a Mathematical Universe Hypothesis
fundamental constants, dimensions as geometry of angular momentum

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In the “Trialogue on the number of fundamental physical constants”, L. Okun, G. Veneziano and M. Duff debated the number of fundamental dimensionful constants in physics. In the model listed here the physical constants $G$, $h$, $e$, $k_B$, $m_e$,... are derived in terms of 2 dimensionless constants; the fine structure constant alpha and Omega ($\Omega^2 = \pi^2/e^{\alpha - 1}$), and 2 dimensionful constants. Solutions are consistent with CODATA 2010, setting alpha = 137.03599636868 gives an exact solution for $\mu_0 = 4\pi/10^7$ and for the Rydberg constant mean value $R = 10973731.568539$. The electron frequency is both dimensionful and unit invariant and so is used to suggest a ‘normalized’ set of Planck units, the formulas for which describe simple geometrical shapes (M=point, T=circle, V=sphere, L=torus) whose radius is Omega suggesting dimensionful-ness may be a geometrical attribute rather than an inherent property. Any 3 units from MLTVP are required to numerically solve the physical constants suggesting the primary dimensionful unit is related to angular momentum.

1 Introduction

In a Scientific American article on the physical constants J. Barrow and J Webb wrote...

‘Some things never change. Physicists call them the constants of nature. Such quantities as the velocity of light, $c$, Newton’s constant of gravitation, $G$, and the mass of the electron, $m_e$, are assumed to be the same at all places and times in the universe. They form the scaffolding around which the theories of physics are erected, and they define the fabric of our universe. Physics has progressed by making ever more accurate measurements of their values. And yet, remarkably, no one has ever successfully predicted or explained any of the constants. Physicists have no idea why they take the special numerical values that they do. In SI units, $c$ is 299,792,458; $G$ is 6.673e-11; and $m_e$ is 9.10938188e-31 -numbers that follow no discernible pattern. The only thread running through the values is that if many of them were even slightly different, complex atomic structures such as living beings would not be possible.

The desire to explain the constants has been one of the driving forces behind efforts to develop a complete unified description of nature, or “theory of everything”. Physicists have hoped that such a theory would show that each of the constants of nature could have only one logically possible value. It would reveal an underlying order to the seeming arbitrariness of nature.’ [2]

2 Mathematical Universe

In the Trialogue, L. Okun gives this definition; There are two kinds of fundamental constants of Nature: dimensionless (like the fine structure constant alpha) and dimensionful (velocity of light, Newton’s gravitational constant...) [1]. In this article I offer a mathematical relationship between the dimensionful constants that suggests dimensions are a function of the geometry of particles rather than inherent properties.

A list of formulas for the physical constants in terms of 2 dimension-full constants $P$ (sqrt of momentum), $V$ (velocity) and 2 dimensionless constants $\alpha$ and $\Omega$ - arguably a natural unit given that it may be defined in terms of 2 other natural units; $\pi$ and $e$;

$$\Omega^2 \sim \frac{\pi^2}{e^{\alpha - 1}}$$ (1)

$$M = \frac{2P^2}{V}$$, (mass) (2)

$$T^2 = \frac{(2\pi\Omega)^{14}\Omega P^9}{V^{12}}$$, (time) (3)

$$L = \frac{TV}{2}$$, (length) (4)

$$A = \frac{8V^3}{\alpha^213}$$, (charge) (5)

The remaining physical constants would then become;

$$G^* = \frac{V^2L}{M}$$, (gravitation constant*) (6)

$$T_p^* = \frac{AV}{\pi}$$, (Planck temperature*) (7)

$$\mu_0^* = \frac{\pi V^2 M}{\alpha LA^2}$$, (vacuum permeability*) (8)

$$e^* = AT$$, (elementary charge*) (9)

$$h^* = 2\pi LVM$$, (Planck constant*) (10)

$$k_B^* = \frac{\pi VM}{A}$$, (Boltzmann constant*) (11)

$$f_e = T\left(\frac{\pi^2}{3\alpha^2 AL}\right)^3$$, (electron frequency*) (12)

$$m_e^* = MF_e$$, (electron mass*) (13)
3 Electron as a black hole

The formula for the electron frequency $f_e$ is unit invariant - it gives the same numerical solution regardless of the units and so can be used to suggest alternative values for MLTPV. The following suggest Planck level units (electron as a black hole) as simple geometrical shapes;

