Notoph-Graviton-Photon Coupling

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Abstract. In the sixties Ogievetskiĭ and Polubarinov proposed the concept of *notoph*, whose helicity properties are complementary to those of *photon*. Later, Kalb and Ramond (and others) developed this theoretical concept. And, at the present times it is widely accepted. We analyze the quantum theory of antisymmetric tensor fields with taking into account mass dimensions of notoph and photon. It appears to be possible the description of both photon and notoph degrees of freedom on the basis of the modified Bargmann-Wigner formalism for the symmetric second-rank spinor.

Next, we proceed to derive equations for the symmetric tensor of the second rank on the basis of the Bargmann-Wigner formalism in a straightforward way. The symmetric multispinor of the fourth rank is used. It is constructed out of the Dirac 4-spinors. Due to serious problems with the interpretation of the results obtained on using the standard procedure we generalize it, and we obtain the spin-2 relativistic equations, which are consistent with the general relativity. The importance of the 4-vector field (and its gauge part) is pointed out.

Thus, we present the full theory which contains the photon, the notoph (the Kalb-Ramond field) and the graviton. The relations of this theory with the higher spin theories are established. In fact, we deduced the gravitational field equations from relativistic quantum mechanics. The relations of this theory with scalar-tensor theories of gravitation and f(R) are discussed. We estimate possible interactions, fermion-notoph, graviton-notoph, photon-notoph, and we conclude that they will be probably seen in experiments in the next few years.

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1. Introduction.

In the series of the papers [1, 2, 3, 4, 5], cf. with Refs. [6, 7, 8], we tried to find connection between the theory of the quantized antisymmetric tensor (AST) field of the second rank (and that of the corresponding 4-vector field) with the $2(2s+1)$ Weinberg-Tucker-Hammer formalism [9, 10].

Several previously published works [11, 12, 13, 14, 15, 16], introduced the concept of the notoph (the Kalb-Ramond field) which is constructed on the basis of the antisymmetric tensor "potentials". It represents itself the non-trivial spin-0 field. The well-known textbooks [17, 18, 19, 20] did *not* discuss the problems, whether the massless quantized AST field and the quantized 4-vector field are transverse or longitudinal fields (in the sense if the helicity $h = \pm 1$ or $h = 0$)? can the electromagnetic potential be a 4-vector in a quantized theory (cf. Ref. [9b,p.251])? how should the massless limit be taken? and many other fundamental problems of the physics of bosons. In my opinion, the most rigorous works are refs. [22, 9, 23, 21], but it is not easy to extract corresponding answers even from them.

First of all, we note that 1) "...In natural units ($c = \hbar = 1$) ... a lagrangian density, since the action is dimensionless, has dimension of [energy]^4"; 2) One can always renormalize the lagrangian density and "one can obtain the same equations of motion... by substituting $L \to (1/M^N)L$, where M is an arbitrary energy scale", cf. [2]; 3) the right physical dimension of the field strength tensor $F^{\mu\nu}$ is [energy]^2; "the transformation $F^{\mu\nu} \to (1/2M)F^{\mu\nu}$ [which was regarded in Ref. [5]] ... requires a more detailed study ... [because] the transformation above changes its physical dimension: it is not a simple normalization transformation". Furthermore, in the first papers on the notoph [12, 13, 14][1] the authors used the normalization of the 4-vector F^μ field[2] to [energy]^2 and, hence, the antisymmetric tensor "potentials" $A^{\mu\nu}$, to [energy]^1. We try to discuss these problems on the basis of the generalized Bargmann-Wigner formalism [22]. Thus, the Proca and Maxwell formalisms are generalized, see, e. g., Ref. [24].

In the Sections 3 and 4 we consider the spin-2 equations. The general scheme for derivation of higher-spin equations has been given in [22]. A field of the rest mass m and the spin $s \geq \frac{1}{2}$ is represented by a completely symmetric multispinor of rank $2s$. The particular cases $s = 1$ and $s = \frac{3}{2}$ have been considered in the textbooks, e. g., Ref. [17]. The spin-2 case can also be of some interest because we can believe that the essential features of the gravitational field are obtained from transverse components of the $(2,0) \oplus (0,2)$ representation of the Lorentz group. Nevertheless, questions of the redundant components of the higher-spin relativistic equations are not yet understood in detail [25].

In the last Sections we discuss the questions of interactions.

2. Photon-Notoph Equations.

For spin 1 we start from[3]

$$[\gamma_{\alpha\beta}p_\alpha p_\beta + Ap_\alpha p_\alpha + Bm^2]\Psi = 0\,, \tag{1}$$

where $p_\mu = -i\partial_\mu$ and $\gamma_{\alpha\beta}$ are the Barut-Muzinich-Williams covariantly - defined 6×6 matrices, $\sum_\mu \gamma_{\mu\mu} = 0$. The determinant of $[\gamma_{\alpha\beta}p_\alpha p_\beta + Ap_\alpha p_\alpha + Bm^2]$ is of the 12th order in p_μ. If we are interested in solutions with $E^2 - \mathbf{p}^2 = m^2$, $c = \hbar = 1$, they can be obtained on using the constraints in the above equation:

$$\frac{B}{A+1} = 1\,, \quad \frac{B}{A-1} = 1\,. \tag{2}$$

We may also have the tachyonic solutions, etc. The particular cases are:

- $A = 0, B = 1 \Leftrightarrow$ we have the Weinberg's equation for $s = 1$ with 3 solutions $E = +\sqrt{\mathbf{p}^2 + m^2}$, 3 solutions $E = -\sqrt{\mathbf{p}^2 + m^2}$, 3 solutions $E = +\sqrt{\mathbf{p}^2 - m^2}$ and 3 solutions $E = -\sqrt{\mathbf{p}^2 - m^2}$. Tachyonic solutions have been reformulated in various ways, for instance, as the ones leading to the spontaneous symmetry breaking, and to the non-zero quantum vacuum.
- $A = 1, B = 2 \Leftrightarrow$ we have the Tucker-Hammer equation for $s = 1$. The solutions are with $E = \pm\sqrt{\mathbf{p}^2 + m^2}$ *only*.

Thus, the addition of the Klein-Gordon equation to (3) may change the physical content even on the *free* level.

[1] It is also known as the longitudinal Kalb-Ramond field, but the consideration of Ogievetskiĭ and Polubarinov permits to study the $m \to 0$ procedure.

[2] It is well known that it is related to the third-rank antisymmetric field tensor.

[3] In the classic works on this formalism the authors worked in the Euclidean metrics. However, there is no any problem to write the equations and other formulas in the pseudo-Euclidean metrics accustomed today; just change the sign of $p_\mu p_\mu$, and other products.

What are the corresponding equations for the antisymmetric tensor field? They can be the Proca equations in the massive case, and the Maxwell equations in the massless case. We have shown in Refs. [1, 2] that one can obtain *four* different equations for antisymmetric tensor fields from the Weinberg's $2(2s+1)-$ component formalism. First of all, we note that Ψ is, in fact, bivector, $\mathbf{E}_i = -iF_{4i}$, $\mathbf{B}_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$,, or $\mathbf{E}_i = -\frac{1}{2}\epsilon_{ijk}\tilde{F}_{jk}$, $\mathbf{B}_i = -i\tilde{F}_{4i}$, or their combinations. One can separate the four cases:

- $\Psi^{(I)} = \begin{pmatrix} \mathbf{E} + i\mathbf{B} \\ \mathbf{E} - i\mathbf{B} \end{pmatrix}$, $P = -1$, where $\mathbf{E}_i$ and $\mathbf{B}_i$ are the components of the tensor.
- $\Psi^{(II)} = \begin{pmatrix} \mathbf{B} - i\mathbf{E} \\ \mathbf{B} + i\mathbf{E} \end{pmatrix}$, $P = +1$, where $\mathbf{E}_i$, $\mathbf{B}_i$ are the components of the tensor.
- $\Psi^{(III)} = \Psi^{(I)}$, but (!) $\mathbf{E}_i$ and $\mathbf{B}_i$ are the corresponding vector and axial-vector components of the *dual* tensor $\tilde{F}_{\mu\nu}$.
- $\Psi^{(IV)} = \Psi^{(II)}$, where $\mathbf{E}_i$ and $\mathbf{B}_i$ are the components of the *dual* tensor $\tilde{F}_{\mu\nu}$.

