Geometry of noninertial bases in relativistic mechanics of continua and Bell's problem solution

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From obtained equations of structure (integrability conditions of continuum equations) the elemental noninertial reference frames (NRF) are investigated.

1. Relativistic global uniformly accelerated Born's hard NRF.
2. Relativistic Born's rigid uniformly rotating RF free of horizon.
3. Rigid vortex-free spherically symmetrical NRF.

All these systems are not described in Minkowski space. On the basis of the global equivalence principle the well-known Bell's problem is solved. The reasonable solution of the problem is absent in the special theory of relativity.

Introduction

When describing the properties of the arbitrary deformable reference frames in the form of the continuous medium either the field of 4-velocity (Euler's standpoint) or the law of continuous medium motion determining the connection between the Euler's and Lagrangian's variables is specified. The space-time is considered either the plane (in case of the special theory of relativity STR) or the Riemannian space (in case of the general theory of relativity GR).

If the gravitational interaction between the particles can be neglected and the external force acting on the body is not a gravitational then the relativistic mechanics of the special theory of relativity STR is applied to describe the medium motion. In STR the fields do not bend the space-time and in IRF and in co-moving NRF of the continuous medium leaving its plane space-time geometry. Perhaps only "space sections" are bent, in general case their geometry is not a Euclidean geometry. Such point of view is routine in the relativity theory (RT). We want to prove the invalidity of such approach connected with the transition from the inertial reference frames (IRF) to the noninertial reference frames.

1. About the difficulties of specifying the Lagrange co-moving to the medium noninertial reference frames (NRF) in special relativity theory (SRT)

This is shown in J. Bell's problem [1], that the thread which connects identical pointlike rockets in uniformly accelerated motion with identical constant accelerations in the cosmonaut system is broken although its length in the inertial reference frame (IRF) does not change. The solution [1] is also used in calculations of bunch motion in linear colliders at the constant electric field [2]. The uncertainty in the specifying of the space "physical" length is one of the difficulties. For example, in the non-inertial reference frame (NRF) co-moving with the bunch, or with the thread in Bell's problem, there is no correct expression for a finite instantaneous length in special relativity (SR). We will use the Minkowski space signature $(+---)$, the Greek indices will vary from 0 to 3 and Latin ones from 1 to 3. The standard expression for finding an element of physical distance dL^2 obtained with the aid of the spatial metric tensor

$$\gamma_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}} \quad (1)$$

is used incorrectly.

In the special theory of relativity (SRT) the correct utilization of this formula at the hypersurface orthogonal to the bunch particles world lines (that is the instantaneous physical space co-moving to the observer medium) resulted in the relation [3-5]

$$L(t) = \frac{c^2}{a_0}\ ln\ (\cosh(\frac{a_0 L_0}{c^2}) + \sinh(\frac{a_0 L_0}{c^2})\sqrt{1+\beta^2}). (2),$$

where $L(t)$ – is the bunch length (or the thread length in J. Bell's problem) in the reference frame co-moving to the bunch as a function of the time IRF t, L_0 is the initial bunch (thread) length, a_0 is the constant acceleration, $\beta = a_0 t/c$. The later formula is original and uncertain in the scientific literature until [3-5].

The standard calculation in accordance with the formula (1) from [6] in [2], [7]

$$L(t) = L_0\sqrt{1 + {a_0}^2 t^2/c^2} = \frac{L_0}{\sqrt{1-v^2(t)/c^2}} (3)$$

in which the curvature of the space similar curve orthogonal to the medium particles world lines is neglected gives at the end of the acceleration process in the Lagrangian comoving NRF the rise of the bunch length in the electronic collider [2] up to 10000 times.