- $M = 1$, (mass)
- $T = 2\pi$, (time)
- $P = \Omega$, (sqrt of momentum)

\[
V = \frac{2\pi P^2}{M} = 2\pi \Omega^2
\]

\[
L = \frac{\pi P^2 T}{M} = 2\pi^2 \Omega^2
\]

\[
A = \frac{8\pi^3}{\alpha P^3} = \frac{2\pi^3 \Omega^3}{\alpha}
\]

\[
f_e = T \left( \frac{\pi^2}{3\alpha^2 AL} \right)^3 = \frac{2\pi}{(2\pi^3\alpha \Omega^3)\pi^3} = \frac{\pi^4}{28^3 3^5 14^4 \alpha^3 Q^7}
\]

\[(2\pi)^{12} \Omega^5 = \alpha^2 \frac{5^4 Q^7}{Q^7}
\]

4 Background

In the following sections the SI constants are defined in terms of the most precise natural constants; $c$ (exact), $\mu_0$ (exact), $R_\infty$ (12-13 digits) and $\alpha$, and $Q$ as the sqrt of Planck momentum ($m_pc = 2\pi Q^2$ [6]) is interpreted as the link between mass and charge.

\[
Q = 1.019 113 411... \sqrt{\frac{kgm}{s}}
\]

5 Mass constants

Defining the mass constants in terms of Planck momentum;

\[
m_p = \frac{2\pi Q^2}{c}
\]

\[
G = \frac{l_p c^3}{2\pi Q^2}
\]

\[
h = 2\pi Q^2 2\pi l_p
\]

\[
t_p = \frac{2l_p}{c}
\]

\[
F_p = \frac{E_p}{l_p} = \frac{2\pi Q^2}{l_p}
\]
6 Ampere $A_Q$

(Proposed) Ampere $A_Q$ [6]

$$A_Q = \frac{8e^3}{\alpha Q^3} \text{ units } = \frac{m^2}{kg s^2 \sqrt{kg m/s}} \quad (27)$$

Planck Temperature $= A_Qc$
Elementary charge $= A_Qt_p$
Magnetic monopole (quark) $= A_Ql_p$
Electron $= t_p(A_Ql_p)^3$
Magnet $= A_Ql_p^2$

7 Elementary charge

$$e = A.s = A_Qt_p \quad e = \frac{8e^3}{\alpha Q^3} \quad \frac{2l_p}{c} = \frac{16l_p e^2}{\alpha Q^3} \quad (28)$$

8 Vacuum permeability

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to exactly $2.10^{-7}$ newton per meter of length.

$$\frac{F_{electric}}{A_Q^2} = \frac{2\pi Q^2}{\alpha t_p}. \frac{\alpha Q^3}{8e^3} \quad \frac{\pi \alpha Q^8}{64l_p e^5} = \frac{2}{10^7} \quad (29)$$

gives:

$$\mu_0 = \frac{\pi^2 \alpha Q^8}{32l_p e^5 c^3} = \frac{4\pi}{10^7} \quad (30)$$

$$\epsilon_0 = \frac{32l_p e^5}{\pi^2 \alpha Q^8} \quad (31)$$

$$k_e = \frac{\pi \alpha Q^8}{128l_p e^3 c^3} \quad (32)$$

$$\alpha = \frac{2h}{\mu_0 e^2 c} = 2.2\pi Q^2.2\pi l_p \cdot \frac{32l_p e^5}{\pi^2 \alpha Q^8} \cdot \frac{a^2 Q^6}{256l_p e^5 c^3} \cdot \frac{1}{c} = \alpha \quad (33)$$

$$\mu_0 \epsilon_0 = \frac{\pi^2 \alpha Q^8}{32l_p e^5 c^3 - \frac{32l_p e^5}{\pi^2 \alpha Q^8} \cdot \frac{1}{c^2} = \frac{1}{c^2} \quad (34)$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \quad (35)$$

9 Planck length $l_p$

$l_p$ in terms of $Q$, $\alpha$, $c$.