The mappings of the WTH equations are:

$$\partial_\alpha\partial_\mu F^{(I)}_{\mu\beta} - \partial_\beta\partial_\mu F^{(I)}_{\mu\alpha} + \frac{A-1}{2}\partial_\mu\partial_\mu F^{(I)}_{\alpha\beta} - \frac{B}{2}m^2 F^{(I)}_{\alpha\beta} = 0\,, \tag{3}$$

$$\partial_\alpha\partial_\mu F^{(II)}_{\mu\beta} - \partial_\beta\partial_\mu F^{(II)}_{\mu\alpha} - \frac{A+1}{2}\partial_\mu\partial_\mu F^{(II)}_{\alpha\beta} + \frac{B}{2}m^2 F^{(II)}_{\alpha\beta} = 0\,, \tag{4}$$

$$\partial_\alpha\partial_\mu \tilde{F}^{(III)}_{\mu\beta} - \partial_\beta\partial_\mu \tilde{F}^{(III)}_{\mu\alpha} - \frac{A+1}{2}\partial_\mu\partial_\mu \tilde{F}^{(III)}_{\alpha\beta} + \frac{B}{2}m^2 \tilde{F}^{(III)}_{\alpha\beta} = 0\,, \tag{5}$$

$$\partial_\alpha\partial_\mu \tilde{F}^{(IV)}_{\mu\beta} - \partial_\beta\partial_\mu \tilde{F}^{(IV)}_{\mu\alpha} + \frac{A-1}{2}\partial_\mu\partial_\mu \tilde{F}^{(IV)}_{\alpha\beta} - \frac{B}{2}m^2 \tilde{F}^{(IV)}_{\alpha\beta} = 0\,. \tag{6}$$

In the Tucker-Hammer case ($A = 1, B = 2$) we can recover the Proca theory from (**??**):

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = m^2 F_{\alpha\beta}\,, \tag{7}$$

($A_\nu = \frac{1}{m^2}\partial_\alpha F_{\alpha\nu}$ should be substituted in $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the result is multiplied by m^2).

We also noted that the massless limit of this theory does *not* coincide with the Maxwell theory in some cases, while it contains the latter as a particular case. In [3, 5, 30] we showed that it is possible to define various massless limits for the Duffin-Kemmer-Proca theory. Another one is the Ogievetskiĭ-Polubarinov *notoph* (which is also called the Kalb-Ramond field), Ref. [12] in the US literature. The transverse components of the AST field can be removed from the corresponding Lagrangian by means of the "new gauge transformation" $A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu$, with the vector gauge function Λ_μ.

The second (II) case is

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = [\partial_\mu\partial_\mu - m^2]F_{\alpha\beta}\,. \tag{8}$$

So, on the mass shell we have $[\partial_\mu\partial_\mu - m^2]F_{\alpha\beta} = 0$, and, hence,

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = 0\,, \tag{9}$$

which rather corresponds to the Maxwell-like case. However, if we calculate dispersion relations for the second case, Eq. (9), it appears that the equation has solutions even if $m \neq 0$.

Now we are interested in the *parity-violating* equations for antisymmetric tensor fields. We investigate the most general mapping of the Weinberg-Tucker-Hammer formulation to the antisymetric tensor field formulation too. Instead of $\Psi^{(I-IV)}$ we shall try to use now

$$\Psi^{(A)} = \begin{pmatrix} \mathbf{E} + i\mathbf{B} \\ \mathbf{B} + i\mathbf{E} \end{pmatrix} = \frac{1+\gamma^5}{2}\Psi^{(I)} + \frac{1-\gamma^5}{2}\Psi^{(II)} \,. \tag{10}$$

As a result, the equation for the AST fields is

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = \frac{1}{2}(\partial_\mu\partial_\mu)F_{\alpha\beta} + [-\frac{A}{2}(\partial_\mu\partial_\mu) + \frac{B}{2}m^2]\tilde{F}_{\alpha\beta} \,. \tag{11}$$

Of course, $\Psi^{(A)\prime} = \begin{pmatrix} \mathbf{B} - i\mathbf{E} \\ \mathbf{E} - i\mathbf{B} \end{pmatrix} = -i\Psi^{(A)}$, and the equation is unchanged. The different choice is

$$\Psi^{(B)} = \begin{pmatrix} \mathbf{E} + i\mathbf{B} \\ -\mathbf{B} - i\mathbf{E} \end{pmatrix} = \frac{1+\gamma^5}{2}\Psi^{(I)} - \frac{1-\gamma^5}{2}\Psi^{(II)} \,. \tag{12}$$

Thus, one has

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = \frac{1}{2}(\partial_\mu\partial_\mu)F_{\alpha\beta} + [\frac{A}{2}(\partial_\mu\partial_\mu) - \frac{B}{2}m^2]\tilde{F}_{\alpha\beta} \,. \tag{13}$$

Of course, one can also use the dual tensor ($\mathbf{E}^i = -\frac{1}{2}\epsilon_{ijk}\tilde{F}_{jk}$ and $\mathbf{B}^i = -i\tilde{F}_{4i}$) and obtain analogous equations:

$$\partial_\alpha\partial_\mu \tilde{F}_{\mu\beta} - \partial_\beta\partial_\mu \tilde{F}_{\mu\alpha} = \frac{1}{2}(\partial_\mu\partial_\mu)\tilde{F}_{\alpha\beta} + [-\frac{A}{2}(\partial_\mu\partial_\mu) + \frac{B}{2}m^2]F_{\alpha\beta} \,, \tag{14}$$

$$\partial_\alpha\partial_\mu \tilde{F}_{\mu\beta} - \partial_\beta\partial_\mu \tilde{F}_{\mu\alpha} = \frac{1}{2}(\partial_\mu\partial_\mu)\tilde{F}_{\alpha\beta} + [\frac{A}{2}(\partial_\mu\partial_\mu) - \frac{B}{2}m^2]F_{\alpha\beta} \,. \tag{15}$$

They are connected with (11,13) by the dual transformations.

The states corresponding to the new functions $\Psi^{(A)}$, $\Psi^{(B)}$ etc are *not* the parity eigenstates. So, it is not surprising that we have $F_{\alpha\beta}$ and its dual $\tilde{F}_{\alpha\beta}$ in the same equations. In total we have already eight equations.

One can also consider the most general case

$$\Psi^{(W)} = \begin{pmatrix} aF_{4i} + b\tilde{F}_{4i} + c\epsilon_{ijk}F_{jk} + d\epsilon_{ijk}\tilde{F}_{jk} \\ eF_{4i} + f\tilde{F}_{4i} + g\epsilon_{ijk}F_{jk} + h\epsilon_{ijk}\tilde{F}_{jk} \end{pmatrix} \,. \tag{16}$$

So, we shall have dynamical equations for $F_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$ with additional parameters $a, b, c, d, \ldots \in \mathbf{C}$. We have a lot of antisymmetric tensor fields here. However,

- the covariant form preserves if there are some restrictions on the parameters, only. Alternatively, we have some additional terms of ∂_4^2 or ∇^2;
- both $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are present in the equations;
- under the definite choice of $a, b, c, d\ldots$ the equations can be reduced to the above equations for the tensor $\mathcal{H}_{\mu\nu}$ and its dual:

$$\mathcal{H}_{\mu\nu} = c_1 F_{\mu\nu} + c_2\tilde{F}_{\mu\nu} + \frac{c_3}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} + \frac{c_4}{2}\epsilon_{\mu\nu\alpha\beta}\tilde{F}_{\alpha\beta} \,; \tag{17}$$

- the parity properties of $\Psi^{(W)}$ are very complicated.

