value?k
- Free electron gyromagnetic constant http://physics.nist.gov/cgi-bin/cuu/Value?gammaebar
- Von Klitzing constant http://physics.nist.gov/cgi-bin/cuu/Value?rk
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