Approach [8] based on the calculation of the distance along the unit vector of some instantaneously comoving reference frame (ICIRF) from the bunch beginning to the end results in practical zero of the bunch length at the and of the acceleration process. In [3-5] at the same conditions the bunch length increases in 1.003 times. In J. Bell's problem provided $a_0 L_0/c^2 = 1$ all formulas from the works above mentioned coincide. And all authors come to conclusion about the thread rupture in J. Bell's problem. However all authors (except [3-5]) connect the string rupture with the Lorentz shrinkages. In our opinion this is erroneous. According to Pauli and Gerglotts which are the founders of the relativistic elasticity theory just deviation from the Born's rigidity but not the Lorentz shrinkage results in the deformations and tensions in the body. To determine the true strains in the body (rod) one must to watch just this body and does not compare its length with other similar rod in some ICIRF. The situation is resemble the comparison of the column length and its sun shadow. Crude error is the connection of the Lorentz transformation and the transition from one IRF to other. Lorentz transformation is the recalculation rule of the geometrical objects (fields) from one IRF to other which do not never coincide by definition. Let, for example, in one IRF the brittle extra slim vitreous rod is present. This rod breaks into pieces when a small deformations are present. Let the set of similar rods is present in other IRF moving with the relativistic velocities. For each observer at the rod his rod is not broken and the other rods must to break into pieces in accordance with the Lorentz shrinkage. This situation is absurd as the integrity or the breaks into pieces is the invariant factor for the rods. This reminds the skeet shooting when one rifleman smashes the plate. And the observer from other IRF seems to be that the plate keeps a whole skin and the rifleman misses. The interpretation of the Lorentz transformations as the transition from one IRF to other is similar to the activity of the passenger to jump a fast express train from the platform.

When constructing the relativistic elasticity theory one trasits to Lagrangian comoving NRF where Lorentz shrinkage is absent by defifnition. Origin of the deformations and tensions in the medium occurs when the medium moves as not rigid in Born's sense body. Deviation from Born's rigidity results in nonzero strain velocity tensor.   In our opinion the thread will be broken if one will toe the line the approach of SRT (special relativity theory) on the basis of conventional transition rules from IRF to NRF but not at the expense of Lorentz shrinkage as it occurs in listed works and at the expense of that in such motion the relativistic Born's string rigidity is violated and the deformations and tensions arise in the string.

2. Connection of the space-time geometry with the continuous medium parameters and the force fields, solution of Bell's problem

In Newtonian mechanics and special relativity theory (SRT) the mass point has zero absolute acceleration relatively inertial reference frame (IRF) when the forces applied to it are absent or their vector sum is equal to zero. In general relativity theory (GR) this rule is not satisfied. The mass point being at rest on the surface of the gravitating sphere in accordance with GR has nonzero first curvature vector (4 – accelaration). Absolute accelaration is directed along the outer normal to the sphere and its value is equal to Newtonian gravity near the surface. Supporting force from the sphere surface brings down the body from its geodetic line having zero first curvature vector only when the supporting force is absent. In accordance with Newton absolute acceleration of the mass point at the sphere surface is equal to zero. For low fields Einstein's equations coincide with Newton theory however conformity principle is not applied relatively absolute accelarations.

The motion or the rest of the probe particles in this field determine the force field character. By defifnition the probe particles do not interact with each other. They interact only with the external field. Let the probe particles are identical and represent some continuous medium. 4 - acceleration, strain velocity tensor and rotational velocity tensor are the continuum characteristics in 4 - space-time. 4 - acceleration enters into the law of motion and at the specified plane metric the field of 4 -velocity and the main medium tensors are determined by integration of motion equation.

The continuous medium in the force field specifies some reference frame (RF). For RF with the specified physical properties one need to know the additional conditions of the main medium tensors depending on 4 - velocities and 4 - accelerations. For example, let us consider the demand concerning the rotation and Born's rigidity. The number of equations for finding of 4 – velocity becomes overdetermined and the integrability conditions must to be fulfill. The later are fulfilled if both 4 – velocities of the medium and the metric coefficients will be desired. For the solution will exist we obtained the integrability conditions (the equations of structure) [11], [9], [16], [17]. The examples of their application are considered in detail in [3], [4], [5], [10], [15]. Equations of structure are exact, they are not directly connected with the Einstein's equations.

For the moving (or being at rest) continuous medium the following correlations are valid.

$$R_{\varepsilon\sigma,\nu\,.}^{\quad\;\;\mu}V_\mu = 2\nabla_{[\varepsilon}\Sigma_{\sigma]\nu} + 2\nabla_{[\varepsilon}\Omega_{\sigma]\nu} + 2\nabla_{[\varepsilon}(V_{\sigma]}F_\nu), (4)$$

for which in the moving continuous medium at the four-dimensional space-time the expressions are correct

