The 2-D Planck universe: 2π, fine structure constant alpha, omega ($\pi^e/e^{(e-1)}$), sqrt of Planck time and sqrt of Planck momentum

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The principal physical constants G, h, c, e, k_B, μ_0, m_e... are defined in terms of the dimensions of mass, length, time, temperature and electric charge (M,L,T,K,A). There are also natural constants such as π and the fine structure constant α which have no dimensions and so are presumed independent of the system of units used. These may be considered as natural or universal constants. Proposed here is a further natural constant which I have denoted Omega. From 2π, α and Ω ($\Omega = \pi^e/e^{(e-1)}$), basic formulas for time, mass, length and charge are proposed in terms of 2 dimensions a (sqrt of Planck time) and b (sqrt of Planck momentum). These base units are then scaled to their Planck unit equivalents from which G, h, c, e, k_B, μ_0, m_e... are solved, the accuracy of each solution is limited by the (12-digit) precision of the Rydberg constant. Results are consistent with CODATA 2010. As the SI units (kg,m,k,A) can be constructed using the sqrt of Planck time and sqrt of Planck momentum, these 2 quantities could be considered as universal dimensions. By comparing the Planck electron with the SI electron, a solution for alpha as α = 137.035 996 368(2) is proposed.

1 Preface

J. Barrow et al noted in a Scientific American article... 'Some things never change. Physicists call them the constants of nature. Such quantities as the velocity of light, c, Newton's constant of gravitation, G, and the mass of the electron, m_e, are assumed to be the same at all places and times in the universe. They form the scaffolding around which the theories of physics are erected, and they define the fabric of our universe. Physics has progressed by making ever more accurate measurements of their values. And yet, remarkably, no one has ever successfully predicted or explained any of the constants. Physicists have no idea why they take the special numerical values that they do. In SI units, c is 299,792,458; G is 6.673e-11; and m_e is 9.10938188e-31 -numbers that follow no discernible pattern. The only thread running through the values is that if many of them were even slightly different, complex atomic structures such as living beings would not be possible. The desire to explain the constants has been one of the driving forces behind efforts to develop a complete unified description of nature, or "theory of everything". Physicists have hoped that such a theory would show that each of the constants of nature could have only one logically possible value. It would reveal an underlying order to the seeming arbitrariness of nature.' [1]

2 Natural units

Natural units are "natural" because the origin of their definition comes only from properties of nature and not from any human construct. From 3 natural units; 2π, the fine structure constant $\alpha \sim 137.036$, $\Omega \sim 2.007$ and from 2 dimensioned quantities a and b, we may construct the following (note: a has the dimension time and b momentum)...

$$\Omega = (+/-)\frac{\pi^{e/2}}{e^{(e-1)/2}} \;...\; \Omega^2 = \frac{\pi^e}{e^{(e-1)}} \tag{1}$$

$$t_u = 2\pi.\frac{1}{a^6},\ unit = T \tag{2}$$

$$c_u = 2\pi.\Omega^2\frac{a}{b^3},\ unit = L/T \tag{3}$$

$$l_u = 2\pi.\pi\Omega^2\frac{1}{a^5 b^3},\ unit = L \tag{4}$$

$$p_u = m_u c_u = 2\pi.\Omega^2\frac{1}{b^8},\ unit = M.L/T \tag{5}$$

$$m_u = \frac{p_u}{c_u} = \frac{1}{ab^5},\ unit = M \tag{6}$$

$$A_u = \frac{8c_u^3}{\alpha\Omega^3} = \frac{2^6\pi^3\Omega^3}{\alpha}a^3b^3,\ unit = A \tag{7}$$

From here we construct 2 units, the magnetic monopole σ_u and the electron frequency $E_\sigma = t_u\sigma_u^3$. Note that the electron frequency dimensions a and b cancel, the electron frequency is therefore a dimensionless value and so may be classed as a natural unit - as its numerical value does not depend on the system of units used (see electron as magnetic monopole).

