

# Planck unit theory: Fine structure constant alpha and sqrt of Planck momentum

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The primary constants;  $G, c, h, e, \alpha, k_B, m_e...$  range in precision from low  $G$  (4-digits) to exact values ( $c, \mu_0$ ). A principle constraint with constructing a Planck unit theory is that the Planck units are limited to the precision of  $G$  and so to 4-digits which in turn limits the usefulness of Planck theories. By postulating the sqrt of Planck momentum  $Q$  as a link between mass and charge, we can formulate the above primary constants as geometrical shapes and then define them in terms of the 4 most accurate constants  $c$  (exact), permeability of vacuum  $\mu_0$  (exact), Rydberg constant (12 digit precision) and the fine structure constant alpha  $\alpha$  (10 digit precision) giving us solutions to the Planck units whose precision is limited only by the precision of alpha. A resultant Planck unit theory emerges which suggests that particles are dimensionless formulas dictating the frequency of Planck events via a periodic (analog) electric wave-state to digital (integer) Planck-time-mass point-state oscillation. The dimensions of our universe then reduce to the 3 units (dimensions) of motion; Planck momentum, Planck time and velocity  $c$ .

## 1 Introduction

J. Barrow et al noted in a Scientific American article... 'Some things never change. Physicists call them the constants of nature. Such quantities as the velocity of light,  $c$ , Newton's constant of gravitation,  $G$ , and the mass of the electron,  $m_e$ , are assumed to be the same at all places and times in the universe. They form the scaffolding around which the theories of physics are erected, and they define the fabric of our universe. Physics has progressed by making ever more accurate measurements of their values. And yet, remarkably, no one has ever successfully predicted or explained any of the constants. Physicists have no idea why they take the special numerical values that they do. In SI units,  $c$  is 299,792,458;  $G$  is 6.673e-11; and  $m_e$  is 9.10938188e-31 -numbers that follow no discernible pattern. The only thread running through the values is that if many of them were even slightly different, complex atomic structures such as living beings would not be possible. The desire to explain the constants has been one of the driving forces behind efforts to develop a complete unified description of nature, or "theory of everything." Physicists have hoped that such a theory would show that each of the constants of nature could have only one logically possible value. It would reveal an underlying order to the seeming arbitrariness of nature.' [1]

## 2 Quintessence momentum

Planck momentum (velocity\*mass) =  $2.\pi.Q^2$  [19]

$$Q = 1.019\ 113\ 4112... \text{ units} = \sqrt{\frac{kg.m}{s}} \quad (1)$$

## 3 Mass constants

Defining in terms of Planck momentum (instead of Planck mass), the mass constants as Planck units become;

$$m_p = \frac{2.\pi.Q^2}{c} \quad (2)$$

$$G = \frac{l_p.c^3}{2.\pi.Q^2} \quad (3)$$

$$h = 2.\pi.Q^2.2.\pi.l_p \quad (4)$$

$$t_p = \frac{2.l_p}{c} \quad (5)$$

$$F_p = \frac{E_p}{l_p} = \frac{2.\pi.Q^2}{t_p} \quad (6)$$

## 4 Ampere $A_Q$

(Proposed) Ampere  $A_Q$  = velocity/mass [19]

$$A_Q = \frac{8.c^3}{\alpha.Q^3}, \text{ units} = \frac{m^2}{kg.s^2.\sqrt{(kg.m/s)}} = \left(\sqrt{\frac{m}{kg.s}}\right)^3 \quad (7)$$

Where:

$$\text{Planck Temperature} = A_Q . c$$

$$\text{Elementary charge} = A_Q . t_p$$

$$\text{Magnetic monopole (quark)} = A_Q.c.t_p = A_Q . l_p$$

$$\text{Electron} = t_p . (A_Q . l_p)^3$$

$$\text{Magnetron} = A_Q . l_p^2$$

## 5 Elementary charge

$$e = A . s = A_Q.t_p$$

$$e = \frac{8.c^3}{\alpha.Q^3} . \frac{2.l_p}{c} = \frac{16.l_p.c^2}{\alpha.Q^3}, \text{ units} = \frac{m^2}{kg.s.\sqrt{(kg.m/s)}} \quad (8)$$