Anther way of constructing the equations of high-spin particles has been given in [22, 17].[4] Bargmann and Wigner claimed explicitly that they constructed $(2s+1)$ states.[5] Below we present the standard Bargmann-Wigner formalism for the spin $s = 1$ (and turn to the standard pseudo-Euclidean metric):

$$[i\gamma^\mu \partial_\mu - m]_{\alpha\beta} \Psi_{\beta\gamma} = 0 \,, \tag{18}$$

$$[i\gamma^\mu \partial_\mu - m]_{\gamma\beta} \Psi_{\alpha\beta} = 0 \,, \tag{19}$$

If one has

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta} A_\mu + (\sigma^{\mu\nu} R)_{\alpha\beta} F_{\mu\nu} \,, \tag{20}$$

with[6]

$$R = e^{i\varphi} \begin{pmatrix} \Theta & 0 \\ 0 & -\Theta \end{pmatrix} \quad \Theta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{25}$$

in the spinorial representation of γ-matrices, we obtain the Duffin-Kemmer-Proca equations:

$$\partial^\alpha F_{\alpha\mu} = \frac{m}{2} A_\mu \,, \tag{26}$$

$$2m F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{27}$$

In order to obtain these equations one should add the equations (18,19) and compare functional coefficients at the corresponding commutators, see Ref. [17]. After the corresponding renormalization $A_\mu \to 2m A_\mu$ (or $F^{\mu\nu} \to (1/2m) F^{\mu\nu}$), we obtain the standard textbook set:

$$\partial^\alpha F_{\alpha\mu} = m^2 A_\mu \,, \tag{28}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{29}$$

It gives the equation (7) for the antisymmetric tensor field. Of course, one can investigate other sets of equations with different normalization of the $F_{\mu\nu}$ and A_μ fields. Are all these sets of equations equivalent? As we see, to answer this question is not trivial. It was argued that the physical normalization is such that in the massless limit the zero-momentum field functions should vanish in the momentum representation (there are no massless particles at rest). Moreover, we advocate the following approach: the massless limit can and must be taken in the end of all calculations only, i. e., for physical quantities.

How can one obtain other equations following from the Weinberg-Tucker-Hammer approach? The recipe for the third equation is simple: use, instead of $(\sigma^{\mu\nu} R) F_{\mu\nu}$, another symmetric matrix $(\gamma^5 \sigma^{\mu\nu} R) F_{\mu\nu}$.

[4] On can also obtain the $s = 0$ Kemmer equations on using the Bargmann-Wigner procedure. One should use the *antisymmetric* second-rank multispinor in this case.

[5] The Weinberg-Tucker-Hammer theory has essentially $2(2s+1)$ components.

[6] The reflection operator R has the properties

$$R^T = -R \,, \quad R^\dagger = R = R^{-1} \,, \tag{21}$$

$$R^{-1} \gamma^5 R = (\gamma^5)^T \,, \tag{22}$$

$$R^{-1} \gamma^\mu R = -(\gamma^\mu)^T \,, \tag{23}$$

$$R^{-1} \sigma^{\mu\nu} R = -(\sigma^{\mu\nu})^T \,. \tag{24}$$

After taking into account the above observations let us repeat the procedure of derivation of the Proca equations from the Bargmann-Wigner equations for a *symmetric* second-rank spinor. However, we now use

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(c_a m A_\mu + c_f F_\mu) + (\sigma^{\mu\nu} R)_{\alpha\rho}(c_A m (\gamma^5)_{\rho\beta} A_{\mu\nu} + c_F I_{\rho\beta} F_{\mu\nu})\,, \tag{30}$$

with the same R and Θ as above. Matrices γ^μ are again chosen in the Weyl (spinorial) representation, i.e., γ^5 is assumed to be diagonal. Constants c_i are some numerical dimensionless coefficients. The properties of the reflection operator R are necessary for the expansion (30) to be possible in such a form, i.e., in order to have the $\gamma^\mu R$, $\sigma^{\mu\nu} R$ and $\gamma^5\sigma^{\mu\nu} R$ to be *symmetric* matrices.

The substitution of the above expansion into the Bargmann-Wigner equations, Ref. [17], gives us the new Proca-like equations:

$$c_a m(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_f(\partial_\mu F_\nu - \partial_\nu F_\mu) = i c_A m^2 \epsilon_{\alpha\beta\mu\nu} A^{\alpha\beta} + 2 m c_F F_{\mu\nu}\,, \tag{31}$$

$$c_a m^2 A_\mu + c_f m F_\mu = i c_A m \epsilon_{\mu\nu\alpha\beta} \partial^\nu A^{\alpha\beta} + 2 c_F \partial^\nu F_{\mu\nu}\,. \tag{32}$$

In the case $c_a = 1$, $c_F = \frac{1}{2}$ and $c_f = c_A = 0$ they are reduced to the ordinary Proca equations.[7] In the general case we obtain dynamical equations which connect the photon, the notoph and their potentials. The divergent (in $m \to 0$) parts of field functions and those of dynamical variables should be removed by corresponding gauge (or Kalb-Ramond gauge) transformations. It is well known that the notoph massless field is considered to be the pure longitudinal field after one takes into account $\partial_\mu A^{\mu\nu} = 0$. Apart from these dynamical equations we can obtain a number of constraints by means of the subtraction of the equations of the Bargmann-Wigner system (instead of the addition as for (31,32)). They read

$$m c_a \partial^\mu A_\mu + c_f \partial^\mu f_\mu = 0\,, \tag{33}$$

$$m c_A \partial^\alpha A_{\alpha\mu} + \frac{i}{2} c_F \epsilon_{\alpha\beta\nu\mu} \partial^\alpha F^{\beta\nu} = 0, \tag{34}$$

that suggests $\widetilde{F}^{\mu\nu} \sim i m A^{\mu\nu}$ and $f^\mu \sim m A^\mu$, as in [12].

Thus, after the suitable choice of the dimensionless coefficients c_i the Lagrangian density for the photon-notoph field can be proposed:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}^{Proca} + \mathcal{L}^{Notoph} = -\frac{1}{8} F_\mu F^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \\ &+ \frac{m^2}{2} A_\mu A^\mu + \frac{m^2}{4} A_{\mu\nu} A^{\mu\nu}\,,\end{aligned} \tag{35}$$

The limit $m \to 0$ may be taken for dynamical variables, in the end of calculations only.