The magnetic constant $\mu_0$ has a fixed value. From eq.21

$$l_p = \frac{\pi^2 \alpha Q^8}{2\mu_0 e^5} \quad (36)$$

$$\mu_0 = 4\pi.10^{-7} \text{ N} / \text{A}^2$$

$$l_p = \frac{5^7 \pi \alpha Q^8}{e^5} \quad (37)$$

giving:

$$h = 2\pi Q^2.2\pi l_p = \frac{2^5 5^7 \pi \alpha Q^{10}}{e^5} \quad (38)$$

$$e = \frac{16l_p e^2}{\alpha Q^3} = \frac{2^4 5^7 \pi Q^5}{e^5} \quad (39)$$

$$G = \frac{l_p e^3}{2\pi Q^2} = \frac{5^7 \alpha Q^6}{e^2} \quad (40)$$

10 Electron as magnetic monopole

The ampere-meter is the SI unit for pole strength (the product of charge and velocity) in a magnet ($Am = ec$). A Magnetic monopole [12] is a hypothetical particle that is a magnet with only 1 pole.

To convert Planck time $t_p$, elementary charge $e$ and the speed of light $c$ to SI units 1s, 1C, 1m/s requires unit-less numbers which are numerically equivalent ($t_s$, $e_s$, $c_s$).

$$\frac{t_p}{t_s} = \frac{5.3912... e^{-44}s}{5.3912... e^{-44}} = 1s$$

$$\frac{e}{e_s} = \frac{1.6021764... e^{-19}C}{1.6021764... e^{-19}} = 1C$$

$$\frac{c}{c_s} = \frac{299792458m/s}{299792458} = 1m/s$$

We may thus form unit-less formulas for $\sigma_e$, a magnetic monopole, and the electron frequency $E_{\sigma}$, $t_s \sigma_{e_s}$. The electron frequency formula appears to describe the number of units of Planck time corresponding to the electron frequency.

$$\sigma_e = \frac{2\pi^2}{3\alpha^2 e_s c_s} \quad \frac{\pi^2 Q^8_s}{24\alpha l_s c_s^3} \quad (41)$$

$$E_{\sigma} = t_s \sigma_{e_s} \quad \frac{\pi^2 Q^8_s}{2^{3} 3^{3} 5^{14} \alpha^2 Q_s^5} = \frac{\pi^4}{2^{3} 3^{3} 5^{14} \alpha^2 Q_s^5} \quad (42)$$

Planck mass:

$$m_p = m_p E_{\sigma} \quad (43)$$

Compton wavelength:

$$\lambda_e = \frac{2\pi l_p}{E_{\sigma}} \quad (44)$$
12 Planck mass black hole

Black hole energy distribution of emission \( T_{mP} \) as described by Planck’s law for \( M = m_P \)

\[
T_{mP} = \frac{h c^3}{16\pi^2 G k_B m} = \frac{T_p}{8\pi}
\]

(54)

Hawking radiation has a blackbody (Planck) spectrum with a temperature \( T \) for an \( M = m_P \) black hole \( (r_s = 2l_p) \) given by

\[
k_B T_{mP} = \frac{hc}{8\pi r_s} = \frac{E_p}{8\pi}
\]

(55)

From eq.46 and eq.50

\[
k_B T_{mP} = \frac{\pi^2 Q^5}{4c^3} \frac{T_p}{8\pi} = \frac{E_p}{8\pi}
\]

(56)

General relativity

\[
\frac{c^4}{8\pi G} = \frac{F_p}{8\pi}
\]

(57)

Bekenstein Hawking entropy \( (S) \) for \( M = m_P \)

\[
A = \frac{16\pi G^2 m_P^3}{c^4}
\]