$$\sigma_u = \frac{\pi^2}{3\alpha^2 A_u l_u} = \frac{\pi^2\Omega^3}{24\alpha l_u c_u^3} = \frac{a^2}{2^7 3\pi^3\alpha\Omega^5} \tag{8}$$

$$E_\sigma = t_u\sigma_u^3 = \frac{2\pi}{(2^7 3\pi^3\alpha\Omega^5)^3} = \frac{1}{2^{20}3^3\pi^8\alpha^3\Omega^{15}} \tag{9}$$

To convert from natural units to Planck units we can use c and μ_0 (eq.28) which have exact values to obtain the required numerical values for a and b;

$$b^{14} = \frac{5^7\alpha}{64\pi^6\Omega^4} \tag{10}$$

$$a = \frac{b^3 c}{2\pi\Omega^2} \tag{11}$$

a = 19 690 340.470 955
b = 1.184 645 784 117

Such that:

$c = c_u$ (m/s)
Planck mass: $m_P = m_u$ (kg)
Planck length: $l_p = l_u$ (m)
Planck time: $t_p = t_u$ (s)
(Planck) Ampere: $A_Q = A_u$ (A)

3 Omega as a Planck unit

The Planck analog of Ω is the square root of Planck momentum denoted here Q. Planck momentum then becomes $m_P c = 2\pi Q^2$ [5].

$$Q = \Omega \frac{1}{b^4} = 1.019\ 113\ 411... \sqrt{\frac{kg.m}{s}} \tag{12}$$

4 Mass constants

Defining the mass constants in terms of Planck momentum instead of Planck mass;

$$m_P = \frac{2\pi Q^2}{c} \tag{13}$$

$$G = \frac{l_p c^3}{2\pi Q^2} \tag{14}$$

$$h = 2\pi Q^2 2\pi l_p \tag{15}$$

$$t_p = \frac{2 l_p}{c} \tag{16}$$

$$F_p = \frac{E_p}{l_p} = \frac{2\pi Q^2}{t_p} \tag{17}$$

5 Ampere A_Q

(Proposed) Ampere A_Q [5]

$$A_Q = \frac{8c^3}{\alpha Q^3} \tag{18}$$

Note:

Planck Temperature = $A_Q c$
Elementary charge = $A_Q t_p$
Magnetic monopole (quark) = $A_Q c t_p = A_Q l_p$
Electron = $t_p (A_Q l_p)^3$
Magneton = $A_Q l_p^2$

6 Elementary charge

$$e = A.s = A_Q t_p$$

$$e = \frac{8c^3}{\alpha Q^3} . \frac{2 l_p}{c} = \frac{16 l_p c^2}{\alpha Q^3} \tag{19}$$

7 Vacuum permeability

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to exactly 2.10^{-7} newton per meter of length.

$$\frac{F_{electric}}{A_Q^2} = \frac{2\pi Q^2}{\alpha t_p} . (\frac{\alpha Q^3}{8c^3})^2 = \frac{\pi \alpha Q^8}{64 l_p c^5} = \frac{2}{10^7} \tag{20}$$

gives:

$$\mu_0 = \frac{\pi^2 \alpha Q^8}{32 l_p c^5} = \frac{4\pi}{10^7} \tag{21}$$

$$\epsilon_0 = \frac{32 l_p c^3}{\pi^2 \alpha Q^8} \tag{22}$$

$$k_e = \frac{\pi \alpha Q^8}{128 l_p c^3} \tag{23}$$

$$\alpha = \frac{2h}{\mu_0 . e^2 c} = 2.2\pi Q^2 .2\pi l_p . \frac{32 l_p c^5}{\pi^2 \alpha Q^8} . \frac{\alpha^2 Q^6}{256 l_p^2 c^4} . \frac{1}{c} = \alpha \tag{24}$$

$$\mu_0 \epsilon_0 = \frac{\pi^2 \alpha Q^8}{32 l_p c^5} . \frac{32 l_p c^3}{\pi^2 \alpha Q^8} = \frac{1}{c^2} \tag{25}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \tag{26}$$