Furthermore, it is logical to introduce the normalization scalar field $\varphi(x)$, and consider the expansion:

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(\varphi A_\mu) + (\sigma^{\mu\nu} R)_{\alpha\beta} F_{\mu\nu}\,. \tag{36}$$

Then, we arrive at the following set

$$2 m F_{\mu\nu} = \varphi(\partial_\mu A_\nu - \partial_\nu A_\mu) + (\partial_\mu \varphi) A_\nu - (\partial_\nu \varphi) A_\mu\,, \tag{37}$$

$$\partial^\nu F_{\mu\nu} = \frac{m}{2}(\varphi A_\mu)\,, \tag{38}$$

[7] We still note that the division by m in the first equation is *not* the well-defined operation in the case if someone is interested in the subsequent limiting procedure $m \to 0$. Probably, in order to avoid this obscure point one may wish to write the Dirac equations in the form $[(i\gamma^\mu \partial_\mu)/m - I]\,\psi(x) = 0$, which follows straightforwardly in the derivation of the Dirac equation on the basis of the Ryder relation [7] and the Wigner rules for the boosts of the field functions from the zero-momentum frame.

which in the case of the constant scalar field $\varphi = 2m$ can also be reduced to the system of the Proca equations. The additional constraints are

$$(\partial^\mu \varphi) A_\mu + \varphi(\partial^\mu A_\mu) = 0\,, \tag{39}$$

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0\,. \tag{40}$$

At the moment it is not yet obvious, how can we account for other equations in the $(1,0)\oplus(0,1)$ representation, e.g. [7b], rigorously. For instance, one can wish to seek the generalization of the Proca equations on the basis of the introduction of two mass parameters m_1 and m_2. But, when we apply the BW procedure to the Dirac equations we cannot obtain new physical content. Another equation in the $(1/2,0)\oplus(0,1/2)$ representation was discussed in Ref. [26]. It has the form:

$$\left[i\gamma^\mu \partial_\mu - m_1 - \gamma^5 m_2\right]\Psi(x) = 0\,. \tag{41}$$

The Bargmann-Wigner procedure for this system of equations (which include the γ^5 matrix in the mass term) yields:

$$.\quad 2m_1 F^{\mu\nu} + 2im_2 \widetilde{F}^{\mu\nu} = \varphi(\partial^\mu A^\nu - \partial^\nu A^\mu) + (\partial^\mu\varphi)A^\nu - (\partial^\nu \varphi)A^\mu\,, \tag{42}$$

$$\partial^\nu F_{\mu\nu} = \frac{m_1}{2}(\varphi A_\mu), \tag{43}$$

with the constraints

$$(\partial^\mu \varphi) A_\mu + \varphi(\partial^\mu A_\mu) = 0\,, \tag{44}$$

$$\partial^\nu \widetilde{F}_{\mu\nu} = \frac{im_2}{2}(\varphi A_\mu)\,. \tag{45}$$

In general, we can now use the four different mass parameters in the equations which are analogous to (18,19). However, the equality of mass factors[8] ($m_1^{(1)} = m_1^{(2)}$ and $m_2^{(1)} = m_2^{(2)}$) is obtained as necessary conditions in the process of calculations in the system of the Dirac-like equations.

In fact, the results of this paper develop the old results of Ref. [12]. According to [12, Eqs.(9,10)] we proceed in constructing the "potentials" for the notoph as follows:[9]

$$A_{\mu\nu}(\mathbf{p}) = N\left[\epsilon_\mu^{(1)}(\mathbf{p})\epsilon_\nu^{(2)}(\mathbf{p}) - \epsilon_\nu^{(1)}(\mathbf{p})\epsilon_\mu^{(2)}(\mathbf{p})\right]\,. \tag{46}$$

We use explicit forms for the polarization vectors (e.g., Refs. [21] and [5, formulas(15a,b)]) boosted to the momentum $\mathbf{p}$:

$$\epsilon^\mu(\mathbf{0},+1) = -\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix}, \epsilon^\mu(\mathbf{0},0) = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \epsilon^\mu(\mathbf{0},-1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}\,, \tag{47}$$

and ($\widehat{p}_i = p_i/|\mathbf{p}|,\ \gamma = E_p/m$), Ref. [21, p.68] or Ref. [19, p.108],

$$\epsilon^\mu(\mathbf{p},\sigma) = L^\mu{}_\nu(\mathbf{p})\epsilon^\nu(\mathbf{0},\sigma)\,, \tag{48}$$

$$L^0{}_0(\mathbf{p}) = \gamma\,,\quad L^i{}_0(\mathbf{p}) = L^0{}_i(\mathbf{p}) = \widehat{p}_i\sqrt{\gamma^2-1}\,, \tag{49}$$

$$L^i{}_k(\mathbf{p}) = \delta_{ik} + (\gamma-1)\widehat{p}_i\widehat{p}_k\,. \tag{50}$$

[8] Here, the superscripts (1) and (2) refers to the first and the second equations, respectively, in the modified Bargmann-Wigner system.

[9] The notation is that of Ref. [12] here.

N, the normalization factor, should be taken into account for possible analyses of propagators and massless limits. After substitutions in the definition (46) one obtains

$$A^{\mu\nu}(\mathbf{p}) = \frac{iN^2}{m}\begin{pmatrix} 0 & -p_2 & p_1 & 0 \\ p_2 & 0 & m+\frac{p_r p_l}{p_0+m} & \frac{p_2 p_3}{p_0+m} \\ -p_1 & -m-\frac{p_r p_l}{p_0+m} & 0 & -\frac{p_1 p_3}{p_0+m} \\ 0 & -\frac{p_2 p_3}{p_0+m} & \frac{p_1 p_3}{p_0+m} & 0 \end{pmatrix}, \tag{51}$$

i.e., it coincides with the longitudinal components of the antisymmetric tensor obtained in Refs. [7a,Eqs.(2.14,2.17)] and [5, Eqs.(17b,18b)] within the normalization and different forms of the spin basis. The $A_{\mu\nu}(\mathbf{p})$ potential reduces to zero in the limiting case ($m \to 0$) under appropriate choice of the normalization $N = m^\alpha$, $\alpha > 1/2$. If $N = \sqrt{m}$ this reduction of the non-transverse state occurs if a $s = 1$ particle moves along with the third axis OZ.[10] It is also useful to compare Eq. (51) with the formula (B2) in Ref. [8] in order to think about correct procedures for taking the massless limits.

Next, the Tam-Happer experiments [27] on two laser beams interaction did not find satisfactory explanation in the framework of the ordinary QED (at least, their explanation is complicated by huge technical calculations). On the other hand, in Ref. [28] a very interesting model has been proposed. It is based on gauging the Dirac field on using the coordinate-dependent parameters $\alpha_{\mu\nu}(x)$ in

$$\psi(x) \to \psi'(x') = \Omega\psi(x)\ , \quad \Omega = \exp\left[\frac{i}{2}\sigma^{\mu\nu}\alpha_{\mu\nu}(x)\right]\ , \tag{52}$$

and, thus, the second "photon" was introduced. The compensating 24-component (in general) field $B_{\mu,\nu\lambda}$ reduces to the 4-vector field as follows (the notation of [28] is used here):

$$B_{\mu,\nu\lambda} = \frac{1}{4}\epsilon_{\mu\nu\lambda\sigma} a_\sigma(x)\,. \tag{53}$$

As readily seen, after comparison of these formulas with those of Refs. [12, 13, 14], the second photon is nothing more than the Ogievetskiĭ-Polubarinov *notoph* within the normalization. Parity properties are dependent not only on the explicit forms of the momentum-space field functions of the $(1/2, 1/2)$ representation, but also on the properties of corresponding creation/annihilation operators. Helicity properties depend on the normalization.

3. The Standard Bargmann-Wigner Formalism Applied for Spin 2.

In this Section we use the commonly-accepted procedure for the derivation of higher-spin equations [22]. We begin with the equations for the 4-rank symmetric spinor:

$$\left[i\gamma^\mu\partial_\mu - m\right]_{\alpha\alpha'}\Psi_{\alpha'\beta\gamma\delta} = 0\,, \tag{54}$$

$$\left[i\gamma^\mu\partial_\mu - m\right]_{\beta\beta'}\Psi_{\alpha\beta'\gamma\delta} = 0\,, \tag{55}$$

$$\left[i\gamma^\mu\partial_\mu - m\right]_{\gamma\gamma'}\Psi_{\alpha\beta\gamma'\delta} = 0\,, \tag{56}$$

$$\left[i\gamma^\mu\partial_\mu - m\right]_{\delta\delta'}\Psi_{\alpha\beta\gamma\delta'} = 0\,. \tag{57}$$

The massless limit (if one needs) should be taken in the end of all calculations.