$$\nabla_\mu V_\nu = \Sigma_{\mu\nu} + \Omega_{\mu\nu} + V_\mu F_\nu, (5)$$

where V_μ is the field of 4 - velocity which meets to the normalizing condition

$$g_{\mu\nu}V^\mu V^\nu = 1, (6)$$

$g_{\mu\nu}$ is the metric tensor in the Euler's reference frame,

$$\Sigma_{\mu\nu} = \nabla_{(\mu}V_{\nu)} - V_{(\mu}F_{\nu)}, (7)$$

$$\Omega_{\mu\nu} = \nabla_{[\mu}V_{\nu]} - V_{[\mu}F_{\nu]}, (8)$$

$$F_{\mu} = V^{\nu}\nabla_{\nu}V_{\mu}. (9)$$

where $\Sigma_{\mu\nu}$ is the strain velocity tensor, $\Omega_{\mu\nu}$ is the rotational velocity tensor, F_{μ} are the first curvature vectors of the medium particles world lines.

Integration of the system (4-9), where $R_{\varepsilon\sigma,\nu\,.}^{\;\;\;\;\;\mu}$ is the curvature tensor expressed by means of the metric tensor in the ordinary way gives the solution of the problem about the space-time geometry in which the NRF with the preset structure is realized. In [9, 10] the theorem concerning the Born's rigid uniformly accelerated medium can be described in the Riemannian space has been proved. Although the equations of structure have not been connected with GR (general relativity theory) but they prescribe a supplementary conditions to the Einstein's equations. The theorem proving that all static spherically symmetrical solutions of GR are simultaneous with the equation of structure has been presented. One-dimensional solution outside the plane infinite massive source in GR is absent and the equation of structure has such solution and induces the metric for the constant uniform static field [10]. The calculation in the Lagrange co-moving NRF results in the metric

$$dS^2 = \exp(\tfrac{2a_0y^1}{c^2})(dy^0)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2, (10)$$

where the acceleration a_0 is considered positive if it is directed along y^1 axis and it is considered negative if it is directed on the contrary.

First the metric(10) has been obtained in [11] and it has been repeated in [12], [13]. One independent component of the curvature tensor calculated in accordance with the metric (10) has the form

$$R_{10,10} = -\frac{{a_0}^2}{c^4}\exp(2a_0y^1/c^2). (11)$$

For the Ricci tensor components $R_{\beta\gamma} = g^{\alpha\gamma}R_{\alpha\beta,\gamma\delta}$ and scalar curvature R we have

$$R_{00} = -R_{10,10}, \quad R_{11} = -\frac{{a_0}^2}{c^4}, \quad R_{10} = 0, \;\; R = 2\frac{{a_0}^2}{c^4}. (12)$$

One might to be directly convinced that NRF (12) is a uniforly accelerated

$$F^1 = \frac{DV^1}{dS} = \frac{dV^1}{dS} + \Gamma^1_{00}(V^0)^2 = \frac{1}{g_{00}}\Gamma^1_{00} = -\frac{g^{11}}{2g_{00}}\frac{\partial g_{00}}{\partial y^1} = \frac{a_0}{c^2}. (13)$$

The rest of the components of 4 – accelerations are equal to zero.

The metric (10) can be interpreted and as the equilibrium of the probe particles in any constant uniform force field. Let the identical probe charges with the same masses are hanged up on the weightless threads at the uniform electrostatic field. It is clear form the physical consideration that the charges are at rest relatively each other (the model of the charged dust) and the tensions of all threads are identical.

Two points of view are permissible.

1. The space-time is a plane and the sum of the forces on each charge is equal to zero.
2. The space-time is a Riemannian with the plane section and the vector of 4 – acceleration is constant and it is calculated in accordance with the formula (13).

Investigation of electrostatics in the Riemannian space is considered in detail in [10] and the system of the solutions of the Einstein - Maxwell equations consistent with the equations of structure has been obtained in [14, 15].

Let us considered the solution of the Bell's problem. We shall develop the second point of view. In the Riemann geometry the particle fixed in the field has nonzero first curvature vector (4 – acceleration) and in the Minkowski space the same particle has a straight world line with zero 4 – acceleration. From the global equivalence principle the locking of the particles in the uniform constant field of forces is equivalent to their occurrence in Born's rigid relativistic global uniformly accelerated NRF.