## 6 Vacuum permeability

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to exactly  $2 \cdot 10^{-7}$  newton per meter of length.

$$\frac{F_{electric}}{A_Q^2} = \frac{2 \cdot \pi \cdot Q^2}{\alpha \cdot l_p} \cdot \left(\frac{\alpha \cdot Q^3}{8 \cdot c^3}\right)^2 = \frac{\pi \cdot \alpha \cdot Q^8}{64 \cdot l_p \cdot c^5} = \frac{2}{10^7} \quad (9)$$

gives:

$$\mu_0 = \frac{\pi^2 \cdot \alpha \cdot Q^8}{32 \cdot l_p \cdot c^5} = \frac{4 \cdot \pi}{10^7} \quad (10)$$

$$\epsilon_0 = \frac{32 \cdot l_p \cdot c^3}{\pi^2 \cdot \alpha \cdot Q^8} \quad (11)$$

$$k_e = \frac{\pi \cdot \alpha \cdot Q^8}{128 \cdot l_p \cdot c^3} \quad (12)$$

$$\alpha = \frac{2 \cdot h}{\mu_0 \cdot e^2 \cdot c} = 2 \cdot 2 \cdot \pi \cdot Q^2 \cdot 2 \cdot \pi \cdot l_p \cdot \frac{32 \cdot l_p \cdot c^5}{\pi^2 \cdot \alpha \cdot Q^8} \cdot \frac{\alpha^2 \cdot Q^6}{256 \cdot l_p^2 \cdot c^4} \cdot \frac{1}{c} = \alpha \quad (13)$$

$$\mu_0 \cdot \epsilon_0 = \frac{\pi^2 \cdot \alpha \cdot Q^8}{32 \cdot l_p \cdot c^5} \cdot \frac{32 \cdot l_p \cdot c^3}{\pi^2 \cdot \alpha \cdot Q^8} = \frac{1}{c^2} \quad (14)$$

$$c = \frac{1}{\sqrt{\mu_0 \cdot \epsilon_0}} = c \quad (15)$$

## 7 Planck length $l_p$

$l_p$  in terms of  $Q$ ,  $\alpha$ ,  $c$ .

The magnetic constant  $\mu_0$  has a fixed value. From eqn.10

$$l_p = \frac{\pi^2 \cdot \alpha \cdot Q^8}{2^7 \cdot \mu_0 \cdot c^5} \quad (16)$$

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \text{ N/A}^2$$

$$l_p = \frac{5^7 \cdot \pi \cdot \alpha \cdot Q^8}{c^5} \quad (17)$$

## 8 Planck Temperature $T_P$

$T_P$  in terms of  $Q$ ,  $\alpha$ ,  $c$ .

$$T_P = \frac{8 \cdot c^4}{\pi \cdot \alpha \cdot Q^3} = \frac{A_Q \cdot c}{\pi}; \text{ units} = K \quad (18)$$

$$T_P = A \cdot m/s$$

Boltzmann's constant  $k_B$  [20].

$$k_B = \frac{E_p}{T_P} = \frac{\pi^2 \cdot \alpha \cdot Q^5}{4 \cdot c^3}; \text{ units} = J/K \quad (19)$$

Stefan-Boltzmann constant  $\sigma$

$$\sigma = \frac{2 \cdot \pi^5 \cdot k_B^4}{15 \cdot h^3 \cdot c^2} = \frac{2 \cdot \pi^2 \cdot m_p}{15 \cdot t_p^3 \cdot T_P^4} \quad (20)$$

Wien's displacement constant  $b$

$$b = \frac{2 \cdot \pi \cdot l_p \cdot T_P}{5} \quad (21)$$

## 9 Planck mass black hole

Black hole energy distribution of emission  $T$  as described by Planck's law for  $M = m_p$

$$T = \frac{h \cdot c^3}{16 \cdot \pi^2 \cdot G \cdot k_B \cdot M} = \frac{T_P}{8 \cdot \pi} \quad (22)$$