(58)

\[
S = \frac{2\pi k_B c^3 A}{4Gh} = 4\pi k_B
\]

(59)

14 Radio wave

Larmor precession frequency \((1 \text{ Tesla}) f_L = 28.025 \text{GHz}, k_Bx \)

is the unit-less Boltzmanns constant and \( \gamma \) is the electron magnetic moment \( \gamma = 1.001 159 652 180 73(28) \).

\[
f_L = \frac{\gamma^2 \mu_B B_{1\text{Tesla}}}{\hbar} = \frac{\gamma^2 \mu_B c^2 m_P}{\pi^2 \alpha_p Q^5 m_e} = \frac{\gamma c_x}{2k_B x} \frac{t_s}{2\pi m_e}
\]

(63)

The presence of the Boltzmann’s constant is a consequence of the SI unit 1 Tesla. A Planck \( B_P (R_P = \text{Planck Rydberg}) \)

\[
B_P = \frac{m_P}{\alpha^2 A_Q l_p^2}
\]

(64)
\[ f_p = \frac{2\mu_B B_p}{\hbar} = \frac{1}{2\pi^2 l_p^3} = \frac{c}{4\pi^2 l_p} = R_p c \]  

(65)

A Planck electron \( B_e (R_{\infty} = \text{Rydberg constant}) \)

\[ B_e = \frac{m_p m_e^2}{\alpha^2 A_0 t_p^2 m_p} = \frac{m_p^2 c^2}{\alpha^2 A_0 t_p^2} \]  

(66)

\[ f_e = \frac{2\mu_B B_e}{\hbar} = \frac{m_e c}{4\pi^2 l_p m_p} = R_{\infty} c \]  

(67)

### 15 Quintessence momentum

The electron mass formula, replacing \( l_p \) (eq.26)

\[ m_e = m_p \frac{\pi^4}{2^{5/3} 3^{14} 5^{10} \alpha^3 Q_e^7} \]  

(68)

The Rydberg constant \( R_{\infty} \)

\[ R_{\infty} = \frac{m_e c^4 \mu_0^2 c^3}{8 \hbar^3} = \frac{\pi^2 c^5}{2^{10} 3^{5} 5^{21} \alpha^8 Q_e^2} \]  

(69)

The Rydberg constant \( R_{\infty} = 10973731.568 \times 539(55) \) [13] with a 12-digit precision is the most accurate of the natural constants. Consequently we may re-define \( Q \) in terms of this constant, \( c \) and \( \alpha \).

\[ Q^{15} = \frac{\pi^2 c^5}{2^{10} 3^{5} 5^{21} \alpha^8 R_{\infty}} \]  

(70)

\[ Q = \left( \frac{\pi^2 c^5}{2^{10} 3^{5} 5^{21} \alpha^8 R_{\infty}} \right)^{\frac{1}{15}} \]  

(71)

Alternately we can solve via the vacuum of permeability;

\[ Q^7 = \left( \frac{64\pi^6}{5^7} \right) \frac{\Omega^{15}}{\alpha^2} \]  

(72)

### 16 Planck time

\( c \) has an exact value, \( Q \) is solved using \( \alpha \) and \( R_{\infty} \). Via the vacuum permeability (eq.26), also an exact value, we can solve \( l_p \):

\[ t_p = \frac{2\pi^5 \alpha Q^8}{e^6} \]  

(73)

### 17 Alpha and Omega

The CODATA \( \alpha = 137.035999074 \).