In release the particles from the bonds they begin to move at the starting IRF in the Minkowski space at the constant uniform electric field and the distance between the particles in IRF is not changed [2] as well as in NRF (10). In Bell's problem when starting of two pointlike rockets with the same constant accelerations in the astronauts' reference frame after the oscillation damping in the thread the world lines of the thread particles will be "parallel" to the world lines of the pointlike rockets in IRF. And perfect weightless accelerometers fixed at the weightless thread and the rockets will show the identical values. Consequently the metric for the thread in the astronauts' reference frame coincides with (10). The thread length in NRF is preserved as well as in IRF since the initial Eulerian coordinates coincide with the Lagrange coordinates. **The thread will not be broken. The paradox arises because of the standard accepted at the moment transition from IRF to NRF.** V.I. Rodichev repeatedly spoke about this [22 - 25]. A. A. Vlasov [26] considering the theory of the growth of the crystal, plasma and biological structures with the preservation of their likeness came to the result that the growth of such structures is possible in non-Euclidean space – time. Other possibilities of the transitions are specified in [9, 16, 17, 11]. With the Bell's problem solution the great difference in calculating the deformations of the electron bunchs in the modern linear colliders in RF and IRF comoving to the bunchs is vanished. The standard calculation in accordance with the formula (3) increased the bunch length at the output of the collider approximately in 10000 times and the calculation in accordance with the formula (2) incresed the length only in 1.003 times. Although formula (2) is suitable for both the great and the small accelerations it does not solve the Bell's paradox in principle. **The paradox is solved only when going out of the Minkowski space to the Riemann space.**

Deduced formula (2) in the SRT (special relativity theory) is correct only in the case of the standard transition from IRF to NRF.

Here we can to quote the [27] with the reference to [28]. "It is very easy to join the words into the expression "the coordinate system of the accelerated observer" however it is more difficult to find the conception for which it can be correspond to. The best that we can to say about this expression – that when careful consideration it is contradictory." We shall point out that the space-time is bent in the accelerated pointlike rockets and the thread only in the limit of the world band. The world lines of the starting IRF particles of the Minkowski space are the straight lines parallel to the time axis and having zero first curvature vectors. From the viewpoint of any NRF these vectors will remain zero as it is impossible to create or to zero out 4 – vectors by means of the transition from NRF to IRF and conversely with the transformation of coordinates containing the time in non-linear form. Namely such transformations of coordinates are considered by the orthodox persons as the transition from IRF to NRF and conversely. From the astronauts' viewpoint the worls lines of the IRF particles seem not to be parallel and the medium particles of the IRF basis move on the geodetic lines relatively NRF (10). The interval element has the form [9, 16, 17]

$$dS^2 = c^2dt^2 - (1 - v^2/c^2)(dx^1)^2 - (dx^2)^2 - (dx^3)^2, (14)$$

containing the Lorentz shrinkages in explicit form and describing the synchronous RF (frame of reference) in the Riemann space-time. The value of velocity v of the IRF basis particles relatively NRF has the form $v = c\sin(a_0t/c)$, and the time parameter t is connected with the time of the Minkowski space T with the relation

$$t=\frac{c}{a_0}\arccos[\exp(1-\sqrt{1+\frac{{a_0}^2T^2}{c^2}})]. (15)$$

3. Relativistic rigid uniformly rotating NRF

Usually when considering the rotating disk one selects the rest-frame in which the cylindrical coordinates r_0, φ_0, z_0, t_0 are introduced and passes to the rotating reference frame r, φ, z, t in accordance with the formulas:

$$r_0=r,\quad \varphi_0=\varphi+\Omega t,\quad z_0=z,\quad t_0=t,$$

where the rotational speed Ω relatively z axis is considered as constant. The interval element has the form

$$dS^2=(1-\frac{\Omega^2r^2}{c^2})c^2dt^2-2\Omega r^2d\varphi dt-dz^2-r^2d\varphi^2-dr^2. (16)$$

The formula holds when $r\Omega/c<1$. In [18-20] other velocity distributions which restrict the linear velocity of the disk at $r\to\infty$ with the value of velocity of light c and at $\Omega r/c=1$ form $v=\Omega r$ are discussed. However only usual distribution law $v=\Omega r$, $\Omega=const$ satisfy to the stiffness criterion both the classic and the relativistic (in Born's sense).