Hawking radiation has a blackbody (Planck) spectrum with a temperature  $T$  for an  $M = m_p$  black hole ( $r_s = 2 \cdot l_p$ ) given by

$$k_B \cdot T = \frac{h \cdot c}{8 \cdot \pi^2 \cdot r_s} = \frac{E_p}{8 \cdot \pi} \quad (23)$$

Using eq19. and eq22.

$$k_B \cdot T = \frac{\pi^2 \cdot \alpha \cdot Q^5}{4 \cdot c^3} \cdot \frac{T_P}{8 \cdot \pi} = \frac{E_p}{8 \cdot \pi} \quad (24)$$

General relativity

$$\frac{c^4}{8 \cdot \pi \cdot G} = \frac{F_p}{8 \cdot \pi} \quad (25)$$

Bekenstein–Hawking entropy (S) for  $M = m_p$

$$A = \frac{16 \cdot \pi \cdot G^2 \cdot m_p^2}{c^4} \quad (26)$$

$$S = \frac{2 \cdot \pi \cdot k_B \cdot c^3 \cdot A}{4 \cdot G \cdot h} = 4 \cdot \pi \cdot k_B \quad (27)$$

Angular momentum  $J =$  Diracs constant

$$\frac{J}{M \cdot c} = l_p \quad (28)$$

Charged

$$r_Q^2 = \frac{e^2 \cdot G}{4 \cdot \pi \cdot \epsilon_0 \cdot c^4} = \frac{l_p^2}{\alpha} \quad (29)$$

## 10 Electron as magnetic monopole

$m_e$  in terms of  $m_p$ ,  $t_p$ ,  $\alpha$ ,  $e$ ,  $c$ . [19]

The ampere-meter is the SI unit for pole strength (the product of charge and velocity) in a magnet ( $A.m = e.c$ ). A Magnetic monopole [4] is a hypothetical particle that is a magnet with only 1 pole. Dimensionless geometrical formulas for a magnetic monopole  $\sigma_e$  and an electron  $E_\sigma$  are proposed.

$$\sigma_e = \frac{2.\pi^2}{3.\alpha^2.e_x.c_x} = \frac{\pi^2.Q_x^3}{24.\alpha.l_x.c_x^3} = A.(m/s).s = A.m \quad (30)$$

$$E_\sigma = t_x.\sigma_e^3 \quad (31)$$

nb. the conversion of Planck time  $t_p$ , elementary charge  $e$  and speed of light  $c$  to SI units  $1s, 1C, 1m/s$  requires dimensionless numbers which are numerically equivalent ( $t_x, e_x, c_x$ ).

$$\frac{t_p}{t_x} = \frac{5.3912...e^{-44}s}{5.3912...e^{-44}} = 1s$$

$$\frac{e}{e_x} = \frac{1.6021764...e^{-19}C}{1.6021764...e^{-19}} = 1C$$

$$\frac{c}{c_x} = \frac{299792458m/s}{299792458} = 1m/s$$

Planck mass:

$$m_e = m_p.E_\sigma \quad (32)$$

Compton wavelength:

$$\lambda_e = \frac{2.\pi.l_p}{E_\sigma} \quad (33)$$

Frequency:

$$T_e = \frac{2.\pi.l_p}{E_\sigma.c} = \frac{t_p}{E_\sigma} = \frac{1}{\sigma_e^3} \cdot \frac{t_p}{t_x} \quad (34)$$

Gravitation coupling constant:

$$\alpha_G = \left(\frac{m_p.E_\sigma}{m_p}\right)^2 = E_\sigma^2 \quad (35)$$

para-positronium lifetime:

$$t_0 = \frac{\alpha^5}{\sigma_e^3} \cdot \frac{t_p}{t_x} \quad (36)$$

ortho-positronium lifetime:

$$t_1 = \frac{9.\pi.\alpha^6}{4.\sigma_e^3.(\pi^2 - 9)} \cdot \frac{t_p}{t_x} \quad (37)$$