8 Planck length l_p

l_p in terms of $Q,\ \alpha,\ c$.
The magnetic constant μ_0 has a fixed value. From eq.21

$$l_p = \frac{\pi^2 \alpha Q^8}{2^7 \mu_0 c^5} \tag{27}$$

$$\mu_0 = 4\pi.10^{-7}\ N/A^2$$

$$l_p = \frac{5^7 \pi \alpha Q^8}{c^5} \tag{28}$$

9 Electron as magnetic monopole

The ampere-meter is the SI unit for pole strength (the product of charge and velocity) in a magnet ($Am = ec$). A Magnetic monopole [10] is a hypothetical particle that is a magnet with only 1 pole.

To convert Planck time t_p, elementary charge e and the speed of light c to SI units $1s, 1C, 1m/s$ requires dimensionless numbers which are numerically equivalent (t_x, e_x, c_x).

$$\frac{t_p}{t_x} = \frac{5.3912...e^{-44} s}{5.3912...e^{-44}} = 1s$$

$$\frac{e}{e_x} = \frac{1.6021764...e^{-19}C}{1.6021764...e^{-19}} = 1C$$
$$\frac{c}{c_x} = \frac{299792458m/s}{299792458} = 1m/s$$

We may thus form dimensionless formulas for a magnetic monopole σ_e and the electron frequency $E_\sigma = t_u\sigma_u^3 = t_x\sigma_e^3$. The electron frequency formula describes the number of units of (Planck) time corresponding to the electron frequency.

$$\sigma_e = \frac{2\pi^2}{3\alpha^2 e_x c_x} = \frac{\pi^2 Q_x^3}{24\alpha l_x c_x^3} \quad (29)$$

$$E_\sigma = t_u\sigma_u^3 = t_x\sigma_e^3 = \frac{1}{2^{20}3^3\pi^8\alpha^3\Omega^{15}} = \frac{\pi^6 Q_x^9}{2^8 3^3\alpha^3 l_x^2 c_x^{10}} \quad (30)$$

Planck mass:

$$m_e = m_P E_\sigma \quad (31)$$

Compton wavelength:

$$\lambda_e = \frac{2\pi l_p}{E_\sigma} \quad (32)$$

Frequency:

$$T_e = \frac{2\pi l_p}{E_\sigma c} = \frac{t_p}{E_\sigma} = \frac{1}{\sigma_e^3}.\frac{t_p}{t_x} \quad (33)$$

Gravitation coupling constant:

$$\alpha_G = (\frac{m_P E_\sigma}{m_P})^2 = E_\sigma^2 \quad (34)$$

para-positronium lifetime:

$$t_0 = \frac{\alpha^5}{\sigma_e^3}.\frac{t_p}{t_x} \quad (35)$$

ortho-positronium lifetime:

$$t_1 = \frac{9\pi\alpha^6}{4\sigma_e^3.(\pi^2 - 9)}.\frac{t_p}{t_x} \quad (36)$$

Up-quark: σ^2
Down quark: σ^{-1}
i.e. UUD: $\sigma^2\ \sigma^2\ \sigma^{-1} = \sigma^3$

10 Reduced formulas

Replacing l_p with eq.27, the natural constants reduce to;

$$h = \frac{2^2 5^7\pi^3\alpha Q^{10}}{c^5} \quad (37)$$

$$e = \frac{2^4 5^7\pi Q^5}{c^3} \quad (38)$$

$$m_e = m_P\ \frac{\pi^4}{2^8 3^3 5^{14}\alpha^5 Q_x^7} \quad (39)$$

The Rydberg constant R_∞

$$R_\infty = \frac{m_e e^4\mu_0^2 c^3}{8h^3} = \frac{\pi^2 c^5}{2^{10}3^3 5^{21}\alpha^8 Q^8 Q_x^7} \quad (40)$$