Let us determine the metric of the rigid relativistic uniformly rotating NRF by means of our method supposing in the formulas the strain velocity tensor $\Sigma_{\mu\nu}=0$ and demanding the constancy of the invariant characterizing the relativisctic generalization of the square of the disk rotational velocity ω.

$$\Omega_{\mu\nu}\Omega^{\mu\nu}=\frac{2\omega^2}{c^2}=const. (17)$$

In the Lagrangian co-moving frame of reference connected with the rotating disk we have

$$dS^2=D(r)c^2dt^2-2P(r)cdtd\varphi-dz^2-r^2d\varphi^2-dr^2, (18)$$

$$F^1=\frac{1}{2D}\frac{dD}{dr},\quad F^2=F^3=F^0=0. (19)$$

Afterwards the cumbersome calculations we have two independent equations

$$\frac{P}{D}\frac{dD}{dr}-\frac{dP}{dr}=-2\frac{\omega}{c}(Dr^2+P^2)^{1/2}. (20)$$

$$\frac{dD}{dr}-=-2\frac{\omega}{c}DP(Dr^2+P^2)^{-1/2}. (21)$$

Condition (17) is equivalent to the constancy of the value of metrically invariant angular velocity vector [20] anf the constancy of the value of the rotary speed in the co-moving tetrads [21].

The values of the relativistic ω and the classic rotary speed Ω are connected with the relation

$$\omega=\Omega(1-\frac{\Omega^2r^2}{c^2})^{-1}. (22)$$

For metric (18) there is a steady-state solution applied in whole sphere $0 \leq r \leq \infty$ but realized in the Riemann space – time.

The solution of the system (20), (21) in the quadratures is absent. Numerical analysis showed that at $\omega r/c = 1$ the metric (18) coincides with the metric (16). Centripetal acceleration in the rotating NRF is determined with the formula

$$a = c^2 F^1 = -\frac{\omega c P}{\sqrt{D r^2 + P^2}}, (23)$$

which at small r passes to the classic and at $r \to \infty$ gives $a = -\omega c$. The calculation of the independent nonzero components of curvature tensor are cumbersome and we omit them (see [9], [11], [16], [17]).

After the simplifications the system [20, 21] is represented in tht form

$$\frac{dv}{dx} + \frac{v}{x}(1 - v^2) = (2 - v^2)(1 - v^2), (24)$$

$$D = \exp(-2 \int v dx),\ v = \frac{U}{\sqrt{1+U^2}},\ U = \frac{P}{r\sqrt{D}},\ x = \frac{\omega r}{c}. (25)$$

Physical interpretation of function $v(x)$ means the dimensionless linear speed of the disk. For small velocities

$$D = \exp(-2 \int v dx) = \exp(-x^2) = 1 - x^2, (26)$$

that is equivalent to the classic expression. It follows from the analysis of (24) that for $x \to \infty$ the equation has the solution $v = 1$. This solution is markedly differed from the classic rigid disk where the velocity field at infinity is indefinitely great. Apparently the diagram of the numerical solution (24) is resemble the diagram of the hyperbolic tangent or the deformed step function for $x > 0$.

It is generally known [6] that at the rotating disk the clock can not be identically synchronized at all points. Therefore synchronizing along the closed circuit and returning to the reference point we shall obtain the time differed from the original time on the value

$$\Delta t = -\frac{1}{c} \oint \frac{g_{02}}{g_{00}} d\varphi = \frac{2\pi}{c} \frac{vr}{\sqrt{1-v^2}} \exp(\int v dx). (27)$$

Let us consider the light rays propagation along the closed circuit relatively to the rotating disk. Let the rays are moved round a circle in the opposite direction and the source is located at the rotating disk. Always the velocity of light is equal c if one uses the metrically invariant time lag [20]. Therefore the rays will reach the source simultaneously. In accordance with the universal time the time difference of the source coming $\Delta t_0 = 2\Delta t$. In the nonrelativistic approximation for the small velocities of the disk $v = x$ the obtained result coincides with the result of the well-known Sagnac experiment.

$$\Delta t_0 = \frac{4\omega S}{c^2}, (28)$$

where S is the area of the disk. In the ultrarelativistic case we have

$$\Delta t_0 = \frac{2\pi}{\omega}\exp(2x). (29)$$

We shall point out that in accordance with the current outlook the clock at the rotating disk can not be identically synchronized at all points. Therefore synchronizing along the closed circuit and returning to the reference point we shall obtain the time differed from the original time. **However this problem has been solved by the authors in [16], [17] by means of the introduction of the “relative tensor of curvature” (terminology of the authors) at the space-time.** But this topic exceeds the limits of this article.