We proceed expanding the field function in the set of symmetric matrices (as in the spin-1 case, cf. Ref. [5]). In the beginning let us use the first two indices:[11]

$$\Psi_{\{\alpha\beta\}\gamma\delta} = (\gamma_\mu R)_{\alpha\beta}\Psi^\mu_{\gamma\delta} + (\sigma_{\mu\nu}R)_{\alpha\beta}\Psi^{\mu\nu}_{\gamma\delta}\,. \tag{58}$$

[10] But, even in this case we cannot have a good behaviour of the 4-vector fields/potentials in the massless limit in the instant form of the relativistic dynamics, cf. [8].

[11] The matrix R can be related to the CP operation in the $(1/2, 0) \oplus (0, 1/2)$ representation.

We would like to write the corresponding equations for functions $\Psi^{\mu}_{\gamma\delta}$ and $\Psi^{\mu\nu}_{\gamma\delta}$ in the form:

$$\frac{2}{m}\partial_\mu \Psi^{\mu\nu}_{\gamma\delta} = -\Psi^{\nu}_{\gamma\delta}\,, \tag{59}$$
$$\Psi^{\mu\nu}_{\gamma\delta} = \frac{1}{2m}\left[\partial^\mu \Psi^{\nu}_{\gamma\delta} - \partial^\nu \Psi^{\mu}_{\gamma\delta}\right]\,. \tag{60}$$

Constraints $(1/m)\partial_\mu \Psi^{\mu}_{\gamma\delta} = 0$ and $(1/m)\epsilon^{\mu\nu}{}_{\alpha\beta}\,\partial_\mu \Psi^{\alpha\beta}_{\gamma\delta} = 0$ can be regarded as the consequence of Eqs. (59,60).

Next, we present the vector-spinor and tensor-spinor functions as

$$\Psi^{\mu}_{\{\gamma\delta\}} = (\gamma^\kappa R)_{\gamma\delta} G_\kappa{}^{\mu} + (\sigma^{\kappa\tau} R)_{\gamma\delta} F_{\kappa\tau}{}^{\mu}\,, \tag{61}$$
$$\Psi^{\mu\nu}_{\{\gamma\delta\}} = (\gamma^\kappa R)_{\gamma\delta} T_\kappa{}^{\mu\nu} + (\sigma^{\kappa\tau} R)_{\gamma\delta} R_{\kappa\tau}{}^{\mu\nu}\,, \tag{62}$$

i. e., using the symmetric matrix coefficients in indices γ and δ. Hence, the total function is

$$\begin{aligned}\Psi_{\{\alpha\beta\}\{\gamma\delta\}} &= (\gamma_\mu R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta} G_\kappa{}^{\mu} + (\gamma_\mu R)_{\alpha\beta}(\sigma^{\kappa\tau} R)_{\gamma\delta} F_{\kappa\tau}{}^{\mu} + \\ &+ (\sigma_{\mu\nu} R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta} T_\kappa{}^{\mu\nu} + (\sigma_{\mu\nu} R)_{\alpha\beta}(\sigma^{\kappa\tau} R)_{\gamma\delta} R_{\kappa\tau}{}^{\mu\nu}\,,\end{aligned} \tag{63}$$

and the resulting tensor equations are:

$$\frac{2}{m}\partial_\mu T_\kappa{}^{\mu\nu} = -G_\kappa{}^{\nu}\,, \tag{64}$$
$$\frac{2}{m}\partial_\mu R_{\kappa\tau}{}^{\mu\nu} = -F_{\kappa\tau}{}^{\nu}\,, \tag{65}$$
$$T_\kappa{}^{\mu\nu} = \frac{1}{2m}\left[\partial^\mu G_\kappa{}^{\nu} - \partial^\nu G_\kappa{}^{\mu}\right]\,, \tag{66}$$
$$R_{\kappa\tau}{}^{\mu\nu} = \frac{1}{2m}\left[\partial^\mu F_{\kappa\tau}{}^{\nu} - \partial^\nu F_{\kappa\tau}{}^{\mu}\right]\,. \tag{67}$$

The constraints are re-written to

$$\frac{1}{m}\partial_\mu G_\kappa{}^{\mu} = 0\,, \quad \frac{1}{m}\partial_\mu F_{\kappa\tau}{}^{\mu} = 0\,, \tag{68}$$
$$\frac{1}{m}\epsilon_{\alpha\beta\nu\mu}\partial^\alpha T_\kappa{}^{\beta\nu} = 0\,, \quad \frac{1}{m}\epsilon_{\alpha\beta\nu\mu}\partial^\alpha R_{\kappa\tau}{}^{\beta\nu} = 0\,. \tag{69}$$

However, we need to make symmetrization over these two sets of indices $\{\alpha\beta\}$ and $\{\gamma\delta\}$. The total symmetry can be ensured if one contracts the function $\Psi_{\{\alpha\beta\}\{\gamma\delta\}}$ with *antisymmetric* matrices $R^{-1}_{\beta\gamma}$, $(R^{-1}\gamma^5)_{\beta\gamma}$ and $(R^{-1}\gamma^5\gamma^\lambda)_{\beta\gamma}$, and equate all these contractions to zero (similar to the $s = 3/2$ case considered in Ref. [17, p. 44]. We obtain additional constraints on the tensor field functions:

$$G_\mu{}^{\mu} = 0\,, \quad G_{[\kappa\,\mu]} = 0\,, \quad G^{\kappa\mu} = \frac{1}{2} g^{\kappa\mu} G_\nu{}^{\nu}\,, \tag{70}$$
$$F_{\kappa\mu}{}^{\mu} = F_{\mu\kappa}{}^{\mu} = 0\,, \quad \epsilon^{\kappa\tau\mu\nu} F_{\kappa\tau,\mu} = 0\,, \tag{71}$$
$$T^{\mu}{}_{\mu\kappa} = T^{\mu}{}_{\kappa\mu} = 0\,, \quad \epsilon^{\kappa\tau\mu\nu} T_{\kappa,\tau\mu} = 0\,, \tag{72}$$
$$F^{\kappa\tau,\mu} = T^{\mu,\kappa\tau}\,, \quad \epsilon^{\kappa\tau\mu\lambda}(F_{\kappa\tau,\mu} + T_{\kappa,\tau\mu}) = 0\,, \tag{73}$$
$$R_{\kappa\nu}{}^{\mu\nu} = R_{\nu\kappa}{}^{\mu\nu} = R_{\kappa\nu}{}^{\nu\mu} = R_{\nu\kappa}{}^{\nu\mu} = R_{\mu\nu}{}^{\mu\nu} = 0\,, \tag{74}$$
$$\epsilon^{\mu\nu\alpha\beta}(g_{\beta\kappa} R_{\mu\tau,\nu\alpha} - g_{\beta\tau} R_{\nu\alpha,\mu\kappa}) = 0 \quad \epsilon^{\kappa\tau\mu\nu} R_{\kappa\tau,\mu\nu} = 0\,. \tag{75}$$

Thus, we encountered with the well-known difficulty of the theory of spin-2 particles in the Minkowski space. We explicitly showed that all field functions become to be equal to zero. Such a situation cannot be considered as a satisfactory one (because it does not give us any physical information), and it can be corrected in several ways.[12]