Up-quark

$$\sigma^2$$

Down quark

$$\sigma^{-1}$$

## 11 Bohr magneton

$$\mu = \frac{e.h.n}{4.\pi.m_e} = \frac{8.m_p.l_p^2.c^3}{\alpha.Q^3.m_e} = \frac{A_Q.l_p^2}{E_\sigma} = \frac{A_Q.l_p.c}{\sigma_e^3} \cdot \frac{t_p}{t_x} \quad (38)$$

$$\mu = A.m^2$$

## 12 Radio wave

$$B_{1Tesla} = \frac{l_x^2.c_x^2.Q^5}{l_p^2.c^2.Q_x^5} \quad (39)$$

$$\mu_B = \frac{e.h}{4.\pi.m_e} \quad (40)$$

Larmor precession frequency (1 Tesla)  $f_L = 28.025\text{GHz}$ ,  $k_{Bx}$  is the dimensionless Boltzmanns constant and  $\gamma$  is the electron magnetic moment  $\gamma = 1.001\ 159\ 652\ 180\ 73(28)$ .

$$f_L = \frac{\gamma.2.\mu_B.B_{1Tesla}}{h} = \frac{\gamma.4.l_x^2.c_x^2.m_p.c}{\pi^2.\alpha.l_p.Q_x^5.m_e} = \frac{\gamma.c_x}{2.k_{Bx}.\sigma_e^3} \cdot \frac{t_x}{t_p} \quad (41)$$

The presence of the Boltzmann's constant is a consequence of the SI unit 1 Tesla. A Planck  $B_p$

$$B_p = \frac{m_p}{\alpha^2.A_Q.t_p^2} \quad (42)$$

$$f_p = \frac{2.\mu_B.B_e}{h} = \frac{1}{2.\pi.\alpha^2.t_p} \quad (43)$$

A Planck electron  $B_e$

$$B_e = \frac{m_p.m_e^2}{\alpha^2.A_Q.t_p^2.m_p^2} = \frac{m_p.t_x^2.\sigma_e^3}{\alpha^2.A_Q.t_p^2} \quad (44)$$

$$f_e = \frac{2.\mu_B.B_e}{h} = \frac{\sigma_e^3}{2.\pi.\alpha^2} \cdot \frac{t_x}{t_p} = \frac{R.c.t_x}{t_p} \quad (45)$$

## 13 Reduced formulas

Replacing  $l_p$  with eqn.16, the natural constants can be reduced to  $Q$ ,  $\alpha$ ,  $c$

$$h = \frac{2^2.5^7.\pi^3.\alpha.Q^{10}}{c^5} \quad (46)$$

$$e = \frac{2^4.5^7.\pi.Q^5}{c^3} \quad (47)$$

$$m_e = m_p \cdot \frac{\pi^4}{2^8.3^3.5^{14}.\alpha^5.Q_x^7} \quad (48)$$

The Rydberg constant  $R_\infty$

$$R_\infty = \frac{m_e.e^4.\mu_0^2.c^3}{8.h^3} = \frac{\pi^2.c^5}{2^{10}.3^3.5^{21}.\alpha^8.Q^8.Q_x^7} \quad (49)$$

#### 14 von Klitzing constant

The von Klitzing constant reduces to  $\alpha$  and  $c$  and so has the potential to provide the most definitive solution for  $\alpha$ .