4. Spherically symmetrical rigid NRF

Let us consider in the Minkowski space the central symmetrical continuum motion occurring from some point in which the origin of coordinates is. Obviously that for the observers in the Lagrange co-moving reference frame the distance between the adjacent medium elements will change with the time i.e. such system is not the rigid one. As all medium points being at the same distance from the centre have identical velocities and accelerations then such medium moves without the rotations. Thus, for such NRF the tensor of the angular velocity is equal to zero, and the strain velocity tensor and the field of the first curvature vectors are differed from zero. If for the considered NRF one demands the fulfillment of the rigidity condition then it follows from the analysis of the structure equation (4) that in the Minkowski space the spherical symmetrical NRF having nonzero radial acceleration and zero strain velocity tensor is absent. In other words in the Minkowski space the rigid radial continuum motion is impossible.

In the Riemannian space such situation is possible. For example, it follows from the condition of static equilibrium in the spherical symmetrical gravitational field described with the Schwarzschild metric. For the observers being at rest at the surface of the motionless gravitating sphere from the GR standpoint the acceleration differs from zero and it is directed from the centre perpendicular to the surface while for the observers keeping of the Newton standpoint the acceleration is equal to zero. And vice versa the free falling body in the Newton gravitational field has nonzero acceleration and in the Schwarzschild field it moves on the geodetic line with zero acceleration. We find the metric of the spherical symmetrical Lagrange co-moving NRF in analogy with GR [6] in the form

$$dS^2 = \exp(\nu)(dy^0)^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - \exp(\lambda)(dr)^2, (30)$$

where ν и λ depend only from r.

Obviously NRF (30) is rigid as the metric factors do not depend from the time and zero components g_{0k} mean that the rotations are absent. The system (5) taking into account the formulated demands and also fulfillment of the co-moving conditions

$$V^k = V_k = 0, \quad V^0 = (g_{00})^{-1/2}, \quad V_0 = (g_{00})^{1/2},$$
$$F^1 = F(r), \qquad F^0 = F^2 = F^3 = 0$$

reduces to one equation

$$F^1 = \frac{1}{2}\frac{d\nu}{dr}\exp(-\lambda). (31)$$

One can to be convinced that the structure equations (4) satisfy to (31) without the additional connections for $\nu(r)$ and $\lambda(r)$ functions. Thus, in accordance with the specified field of the first curvature vectors F^1 it is impossible uniquely to determine the metric (30) without the additional factors.

From the physical encyclopaedia “reference frames (RF) are the collections of the coordinate and clock system connected with the body relatively which the motion (or the equilibrium) of any other mass points or bodies is studied”... . Therefore to investigate the motion (equilibrium) of other bodies the analytical specifying of the body properties is necessary – basis of RF itself. And what means the RF in vacuum? The physical encyclopaedia ignores. In the Schwarzschild field vacuum is present outside the body. In accordance with GR in vacuum in the static field (as well as alternating field) we understand that RF “… is the collection of the infinite number of the bodies filling all space like some medium” [6]. Let us consider some simplest possibilities.

a). Let the observers locating at the earth surface measure the gravitational field by means of the accelerometers. The earth rotation is not taken into account, its density is considered as the constant, and the earth from is the spherical. They will find that the acceleration field is directed on the radius from the centre perpendicular to the surface. In order to measure the field far from the surface we use the set of the radial weightless rigid rods. We install the system of the accelerometers along the rods. Collection of the rods and the accelerometers specifies the basis of the radial accelerated rigid reference frame. Really with the removing form the earth surface the acceleration field will decrease in accordance with the Newton’s law of gravitation (in the zero approximation). If the observers consider that its space is flat and the law of gravitation is exact then the metrics (30) will have the form [11].

$$dS^2 = \exp(-r_g/r)(dy^0)^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - (dr)^2, (32)$$

where $r_g = 2kM/c^2$ is the gravitational radius. When derivation (32) we took into account in accordance with the definition of the plane space $\lambda = 0$ and ν has been found from (31) and Newton’s law of gravitation.

So, though the space metrics is flat, the space-time metric (31) is the Riemannian one. Thus, the Newton’s gravitation theory in the flat space permits two logically consistent interpretations.

In accordance with the generally accepted interpretation in the Newton’s theory both the space and the space-time are the flat. At the same time on the body being at the earth surface two forces act: the gravity and the support reaction force which in sum give zero and therefore the body has no acceleration.