11 Quintessence momentum

The Rydberg constant R_∞ = 10973731.568 539(55) [11] with a 12-digit precision is the most accurate of the natural constants. consequently we may re-define Q in terms of this constant, c and α.

$$Q^{15} = \frac{\pi^2 c^5}{2^{10}3^3 5^{21}\alpha^8 R_\infty} \quad (41)$$

$$Q = (\frac{\pi^2 c^5}{2^{10}3^3 5^{21}\alpha^8 R_\infty})^{\frac{1}{15}} \quad (42)$$

12 Fine structure constant alpha

The CODATA α = 137.035999074, however this value depends on certain QED assumptions. Here I calculate a value for alpha using the Rydberg constant as this is the most precisely measured fundamental constant.

Combining R_∞ = 10973731.568539 (the CODATA mean value) with electron frequencies $f_e = t_u\sigma_u^3$ and $f_e = t_x\sigma_e^3$ gives a solution for α and Q as;

α = 137.035 996 368(2)
Q = 1.019 113 421 977

Experimental results:
The von Klitzing constant reduces to α and c and so has the potential to provide the most definitive solution for α.

$$R_K = \frac{h}{e^2} = \frac{\pi\alpha c}{5000000} \quad (43)$$

R_K = 25812.807 557(18) [4]

α = 137.035 999 677(96)

Atom-recoil measurements:
$h/R_b = 4.5913592729e - 9$amu
$87R_b$ = 86.909180527amu
m_e = 0.000548579909067amu
$\alpha = 1/\sqrt{((2R_\infty/c)(h/R_b)(87R_b/m_e))}$
α = 137.035 998 998

h/neutron ratio [7]
$h/m_n = 3.956033332e - 7$amu
m_n = 1.00866491600amu
$\alpha = 1/\sqrt{((2R_\infty/c)(h/m_n)(m_n/m_e))}$
α = 137.036 010 857

13 Planck Temperature T_P

T_P in terms of $Q,\ \alpha,\ c$.

$$T_P = \frac{8c^4}{\pi\alpha Q^3} = \frac{A_Q c}{\pi}; units = K \quad (44)$$

$$T_P = Am/s$$

Boltzmann's constant k_B [9]

$$k_B = \frac{E_p}{T_P} = \frac{\pi^2 \alpha Q^5}{4c^3} = \frac{\pi m_P c}{A_Q}; units = J/K \tag{45}$$

Stefan-Boltzmann constant σ [6]

$$\sigma = \frac{2\pi^5 k_B^4}{15h^3c^2} = \frac{2\pi^2 m_P}{15t_p^3 T_P^4} \tag{46}$$

Wien's constant b (w = 4.965114231744276...)

$$b = \frac{2\pi l_p T_P}{w} = \frac{2l_p A_Q c}{w} \tag{47}$$

$$b = Am^2/s$$

14 Planck mass black hole

Black hole energy distribution of emission T_{mP} as described by Planck's law for M = m_P

$$T_{mP} = \frac{hc^3}{16\pi^2 G k_B m} = \frac{T_P}{8\pi} \tag{48}$$

Hawking radiation has a blackbody (Planck) spectrum with a temperature T for an M = m_P black hole ($r_s = 2l_p$) given by

$$k_B t_{mP} = \frac{hc}{8\pi^2 r_s} = \frac{E_p}{8\pi} \tag{49}$$

From eq.45 and eq.48

$$k_B t_{mP} = \frac{\pi^2 \alpha Q^5}{4c^3} \frac{T_P}{8\pi} = \frac{E_p}{8\pi} \tag{50}$$