$$R_K = \frac{h}{e^2} = \frac{\pi \cdot \alpha \cdot c}{5000000} \quad (50)$$

$$R_K = 25812.807\ 557(18) \quad [17]$$

$$\alpha = 137.035\ 999\ 677(96)$$

#### 15 Fine structure constant alpha

The von Klitzing alpha (above) is close to the following;

$$\alpha = (17^4 + 11)/(c \cdot \phi \cdot \mu_0) = 137.035\ 999\ 768\ 293\dots$$

and James Gilson [18]

$$\alpha = 137.035\ 999\ 786\ 699\dots$$

Atom-recoil measurements:

$$h/R_b = 4.5913592729e - 9\text{amu}$$

$$87R_b = 86.909180527\text{amu}$$

$$m_e = 0.000548579909067\text{amu}$$

$$\alpha = 1/\sqrt{(2 \cdot R_\infty/c) \cdot h/R_b \cdot 87R_b/m_e}$$

$$\alpha = 137.035\ 998\ 998$$

h/neutron ratio [24]

$$h/m_n = 3.956033332e - 7\text{amu}$$

$$m_n = 1.00866491600\text{amu}$$

$$\alpha = 1/\sqrt{(2 \cdot R_\infty/c) \cdot h/m_n \cdot m_n/m_e}$$

$$\alpha = 137.036\ 010\ 857$$

#### 16 Quintessence momentum

The Rydberg constant  $R_\infty = 10\ 973\ 731.568\ 539(55)$  [5] with a 12-digit precision is the most accurate of the natural constants. consequently we may re-define  $Q$  in terms of this constant,  $c$  and  $\alpha$ .

$$Q^{15} = \frac{\pi^2 \cdot c^5}{2^{10} \cdot 3^3 \cdot 5^{21} \cdot \alpha^8 \cdot R_\infty} \quad (51)$$

$$Q = \left( \frac{\pi^2 \cdot c^5}{2^{10} \cdot 3^3 \cdot 5^{21} \cdot \alpha^8 \cdot R_\infty} \right)^{\frac{1}{15}} \quad (52)$$

#### 17 Universe frequency

We can use eqn.22 to solve the CMB cosmic microwave background temperature [22]. This then permits us to determine the universe age (in units of Planck time) when the CMB temperature reaches the lowest possible temperature and the universe can expand no further. The dimensionless universe frequency formula then becomes;

$$f_{universe} = \left( \frac{T_p}{8 \cdot \pi} \right)^4 = 0.10137\ 10^{124} \quad (53)$$

#### 18 Numerical solutions

CODATA 2010 values

$$\alpha = 137.035\ 999\ 074(44) \quad [7]$$

$$R_\infty = 10\ 973\ 731.568\ 539(55) \quad [5]$$

$$h = 6.626\ 069\ 57(29) e - 34 \quad [6]$$

$$e = 1.602\ 176\ 565(35) e - 19 \quad [9]$$

$$m_e = 9.109\ 382\ 91(40) e - 31 \quad [10]$$

$$G = 6.673\ 84(80) e - 11 \quad [12]$$

$$\mu_0 = 4 \cdot \pi / 10^7$$

$$k_B = 1.380\ 6488(13) e - 23 \quad [13]$$

using  $\alpha = 137.035\ 999\ 074$

$$R_\infty = 10\ 973\ 731.568\ 539$$

$$\mu_0 = 4 \cdot \pi / 10^7$$

gives

$$h = 6.626\ 069\ 148 e - 34$$

$$e = 1.602\ 176\ 513 e - 19$$

$$m_e = 9.109\ 382\ 323 e - 31$$

$$G = 6.672\ 497\ 199 e - 11$$

$$k_B = 1.379\ 510\ 149 e - 23$$

Results agree precisely with CODATA 2010 values except for  $G$  and  $k_B$ .

$G$ : The same inputs were used to solve Planck constant, the electron wavelength and electron mass and so by extension the Rydberg constant; and these 4 values all have the requisite precision. We may also note that the calculated  $G$  agrees with the Sandia National Laboratories  $G$ ;

$$\text{Parks et al } G = 6.672\ 34(21) e - 11 \quad [14]$$

$k_B$ : This value assumes idealized 'Planck' conditions. Accuracy is within 1.0008254 of CODATA 2010 value. However when used to solve the Larmor frequency, accuracy improves to  $28.024954/28.024944 = 1.000000357$ .