In our interpretation on the body being at rest relatively the earth surface only one force acts – the support reaction force which adds to the body the acceleration measured with the accelerometer (floor scales) and calculated in accordance with the (31) using the metric (32). If the support is removed then the body will move on the geodesic line in the space-time with the metric (32) while in general interpretation when the support is absent the body will move in the flat space-time under the action of the gravity.

Modified Newton’s interpretation is closer the Einstein’s interpretation then the Newton’s one. Using [31] one can to show that the calculation of the pericentre displacement over one rotation in accordance with the metric (32) is one-third of one in accordance with the Schwarzschild metric. The change of the light ray direction when passing near the central body in accordance with (32) is one-second of the Schwarzschild one. So, the proposed model does not pretend on the GR substitution, it fixes the more close connection between the Newton’s and the Einstein’s theories showing that one can to consider the Newton’s theory in the Riemannian space-time. If in the Newton’s approximation the considered interpretation coincides with the experimental data then the model does not take into account more thin effects which are explained in GR.

b). When derivation (32) one supposes $\lambda = 0$ that corresponds to the model of the flat space section. The system of the rigid non-deformable rods on which the sound spreads with the infinitely large velocity that contradicts to the finite velocity of the interaction spreading has been selected as the reference frame outside the earth. We shall consider that the basis structure of the radial accelerated NRF outside the earth is equivalent to some elastic medium subjected to the deformations and consequently the tensions but having zero strain velocity tensor.

It is conveniently to determine the connection between the deformation and stress tensors in the Lagrange co-moving NRF considering the elastic medium for which the Hooke law in the form [29] specified at the hypersurface orthogonal to the world lines [30] is valid

$$P^{ij} = \tilde{\lambda} I_1 \gamma^{ij} + 2\mu\gamma^{ik}\gamma^{jl}\varepsilon_{kl}, \quad I_1(\varepsilon) = \gamma^{kl}\varepsilon_{kl} = \frac{1}{2}(1-\exp(-\lambda)), (33),$$

where I_1 is the first invariant of the deformation tensor, $\tilde{\lambda}$ is the Lamé coefficient, $\gamma^{ij} = -g^{ij}$ is the metric of the space section (30).

$$\varepsilon_{ij} = \frac{1}{2}(\gamma_{ij} - \gamma'_{ij})$$

γ'_{ij} is the metric tensor of the plane space in the spherical coordinates.

The elastic medium must to satisfy the continuity equation.

$$\nabla_\mu(\rho V^\mu) = 0$$

The solution of the continuity equation results in the relation [29], [30]

$$\rho = \rho_0 \exp(-\lambda/2), (34)$$

where ρ_0 is the “medium” density in the unstrained state.

The “motion” equations of the elastic medium in the Lagrange NRF have the form similar to the equilibrium condition of the elastic medium in the Newtonian gravitational field at the classic consideration

$$\nabla_j P^{ij} = -\rho a^j, (35)$$

where a^j are the “unphysical” – affine components of the acceleration and the raising and the lowering of tensor indexes and the calculation of the covariant derivative is realized by means of the space metric γ_{ij}. As the metric (30) is orthogonal then to construct the tetrad field one can to combine the vectors of ortho reference mark $\vec{e}_\alpha$ with the vectors of the affine reference mark and to write the tetrad field in the form of the Lame calibration [32]. [33].

$$e^{\mu}_{(\alpha)} = \frac{\delta^{\mu}_{\alpha}}{\sqrt{|g_{\alpha\alpha}|}}, \qquad e^{(\alpha)}_{\mu} = \delta^{\alpha}_{\mu}\sqrt{|g_{\alpha\alpha}|},$$

where the summation of α is absent. The tetrad tensor components coincide with the “physical”. Supposing that the physical or the tetrad acceleration components correspond (as in the case a).) to the Newtonian value, we have from (34) and (35) in the spherical coordinates the expression

$$\exp(-\lambda)\frac{d\lambda}{dr} = -2\frac{\rho_0 k M}{(\tilde{\lambda}+2\mu)r^2}, (36)$$

The integration of this expression provided that at the infinity the space is plane ($\lambda = 0$) results in the relation

$$\exp(-\lambda) = (1 - \frac{2kM}{c_0^2 r}), \qquad c_0^2 = \frac{\tilde{\lambda}+2\mu}{\rho_0}, (37)$$

where c_0 is the longitudinal sound velocity.