4. The Generalized Bargmann-Wigner Formalism for Spin 2.
We shall modify the formalism in the spirit of Ref. [30]. The field function (58) is now presented as

$$\Psi_{\{\alpha\beta\}\gamma\delta} = \alpha_1(\gamma_\mu R)_{\alpha\beta}\Psi^\mu_{\gamma\delta} + \alpha_2(\sigma_{\mu\nu}R)_{\alpha\beta}\Psi^{\mu\nu}_{\gamma\delta} + \alpha_3(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}\widetilde{\Psi}^{\mu\nu}_{\gamma\delta}\,, \tag{76}$$

with

$$\Psi^\mu_{\{\gamma\delta\}} = \beta_1(\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\ \mu} + \beta_2(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu} + \beta_3(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\mu}\,, \tag{77}$$

$$\Psi^{\mu\nu}_{\{\gamma\delta\}} = \beta_4(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\ \mu\nu} + \beta_5(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu} + \beta_6(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\mu\nu}\,, \tag{78}$$

$$\widetilde{\Psi}^{\mu\nu}_{\{\gamma\delta\}} = \beta_7(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_\kappa{}^{\ \mu\nu} + \beta_8(\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\mu\nu} + \beta_9(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}D_{\kappa\tau}{}^{\mu\nu}\,. \tag{79}$$

Hence, the function $\Psi_{\{\alpha\beta\}\{\gamma\delta\}}$ can be expressed as a sum of nine terms:

$$\begin{aligned}
&\Psi_{\{\alpha\beta\}\{\gamma\delta\}} = \alpha_1\beta_1(\gamma_\mu R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\ \mu} + \alpha_1\beta_2(\gamma_\mu R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu} + \\
+&\ \alpha_1\beta_3(\gamma_\mu R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\mu} + +\alpha_2\beta_4(\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\ \mu\nu} + \\
+&\ \alpha_2\beta_5(\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu} + \alpha_2\beta_6(\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\mu\nu} + \\
+&\ \alpha_3\beta_7(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_\kappa{}^{\ \mu\nu} + \alpha_3\beta_8(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\mu\nu} + \\
+&\ \alpha_3\beta_9(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}D_{\kappa\tau}{}^{\mu\nu}\,.
\end{aligned} \tag{80}$$

The corresponding dynamical equations are given by[13]

$$\frac{2\alpha_2\beta_4}{m}\partial_\nu T_\kappa{}^{\ \mu\nu} + \frac{i\alpha_3\beta_7}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\widetilde{T}_{\kappa,\alpha\beta} = \alpha_1\beta_1 G_\kappa{}^{\ \mu}\,, \tag{81}$$

$$\begin{aligned}
&\frac{2\alpha_2\beta_5}{m}\partial_\nu R_{\kappa\tau}{}^{\mu\nu} + \frac{i\alpha_2\beta_6}{m}\epsilon_{\alpha\beta\kappa\tau}\partial_\nu\widetilde{R}^{\alpha\beta,\mu\nu} + \frac{i\alpha_3\beta_8}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\widetilde{D}_{\kappa\tau,\alpha\beta} - \\
-&\ \frac{\alpha_3\beta_9}{2}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\alpha\beta} = \alpha_1\beta_2F_{\kappa\tau}{}^{\mu} + \frac{i\alpha_1\beta_3}{2}\epsilon_{\alpha\beta\kappa\tau}\widetilde{F}^{\alpha\beta,\mu}\,,
\end{aligned} \tag{82}$$

$$2\alpha_2\beta_4T_\kappa{}^{\ \mu\nu} + i\alpha_3\beta_7\epsilon^{\alpha\beta\mu\nu}\widetilde{T}_{\kappa,\alpha\beta} = \frac{\alpha_1\beta_1}{m}(\partial^\mu G_\kappa{}^{\ \nu} - \partial^\nu G_\kappa{}^{\ \mu})\,, \tag{83}$$

$$\begin{aligned}
&2\alpha_2\beta_5R_{\kappa\tau}{}^{\mu\nu} + i\alpha_3\beta_8\epsilon^{\alpha\beta\mu\nu}\widetilde{D}_{\kappa\tau,\alpha\beta} + i\alpha_2\beta_6\epsilon_{\alpha\beta\kappa\tau}\widetilde{R}^{\alpha\beta,\mu\nu} - \frac{\alpha_3\beta_9}{2}\epsilon^{\alpha\beta\mu\nu}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\alpha\beta} = \\
=&\ \frac{\alpha_1\beta_2}{m}(\partial^\mu F_{\kappa\tau}{}^{\nu} - \partial^\nu F_{\kappa\tau}{}^{\mu}) + \frac{i\alpha_1\beta_3}{2m}\epsilon_{\alpha\beta\kappa\tau}(\partial^\mu\widetilde{F}^{\alpha\beta,\nu} - \partial^\nu\widetilde{F}^{\alpha\beta,\mu})\,.
\end{aligned} \tag{84}$$

The essential constraints are:

$$\alpha_1\beta_1G^\mu{}_{\ \mu} = 0\,, \quad \alpha_1\beta_1G_{[\kappa\mu]} = 0\,, \tag{85}$$

[12] The reader can compare our results of this Section with those of Ref. [29]. I became aware about their consideration from Dr. D. V. Ahluwalia (personal communications, May 5, 1998). I consider their discussion of the standard formalism in the Sections I and II, as insufficient.

[13] All indices in this formula are already pure vectorial and have nothing to do with previous notation. The coefficients α_i and β_i may, in general, carry some dimension.

$$2i\alpha_1\beta_2 F_{\alpha\mu}{}^{\mu} + \alpha_1\beta_3\epsilon^{\kappa\tau\mu}{}_{\alpha}\widetilde{F}_{\kappa\tau,\mu} = 0\,, \tag{86}$$

$$2i\alpha_1\beta_3 \widetilde{F}_{\alpha\mu}{}^{\mu} + \alpha_1\beta_2\epsilon^{\kappa\tau\mu}{}_{\alpha}F_{\kappa\tau,\mu} = 0\,, \tag{87}$$

$$2i\alpha_2\beta_4 T^{\mu}{}_{\mu\alpha} - \alpha_3\beta_7\epsilon^{\kappa\tau\mu}{}_{\alpha}\widetilde{T}_{\kappa,\tau\mu} = 0\,, \tag{88}$$

$$2i\alpha_3\beta_7 \widetilde{T}^{\mu}{}_{\mu\alpha} - \alpha_2\beta_4\epsilon^{\kappa\tau\mu}{}_{\alpha}T_{\kappa,\tau\mu} = 0\,, \tag{89}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_6\widetilde{R}_{\kappa\tau,\mu\nu} + \alpha_3\beta_8\widetilde{D}_{\kappa\tau,\mu\nu}\right] + 2\alpha_2\beta_5 R^{\mu\nu}{}_{\mu\nu} + 2\alpha_3\beta_9 D^{\mu\nu}{}_{\mu\nu} = 0\,, \tag{90}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_5 R_{\kappa\tau,\mu\nu} + \alpha_3\beta_9 D_{\kappa\tau,\mu\nu}\right] + 2\alpha_2\beta_6 \widetilde{R}^{\mu\nu}{}_{\mu\nu} + 2\alpha_3\beta_8 \widetilde{D}^{\mu\nu}{}_{\mu\nu} = 0\,, \tag{91}$$

$$2i\alpha_2\beta_5 R_{\beta\mu}{}^{\mu\alpha} + 2i\alpha_3\beta_9 D_{\beta\mu}{}^{\mu\alpha} + \alpha_2\beta_6\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{R}^{\lambda\mu}{}_{\mu\nu} + \alpha_3\beta_8\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{D}^{\lambda\mu}{}_{\mu\nu} = 0\,, \tag{92}$$