General relativity

$$\frac{c^4}{8\pi G} = \frac{F_p}{8\pi} \tag{51}$$

Bekenstein–Hawking entropy (S) for M = m_P

$$A = \frac{16\pi G^2 m_P^2}{c^4} \tag{52}$$

$$S = \frac{2\pi k_B c^3 A}{4Gh} = 4\pi k_B \tag{53}$$

15 Bohr magneton

$$\mu = \frac{ehn}{4\pi m_e} = \frac{8m_P l_p^2 c^3}{\alpha Q^3 m_e} = \frac{A_Q l_p^2}{E_\sigma} = \frac{A_Q l_p c}{\sigma_e^3}.\frac{t_p}{t_x} \tag{54}$$

$$\mu = Am^2$$

16 Radio wave

$$B_{1Tesla} = \frac{l_x^2 c_x^2 Q^5}{l_p^2 c^2 Q_x^5} \tag{55}$$

$$\mu_B = \frac{eh}{4\pi m_e} \tag{56}$$

Larmor precession frequency (1 Tesla) f_L = 28.025GHz, k_{Bx} is the dimensionless Boltzmanns constant and γ is the electron magnetic moment γ = 1.001 159 652 180 73(28).

$$f_L = \frac{\gamma 2\mu_B B_{1Tesla}}{h} = \frac{\gamma 4 l_x^2 c_x^2 m_P c}{\pi^2 \alpha l_p Q_x^5 m_e} = \frac{\gamma c_x}{2k_{Bx}\sigma_e^3}.\frac{t_x}{t_p} \tag{57}$$

The presence of the Boltzmann's constant is a consequence of the SI unit 1 Tesla. A Planck B_P (R_P = Planck Rydberg)

$$B_P = \frac{m_P}{\alpha^2 A_Q t_p^2} \tag{58}$$

$$f_P = \frac{2\mu_B B_P}{h} = \frac{1}{2\pi\alpha^2 t_p} = \frac{c}{4\pi\alpha^2 l_p} = R_P c \tag{59}$$

A Planck electron B_e (R_∞ = Rydberg constant)

$$B_e = \frac{m_P m_e^2}{\alpha^2 A_Q t_p^2 m_P^2} = \frac{m_P t_x^2 \sigma_e^3}{\alpha^2 A_Q t_p^2} \tag{60}$$

$$f_e = \frac{2\mu_B B_e}{h} = \frac{m_e c}{4\pi\alpha^2 l_p m_P} = R_\infty c \tag{61}$$

17 Dimensional analysis of SI units

It is more accurate (although more complicated) to replace a and b with unitary dimensions T and P. We then see that of the dimensions that we have chosen to describe our universe (mass, length, time, charge...), only time is a fundamental property, the rest are composite units;

$$T = \frac{1}{a^3},\ unit = \sqrt{(time)} \tag{62}$$

$$P = \frac{1}{b^4},\ unit = \sqrt{(momentum)} \tag{63}$$

$$L = T^{(5/3)} P^{(3/4)},\ unit = length \tag{64}$$

$$M = T^{(1/3)} P^{(5/4)},\ unit = mass \tag{65}$$

such that;

$$t_u = 2\pi.T^2 \tag{66}$$

$$c_u = 2\pi\Omega^2.\frac{P^{(3/4)}}{T^{(1/3)}} \tag{67}$$

$$l_u = 2\pi^2\Omega^2.T^{(5/3)} P^{(3/4)} \tag{68}$$

$$m_u = T^{(1/3)} P^{(5/4)} \tag{69}$$

$$A_u = \frac{8c_u^3}{\alpha\Omega^3} = \frac{2^6\pi^3\Omega^3}{\alpha}.\frac{1}{TP^{(3/4)}} \tag{70}$$

$$T_{p-u} = \frac{A_u c_u}{\pi} = \frac{2^7\pi^3\Omega^5}{\alpha}.\frac{1}{T^{(4/3)}} \tag{71}$$