Taking into account that the first curvature vector

$$F^1 = c^{-2}a^1 = (\gamma_{11})^{-1/2} kM/(cr)^2$$

using (31) and (36) we obtain the equation for ν. The integration of this equation provided that at infinity $\nu = 0$ forms

$$\nu = 2(\frac{c_0}{c})^2(\sqrt{1 - \frac{2kM}{{c_0}^2 r}} - 1). (38)$$

The limit of the expressions (38) and (39) when $c_0 \to \infty$ results in the metric (32) that corresponds to the model of perfectly rigid body in the Newtonian sense. We shall refer to as relativistic rigid body such body in which the longitudinal sound velocity is equal to the velocity of light in vacuum [11]. The expression (38) exactly coincides with the γ_{11} component of the Schwarzschild metric in the standard form and g_{00} component of this metrics is obtained from (38) if one expands $\exp(\nu)$ into a series and keeps only the first infinitesimal order on (r_g/r).

Thus, for the spherically – symmetrical rigid NRF where the basis is the relativistic rigid body and the acceleration corresponds to the Newtonian one the metric has the form (30) where ν is determined from (38) when the sound velocity c_0 is equal to the velocity of light c in vacuum and λ is obtained at the same conditions from (37). We represent the final output in the form

$$dS^2 = \exp\{2\sqrt{1 - \frac{2kM}{{c_0}^2 r}} - 2\}(dy^0)^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - \frac{dr^2}{1-\frac{2kM}{c_0^2 r}}. (39)$$

The calculation of the known GR effects in accordance with the metric (39) when $c_0 = c$ only insignificantly differs from the calculation using the Schwarzschild's metric. The difference is in the pericenter shift calculation which is equal to 5/6 from the Schwarzschild's one. The change of the light beam direction when passing close by the central body coincides with the Schwarzschild's one. Therefore the modified model is considerably nearer to GR than (32).

Conslusion

1. It is proved that in the Minkowski space the translatory globally uniformly accelerated and rigid in the Born's sense continuum motion is impossible. If besides the continuum motion equations one imposes the supplementary conditions for the rigidity or the rotations of the continuum following from physical considerations then these conditions "remove" the moving medium from the plane space-time.

2. The metric of the rigid in the Born's sense globally uniformly accelerated continuum realized in the Riemannian space-time has been presented. The metric integrates the properties of the Möller's metric (the rigidity in the Born's sense) and the properties of the Logunov's metric (the

global uniformly acceleration). It should be noted that the proper time which was obtained by Einstein [34] in 1907 and which was named exact is obtained from the metric (10) for the fixed Lagrangian particle.

$$\tau_s = \exp(\frac{a_0 y^1}{c^2})\tau,$$

where τ_s is the proper time for given space point, τ is the universal time. But Einstein renounced of the exact expression for the approximation (Möller's).

3. The relativistic rigid in the Born's sense uniformly rotating NRF without the restriction of the radius value and having at infinity the linear velocity which is equal to the velocity of light and finite acceleration but realized in the Riemannian space time has been obtained.

4. Equations of structure being exact restrict the domain of applicability of the Einstein's equations as they give the supplementary conditions which are not always compatible with the GR solutions.

5. The spherically symmetrical rigid NRF having no the analog in the Minkowski space which is equivalent to the balance of the gravitational forces to the elastic forces has been constructed. If in the elastic medium the transverse sound velocity concides with the velocity of light in free space then the body is the relativistic rigid and the obtained equilibrium solution is described with the metric closed to the Schwarzschild's metric. For the classic solids the velocity of sound goes to infinity but the equilibrium space-time metric remains the Riemannian with the plane space. It turns out that the connection between the Newton's and Einstein's theories is much close then commonly thought.

6. The Bell inequality was solved.

7. The time-series identification of the physical frame of reference as the reference body with the specified physical properties resulted in essential approaching of the Newton's and Einstein's gravitation theories. Assignment of the physical properties to the reference frames in a manner is equivalent to the introduction of the quantum-mechanical principle of complementarity to the Newton's gravitation theory. In this approach the space-time geometry depends on the means by the instrumentality of which it is observed. Similarly the quantum mechanics it is impossible to describe the atomic systems independently of watching facilities.

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