The von Klitzing constant reduces to \( \alpha \) and \( c \) and so has the potential to provide the most definitive solution for \( \alpha \).

\[ R_K = \frac{\hbar}{e^2} = \frac{\pi ac}{500000} \]  

(74)

\[ R_K = 25812.807 \times 557(18) \]  

(5)

\[ \alpha = 137.035 \times 999 \times 677(96) \]

Atom-recoil measurements:

\[ h/R_p = 4.5913592729 \times 9 \text{amu} \]

\[ 87R_p = 86.909180527 \text{amu} \]

\[ m_e = 0.00054857999067 \text{amu} \]

\[ \alpha = 1/\sqrt{(2R_{\infty}/c)(h/R_p)(87R_p/m_e)) \]

\[ \alpha = 137.035 \times 998 \times 998 \]

\[ h/\text{neutron ratio} \]  

\[ h/m_n = 3.956033332 \times 7 \text{amu} \]

\[ m_n = 1.0086649100 \text{amu} \]

\[ \alpha = 1/\sqrt{(2R_{\infty}/c)(h/m_n)(m_n/m_e)) \]

\[ \alpha = 137.036 \times 010 \times 857 \]

\[ \Omega^{15} = \frac{5^{14} \alpha^2 Q^7}{2^{12} \pi^{12}} \]  

(75)

\[ \Omega^{225} = \frac{5^{63} c^{35}}{2^{250} 3^{21} \pi^{106} \alpha^{26} R_{\infty}^7} \]  

(76)

Alternately setting Omega;

\[ \Omega^2 = \frac{\pi^e}{e^{(e-1)}} \]  

(77)

gives \( \alpha = 137.035 \times 996 \times 36868... \)

### 18 Geometrical forms

The Planck-level units proposed suggest geometrical shapes. For example, we may interpret the geometry of 2V (velocity) as the surface area of a sphere \( 4\pi\alpha^2 \cdot 2L \) (length) as the surface area of a torus \( 4\pi\alpha^2 \cdot 2L \), electron frequency as volume of a torus \( 4\pi^2 \cdot r^2 \). Mass (M) as an integer in terms of units of Planck mass and Time (T) in radians. For example, with the Bohr radius \( r \) by replacing \( L = \text{Planck length} l_p \) with \( L = 2\pi\Omega^2 \), we may incorporate \( n \) in the radial axis whereby length in our 3-D sense reduces to a geometrical shape - this torus surface area.

\[ r_{\text{Bohr}} = an^2\lambda_c = \frac{an^2 l_p}{c} = \frac{an^2 2\pi^2 \Omega^2}{c} = \frac{\alpha}{f_e} 2\pi^2 (n\Omega)^2 \]  

(78)

\[ \frac{1}{f_e} = 4\pi^2 \lambda_c \cdot \left( r_c = 2^6 \pi^3 \alpha^2 \Omega^5 \right) \]  

(79)

### 19 Summary

I have outlined a solution to the physical constants \( G, h, e, k_B, \mu_0, m_e... \) in terms of the geometry of \( Q \) (the sqrt of Planck momentum), \( c \) and \( l_p \) (PVL). The final precision of the numerical solutions (online calculator [61]) however derives from the
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4 most accurate constants; c, alpha, the vacuum permeability (from which I derived \(l_p\) and \(t_p\)) and the Rydberg constant (which gives \(Q\)).

As a minimum 3 units from MLTPV are required to numerically solve the constants it may be that the fundamental dimensionfulness is a form of angular momentum, as for example Planck’s constant \(h\) which incorporates MVL or an equivalent combination thereof.

The construction of the electron frequency \(f_e\) from Planck units suggests an electron as black hole [28]. The formulas for electron mass, wavelength etc suggest that particle frequency dictates the frequency of the corresponding Planck units. For example, electron mass becomes the frequency of occurrence of a unit of Planck mass as dictated by the electron frequency [10].

The numerical solution for \(\Omega\) was chosen for its symmetrical elegance, for it resembles Euler’s identity \((e^{i\pi} + 1 = 0)\), perhaps the most famous formula in all mathematics. As the range of possible values for \(\Omega\) is extremely limited, the value of \(\Omega\) is fixed by the value of \(\alpha\) (and vice-versa), this formula \(\pi e/e - 1\) is quite likely the only non-complex solution within this range. As \(\pi\) and \(e\) can be constructed from integers in series, constants based upon \(\pi\) and \(e\) could be natural occurring, the precision of \(\pi\) and \(e\) and so the precision of the physical constants increasing as the universe grows.

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