$$\mu_{0-u} = \frac{\pi^2\alpha Q_u^8}{32 l_u c_u^5} = \frac{\alpha}{2^{11}\pi^5\Omega^4}.P^{(7/2)} \tag{72}$$

$$e_u = \frac{16 l_u c_u^2}{\alpha Q_u^3} = \frac{2^7\pi^4\Omega^3}{\alpha}.\frac{T}{P^{(3/4)}} \tag{73}$$

$$h_u = 2\pi Q_u^2 2\pi l_u = 8\pi^4\Omega^4.T^{(5/3)}P^{(11/4)} \tag{74}$$

$$k_{B-u} = \frac{\pi^2\alpha Q_u^5}{4c_u^3} = \frac{\alpha}{2^5\pi\Omega}.TP^{(11/4)} \tag{75}$$

$$S_{B-u} = \frac{2\pi^2 m_u}{15 t_u^3 T_{ku}^4} = \frac{\alpha^4}{2^{30}15\pi^{13}\Omega^{20}}.\frac{P^{(5/4)}}{T^{(1/3)}} \tag{76}$$

We may then note that some values that would appear to have dimensions in terms of the SI units are in fact dimensionless. The electron frequency $f_e = s/m^3A^3 = 1$ has been studied. Other examples of dimensionless quantities;

$$\frac{kg^3 s^4}{m^6 A} = 1 \tag{77}$$

$$\frac{kg^3 s^3 A^2}{m^3} = 1 \tag{78}$$

18 Numerical solutions

Results (see online calculator [5]) agree precisely with CODATA 2010 values except for G and k_B.
CODATA 2010 values
α = 137.035 999 074(44) [13]
R_∞ = 10 973 731.568 539(55) [11]
h = 6.626 069 57(29) $e-34$ [12]
e = 1.602 176 565(35) $e-19$ [15]
m_e = 9.109 382 91(40) $e-31$ [16]
G = 6.673 84(80) $e-11$ [18]
$\mu_0 = 4.\pi/10^7$
k_B = 1.380 6488(13) $e-23$ [19]

using α = 137.035 999 074
R_∞ = 10 973 731.568 539 gives
h = 6.626 069 148 $e-34$
e = 1.602 176 513 $e-19$
m_e = 9.109 382 323 $e-31$
G = 6.672 497 199 $e-11$
k_B = 1.379 510 149 $e-23$
f_L = 28.024 953 555 GHz

using α = 137.035 996 369 gives
h = 6.626 069 715 $e-34$
e = 1.602 176 598 $e-19$
m_e = 9.109 382 742 $e-31$
G = 6.672 497 489 $e-11$
k_B = 1.379 510 194 $e-23$
f_L = 28.024 953 739 GHz

G: The same inputs were used to solve Planck constant, the electron wavelength and electron mass and so by extension the Rydberg constant; and these 4 values all have the requisite precision. We may also note that the calculated G agrees with the Sandia National Laboratories G;
Parks et al G = 6.672 34(21) $e-11$ [2]

k_B: Accuracy is within 1.000825 of CODATA 2010 value. However dividing the Larmor frequency f_L (which uses k_B) by the free electron gyromagnetic ratio $\gamma_e/(2\pi)$, the accuracy improves to 28.024954/28.024944 = 1.000000357.

19 The T-shirt

The electron frequency formula presented here is a ratio of dimensioned quantities and so 2π, Ω and α may also be so. The above further suggests that these dimensioned quantities are time and momentum, as such the dimensions we are familiar with; mass, length, charge... may actually be artificial constructs. I have previously argued [6] that time is a function (and measure) of the expansion of the universe, if so then our physical universe, from matter to the fabric of space itself, is simply the geometry of momentum [8].

...our ambition is to find an answer so elegant and simple that it will fit easily on the front of a T-shirt.
- Nobel laureate Leon Lederman

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