Refer to the website [19] for further details and a complete list of calculated constants.

#### 19 Summary

The 3 units of motion; Planck momentum, Planck time and  $c$  formed the mass domain. From the sqrt of Planck momentum  $Q$ ,  $c$  and alpha was formed an ampere. From this ampere, Planck time and  $c$  was formed the charge domain which includes particles and particle properties. As only the ampere formula was hypothesised, i.e.: the other formulas were all derived, and as the ampere has a simple cubic geometry, and as all results are within CODATA precision, I argue that the significance of this approach cannot be easily dismissed as numerology but rather deserves further analysis.

A universe whose dimensions are motion also suggests a Planck unit theory and a Mathematical Universe Hypothesis MUH [21].

Maple code:

```

pi := 3.1415926535897932384626 :
c := 299792458 :
a := 137.035999074 :
R := 10973731.568539 :

Q := (pi^2 * c^5 / (2^10 * 3^3 * 5^21 * a^8 * R))^(1/15) :
lp := (5^7 * pi * a * Q^8 / c^5) :
mP := (2 * pi * Q^2 / c) :
tp := 2 * lp / c :
Aq := 8 * c^3 / (a * Q^3) :

```

```

G = c^2 * lp / mP
e = 16 * lp * c^2 / (a * Q^3)
h = 2 * pi * Q^2 * 2 * pi * lp
me = mP * tp * (pi^2 * Q^3 / (24 * a * lp * c^3))^3
kB = pi^2 * a * Q^5 / (4 * c^3)

```

21. M. J. Macleod, Plato's Cave: the geometry of momentum (2013 edition)  
<http://www.platoscode.com>
22. M.J.Macleod, Time and the black hole universe  
<http://vixra.org/abs/1310.0191>
23. M.J.Macleod, Nuclear, Atomic, Molecular, Gravitational Orbitals as Anti-Photons  
<http://vixra.org/abs/1308.0140>
24. E Kruger et al 1998 Metrologia 35 203

## References

1. SciAm 06/05, P57: Constants, J Barrow, J Webb
2. [hyperphysics.phy-astr.gsu.edu/hbase/electric/elefie.html](http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefie.html)
3. <http://en.wikipedia.org/wiki/Planck-force>
4. [en.wikipedia.org/wiki/Magnetic-monopole](http://en.wikipedia.org/wiki/Magnetic-monopole)
5. <http://physics.nist.gov/cgi-bin/cuu/Value?ryd>
6. <http://physics.nist.gov/cgi-bin/cuu/Value?ha>
7. <http://physics.nist.gov/cgi-bin/cuu/Value?alphinv>
8. <http://physics.nist.gov/cgi-bin/cuu/Value?plkl>
9. <http://physics.nist.gov/cgi-bin/cuu/Value?e>
10. <http://physics.nist.gov/cgi-bin/cuu/Value?me>
11. <http://physics.nist.gov/cgi-bin/cuu/Value?mu0>
12. <http://physics.nist.gov/cgi-bin/cuu/Value?bg>
13. <http://physics.nist.gov/cgi-bin/cuu/Value?k>
14. Parks, H. V, Faller, J. E. Phys. Rev. Lett.  
<http://xxx.lanl.gov/abs/1008.3203> (2010)
15. A. M. Jeffery, R. E. Elmquist, L. H. Lee, J. Q. Shields,  
and R. F. Dziuba, IEEE Trans. Instrum. Meas. 46, 264,  
(1997).
16. [http://www.nature.com/nature/journal/v473/n7348/  
full/nature10104.html](http://www.nature.com/nature/journal/v473/n7348/full/nature10104.html)
17. A. Tzalenchuk et al, Towards a quantum resistance standard based on epitaxial graphene, Nature Nanotechnology 5, 186 - 189 (2010)
18. <http://www.fine-structure-constant.org/page5A.html>
19. <http://www.planckmomentum.com/>
20. private correspondence (Marian Gheorghe)