$$2i\alpha_1\beta_2 F^{\lambda\mu}{}_{\mu} - 2i\alpha_2\beta_4 T_{\mu}{}^{\mu\lambda} + \alpha_1\beta_3\epsilon^{\kappa\tau\mu\lambda}\widetilde{F}_{\kappa\tau,\mu} + \alpha_3\beta_7\epsilon^{\kappa\tau\mu\lambda}\widetilde{T}_{\kappa,\tau\mu} = 0\,, \tag{93}$$

$$2i\alpha_1\beta_3 \widetilde{F}^{\lambda\mu}{}_{\mu} - 2i\alpha_3\beta_7 \widetilde{T}_{\mu}{}^{\mu\lambda} + \alpha_1\beta_2\epsilon^{\kappa\tau\mu\lambda}F_{\kappa\tau,\mu} + \alpha_2\beta_4\epsilon^{\kappa\tau\mu\lambda}T_{\kappa,\tau\mu} = 0\,, \tag{94}$$

$$\begin{aligned}
&\alpha_1\beta_1(2G^{\lambda}{}_{\alpha} - g^{\lambda}{}_{\alpha}G^{\mu}{}_{\mu}) - 2\alpha_2\beta_5(2R^{\lambda\mu}{}_{\mu\alpha} + 2R_{\alpha\mu}{}^{\mu\lambda} + g^{\lambda}{}_{\alpha}R^{\mu\nu}{}_{\mu\nu}) + \\
+\;& 2\alpha_3\beta_9(2D^{\lambda\mu}{}_{\mu\alpha} + 2D_{\alpha\mu}{}^{\mu\lambda} + g^{\lambda}{}_{\alpha}D^{\mu\nu}{}_{\mu\nu}) + 2i\alpha_3\beta_8(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{D}^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}\widetilde{D}_{\kappa\tau,\mu\alpha}) - \\
-\;& 2i\alpha_2\beta_6(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{R}^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}\widetilde{R}_{\kappa\tau,\mu\alpha}) = 0\,,
\end{aligned} \tag{95}$$

$$\begin{aligned}
&2\alpha_3\beta_8(2\widetilde{D}^{\lambda\mu}{}_{\mu\alpha} + 2\widetilde{D}_{\alpha\mu}{}^{\mu\lambda} + g^{\lambda}{}_{\alpha}\widetilde{D}^{\mu\nu}{}_{\mu\nu}) - 2\alpha_2\beta_6(2\widetilde{R}^{\lambda\mu}{}_{\mu\alpha} + 2\widetilde{R}_{\alpha\mu}{}^{\mu\lambda} + \\
+\;& g^{\lambda}{}_{\alpha}\widetilde{R}^{\mu\nu}{}_{\mu\nu}) + +2i\alpha_3\beta_9(\epsilon_{\kappa\alpha}{}^{\mu\nu}D^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}D_{\kappa\tau,\mu\alpha}) - \\
-\;& 2i\alpha_2\beta_5(\epsilon_{\kappa\alpha}{}^{\mu\nu}R^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}R_{\kappa\tau,\mu\alpha}) = 0\,,
\end{aligned} \tag{96}$$

$$\begin{aligned}
&\alpha_1\beta_2(F^{\alpha\beta,\lambda} - 2F^{\beta\lambda,\alpha} + F^{\beta\mu}{}_{\mu}\,g^{\lambda\alpha} - F^{\alpha\mu}{}_{\mu}\,g^{\lambda\beta}) - \\
-\;& \alpha_2\beta_4(T^{\lambda,\alpha\beta} - 2T^{\beta,\lambda\alpha} + T_{\mu}{}^{\mu\alpha}g^{\lambda\beta} - T_{\mu}{}^{\mu\beta}g^{\lambda\alpha}) + \\
+\;& \frac{i}{2}\alpha_1\beta_3(\epsilon^{\kappa\tau\alpha\beta}\widetilde{F}_{\kappa\tau}{}^{\lambda} + 2\epsilon^{\lambda\kappa\alpha\beta}\widetilde{F}_{\kappa\mu}{}^{\mu} + 2\epsilon^{\mu\kappa\alpha\beta}\widetilde{F}^{\lambda}{}_{\kappa,\mu}) - \\
-\;& \frac{i}{2}\alpha_3\beta_7(\epsilon^{\mu\nu\alpha\beta}\widetilde{T}^{\lambda}{}_{\mu\nu} + 2\epsilon^{\nu\lambda\alpha\beta}\widetilde{T}^{\mu}{}_{\mu\nu} + 2\epsilon^{\mu\kappa\alpha\beta}\widetilde{T}_{\kappa,\mu}{}^{\lambda}) = 0\,.
\end{aligned} \tag{97}$$

They are the results of contractions of the field function (80) with six antisymmetric matrices, as above. Furthermore, one should recover the relations (70-75) in the particular case when $\alpha_3 = \beta_3 = \beta_6 = \beta_9 = 0$ and $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \beta_4 = \beta_5 = \beta_7 = \beta_8 = 1$.

As a discussion, we note that in such a framework we have physical content because only certain combinations of field functions can be equal to zero. In general, the fields $F_{\kappa\tau}{}^{\mu}$, $\widetilde{F}_{\kappa\tau}{}^{\mu}$, $T_{\kappa}{}^{\mu\nu}$, $\widetilde{T}_{\kappa}{}^{\mu\nu}$, and $R_{\kappa\tau}{}^{\mu\nu}$, $\widetilde{R}_{\kappa\tau}{}^{\mu\nu}$, $D_{\kappa\tau}{}^{\mu\nu}$, $\widetilde{D}_{\kappa\tau}{}^{\mu\nu}$ can correspond to different physical states and the equations above describe couplings one state with another.

Furthermore, from the set of equations (81-84) one obtains the *second*-order equation for the symmetric traceless tensor of the second rank ($\alpha_1 \neq 0$, $\beta_1 \neq 0$):

$$\frac{1}{m^2}\left[\partial_\nu \partial^\mu G_\kappa{}^{\nu} - \partial_\nu \partial^\nu G_\kappa{}^{\mu}\right] = G_\kappa{}^{\mu}\,. \tag{98}$$

After the contraction in indices κ and μ this equation is reduced to

$$\partial_\mu G^\mu{}_\alpha = F_\alpha\,, \tag{99}$$

$$\frac{1}{m^2}\partial_\alpha F^\alpha = 0\,, \tag{100}$$

i. e., to the equations connecting the analogue of the energy-momentum tensor and the analogue of the 4-vector potential (the additional notoph field as opposed to the Logunov theory?). As we showed in our recent work [30] the longitudinal potential may have importance in the construction of electromagnetism (see also the works on the notoph and notivarg concept [31]). Moreover, according to the Weinberg theorem [9] for massless particles it is the gauge part of the 4-vector potential $\sim \partial_\mu \chi$, which is the physical field. The case, when the longitudinal potential is equated to zero, can be considered as a particular case only. This case may be relevant to some physical situation but hardly to be considered as a basis for unification. Further investigations may provide additional foundations to "surprising" similarities of gravitational and electromagnetic equations in the low-velocity limit, Refs. [32, 33, 34, 36].

5. Interactions with Fermions.

The possibility of terms as $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ appears to be related to the matters of chiral interactions [38, 39]. As we are now convinced, the Dirac field operator can be always presented as a superposition of the self- and anti-self charge conjugate field operators (cf. Ref. [37]). The anti-self charge conjugate part can give the self charge conjugate part after multiplying by the γ^5 matrix, and *vice versa*. We derived[14]

$$[i\gamma^\mu D^*_\mu - m]\psi^s_1 = 0\,, \tag{102}$$

or[15]

$$[i\gamma^\mu D_\mu - m]\psi^a_2 = 0\,. \tag{104}$$

Both equations lead to the terms of interaction such as $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ provided that the 4-vector potential is considered as a complex function(al). In fact, from (102) we have:

$$i\sigma^\mu \nabla_\mu \chi_1 - m\phi_1 = 0\,, \tag{105}$$

$$i\tilde{\sigma}^\mu \nabla^*_\mu \phi_1 - m\chi_1 = 0\,. \tag{106}$$

And, from (104) we have

$$i\sigma^\mu \nabla^*_\mu \chi_2 - m\phi_2 = 0\,, \tag{107}$$

$$i\tilde{\sigma}^\mu \nabla_\mu \phi_2 - m\chi_2 = 0\,. \tag{108}$$

[14] The anti-self charge conjugate field function ψ_2 can also be used. The equation has then the form:

$$[i\gamma^\mu D^*_\mu + m]\psi^a_2 = 0\,. \tag{101}$$

[15] The self charge conjugate field function ψ_1 also can be used. The equation has the form:

$$[i\gamma^\mu D_\mu + m]\psi^s_1 = 0\,. \tag{103}$$

As readily seen, in the cases of alternative choices we have opposite charges in the terms of the type $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ and in the mass terms.

The meanings of σ^μ and $\tilde{\sigma}^\mu$ are obvious from the definition of γ matrices. The derivatives are defined:

$$D_\mu = \partial_\mu - ie\gamma^5 C_\mu + eB_\mu\,, \quad \nabla_\mu = \partial_\mu - ieA_\mu\,, \tag{109}$$

and $A_\mu = C_\mu + iB_\mu$. Thus, relations with the magnetic monopoles can also be established.

From the above system we extract the terms as $\pm e^2\sigma^i\sigma^j A_i A_j^*$, which lead to the discussed terms [38, 39].[16] Furthermore, one can come to the same conclusions not applying to the constraints on the creation/annihilation operators (which we have previously chosen for clarity and simplicity in Ref. [39]). It is possible to work with self/anti-self charge conjugate fields and the Majorana *anzatzen*. Thus, in the considered cases it is the γ^5 transformation which distinguishes various field configurations (helicity, self/anti-self charge conjugate properties etc) in the coordinate representation.

6. Boson Interactions.

The most general relativistic-invariant Lagrangian for the symmetric 2nd-rank tensor is

$$\begin{aligned}\mathcal{L} = &-\alpha_1(\partial^\alpha G_{\alpha\lambda})(\partial_\beta G^{\beta\lambda}) - \alpha_2(\partial_\alpha G^{\beta\lambda})(\partial^\alpha G_{\beta\lambda})\\ &- \alpha_3(\partial^\alpha G^{\beta\lambda})(\partial_\beta G_{\alpha\lambda}) + m^2 G_{\alpha\beta}G^{\alpha\beta}\,.\end{aligned} \tag{110}$$

It leads to the equation

$$\left[\alpha_2\partial^2 + m^2\right] G^{\{\mu\nu\}} + (\alpha_1+\alpha_3)\partial^{\{\mu|}(\partial_\alpha G^{\alpha|\nu\}}) = 0\,. \tag{111}$$

In the case $\alpha_2 = 1 > 0$ and $\alpha_1 + \alpha_3 = -1$ it coincides with Eq. (98). There is no any problem to obtain the dynamical invariants for the fields of the spin 2 from the above Lagrangian. The mass dimension of $G^{\mu\nu}$ is $[energy]^1$.

We now present possible relativistic interactions of the symemtric 2nd-rank tensor. They should be the following ones:

$$\mathcal{L}^{int}_{(1)} \sim G_{\mu\nu}F^\mu F^\nu\,, \tag{112}$$

$$\mathcal{L}^{int}_{(2)} \sim (\partial^\mu G_{\mu\nu})F^\nu\,, \tag{113}$$

$$\mathcal{L}^{int}_{(3)} \sim G_{\mu\nu}(\partial^\mu F^\nu)\,. \tag{114}$$

The term $\sim (\partial_\mu G^\alpha{}_\alpha)F^\mu$ vanishes due to the constraint of tracelessness. Obviously, these interactions cannot be obtained from the free Lagrangian (110) just by the covariantization of the derivative $\partial_\mu \to \partial_\mu + gF_\mu$.

It is also interesting to note that thanks to the possible terms

$$V(F) = \beta_1(F_\mu F^\mu) + \beta_2(F_\mu F^\mu)(F_\nu F^\nu) \tag{115}$$

we can give the mass to the G_{00} component of the spin-2 field. This is due to the possibility of the Higgs spontaneous symmetry breaking [40]

$$F^\mu(x) = \begin{pmatrix} v + \partial_0\chi(x)\\ g^1\\ g^2\\ g^3\end{pmatrix}\,, \tag{116}$$

[16] I am grateful to Prof. S. Esposito for the e-mail communications (1997-98) on the alternative proof of the considered interaction. We would like to note that the terms of the type $\sigma\cdot[\mathbf{A}\times\mathbf{A}^*]$ can be reduced to $(\sigma\cdot\nabla)V$, where V is the scalar potential.

with v being the vacuum expectation value, $v^2 = (F_\mu F^\mu) = -\beta_1/2\beta_2 > 0$. Other degrees of freedom of the 4-vector field are removed since they can be interpreted as the Goldstone bosons. It was stated that "for any continuous symmetry which does not preserve the ground state, there is a massless degree of freedom which decouples at low energies. This mode is called the Goldstone (or Nambu-Goldstone) particle for the symmetry". As usual, the Higgs mechanism and the Goldstone modes should be important in giving masses to the three vector bosons.[17] As one can easily see, this expression does not permit an arbitrary phase for F^μ, which is possible only if the 4-vector would be the complex one.

Next, due to the Lagrangian interaction of fermions with notoph are of the order e^2 since the beginning (as opposed to the interaction with the 4-vector potential A_μ), it is more difficult to observe it. However, as far as I know the theoretical precision calculus in QED (the Landé factor, the anomalous magnetic moment, the hyperfine splittings in positronium and muonium, and the decay rate of o-Ps and p-Ps) are about the order corresponding to the 4th-5th loops, where the difference may appear with the experiments [41, 42].

7. Conclusions.

We considered the Bargmann-Wigner formalism to derive the equations for the AST field and for the symmetric tensor of the 2nd rank. We introduced additional scalar normalization field in the Bargmann-Wigner formalism in order to take into account possible physical significance of the Ogievetskiĭ-Polubarinov–Kalb-Ramond modes. We introduced the additional symmetric matrix in the Bargmann-Wigner expansion ($\gamma^5\sigma^{\mu\nu}R$) in order to take into account the dual fields. The consideration is similar to Ref. [43].

Furthermore, we discussed the interactions of notoph, photon and graviton (and, probably, notivarg[18]). For instance, the interaction notoph-graviton may give the mass to spin-2 particles in the way which is similar to the spontaneous-symmetry-breaking Higgs formalism.

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[17] It is interesting to note the following statement (given without references in wikipedia.org): "In general, the phonon is effectively the Nambu-Goldstone boson for spontaneously broken Galilean/Lorentz symmetry. However, in contrast to the case of internal symmetry breaking, when spacetime symmetries are broken, the order parameter need not be a scalar field, but may be a tensor field, and the corresponding independent massless modes may now be fewer than the number of spontaneously broken generators, because the Goldstone modes may now be linearly dependent among themselves: e.g., the Goldstone modes for some generators might be expressed as gradients of Goldstone modes for other broken generators."

[18] In order to analize its dynamical invariants and interactions one should construct the Lagrangian from the analogs of the Riemann tensor $\widetilde{D}^{\mu\nu,\alpha\beta}$.

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