Improving Koide Formula

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Abstract. Based on the methods, author used in previously published articles at viXra.org, Koide formula is corrected, to the results 2/3. The mass of tau lepton is also calculated.

Introduction

Famous Koide formula [1] connects the masses of charged leptons with a simple equation:

\[
\frac{m_e + m_\mu + m_\tau}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}} \approx \frac{2}{3}
\]

We use the following CODATA values [2]:
The inverse of the fine structure constant \( \alpha = 137.035 \, 999 \, 074 (44) \),
Mass ratio of protons and electrons \( \mu = 1836.152 \, 672 \, 45 (75) \)
Mass of proton \( m_p = 1.672 \, 621 \, 777 (74) \text{e-27} \)
Electron mass \( m_e = 9.109 \, 382 \, 91 (40) \text{e-31 kg} \)
Muon mass \( m_\mu = 1.883 \, 531 \, 475(96) \text{e-28 kg} \)
Mass of Tau particle \( m_\tau = 3.167 \, 47 (29) \text{e-27 kg} \)

We have:

\[
\frac{m_e + m_\mu + m_\tau}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}}^2 = 0.666658
\]

Analysis

Let us start with Boscovich force curve:

Figure 1. General (a) and particular (b, c) shapes of curves that present the attractive and repulsive forces (F) (bottom and upper ordinates, respectively) vs. distance (r) (abscissa) between the elementary points and particles of matter /6/.
Bošković emphasized the importance of distances at which the curve crosses the abscissa: R, N, I and E represent the stable, but P, L and G are the unstable positions. It is a logical assumption that the relation is simpler between the particles of the same order, and even simpler between the neighboring points.

Suppose that all leptons in stable positions, i.e. at point E curve (a). If that were indeed the case, then it follows that they have nothing to do with the rest. Therefore, we assume that at least one member of the Koide formula should have a position beyond the point E. Suppose that, the virtual particles is in the G-spot which is unstable equilibrium. Therefore, it oscillates around the G. Let’s call this virtual particle wave lepton. Determine its mass using the formula:

\[ m_w = m_e / \xi = 7.944055747E-31 \]  

where:

\[ \xi = 2\pi \alpha^{*} 2 \alpha^{((2/3) \times ((\mu/\alpha+1)/((\mu/\alpha+2)+1)))/\mu} = 1.146691715 \]

Now let us introduce the first correction Koide formula assuming that in (1) instead of the electron, in the root should stand virtual lepton mass \( m_w \). Then we have:

\[ x = (m_e + m_\mu + m_\tau) / (\sqrt{m_w} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 0.6678466747 \]

Here I owe clarification, why then did not use the same value for Muon and Tau particle in the root? Because the dominant influence of the electron and the virtual lepton is surrounded by two other corpuscular leptons. The situation, in which, the \( m_w \) is assigned to Muon, is several orders of magnitude less frequent. It is even rarer for Tau lepton, for tens of orders of magnitude. From the previous discussion, the correction in this way will not be exact, because we have neglected to repair Muon and Tau. But since it has to start somewhere, and so we will not worry about the error, if any, that is after dozens of significant digits.

**Correction**

Assume here that the Koide formula ideal applies to quasi-universe that has no substance, that is, perfectly empty. The universe really is almost empty, but of course if it is totally empty, it would not have been. Therefore, here we introduce the assumption that the difference is a result of the existence of matter. Then it is reasonable to assume that \( \Delta = f (m_\mu) \). That is, \( \Delta \) can be easily expressed in terms of the proton. It is also reasonable to assume that the correction can be in terms of the Planck mass.

Difference \( \Delta = x - 2/3 = 0.667846674704 - 2/3 = 0.001180008 \)

is the one that needs to be corrected in order to determine the Koide formula to be ideally correct. Previously we replace the electron in the root with the virtual mass of wave leptons. Therefore, a definition:

\[ g = \log(M_\mu/m_{pl,2}) = 202.31422768 \]
Where:
Mass of the universe $M_u = 1.73944912\times10^{53}$ kg,
Planck mass $m_{pl} = 2.1765099035\times10^{-8}$ [3]

We calculate:

$$y = \frac{1}{\pi^2} \frac{2}{3} g = \frac{1}{847.4518552} = 0.001180008 \quad (8)$$

We can see that $y = \Delta$. If you have obtained a replacement, from (5) and (8) we get:

$$x - \Delta = x - y = \frac{m_e + m_\mu + m_\tau}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}} - y = 0.667846674704 - 0.001180008 = 0.6666666666... \quad (9)$$

Instead of (7), we can determine $g$ in the following way:

$$g = \frac{3c y}{8 + 3 t/4 - (z_{p}+1)/4} = 202.3142277 \quad (10)$$

Where:

$$t = \log(2\pi, 2) = 2.6514961295 \quad (11)$$

Cycle:

$$c_y = e^{\frac{2\pi}{8}} = 535.491655525$$

Proton shift:

$$z_{p} = \log(m_p/m(c_y/2)) = 1.935061 \quad \text{from [3]},$$

Or proton shift:

$$z_{p} = \frac{\mu}{\hat{\mu}+1}/\frac{\mu}{\hat{\mu}+2}+1 = 0.9350609435 \quad (12) \quad \text{from [4, table 1]}$$

That is, if we put (10) into (8) yields:

$$y = \frac{1}{2\pi^2 (2/3)^g} = 1/[2\pi^2 (cy/4 + t/2 - z_{p}/6)] = 0.001180008 \quad (13)$$

That is from (5), (6), (8) i (12) Koide formula with corrections is:

$$x = \left( m_e + m_\mu + m_\tau \right) / \left( \sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau} \right)^2 - 1/[2\pi (cy/4 + t/2 - z_{p}/6)] = 2/3 \quad (14)$$

That is, we have got Koide formula with the correction. It contains, apart from the masses of charged leptons, even mathematical constant, the fine structure constant and the proton. Such that:

$$x = f(m_e, m_\mu, m_\tau, m_\nu, \mu, \hat{\mu}) = 2/3$$

That is, since the $m_{\nu} = f(\mu, \hat{\mu})$ and $\mu = m_p / m_e$ we can write:

$$x = f(m_e, m_\mu, m_\tau, m_\nu, \hat{\mu}) = 2/3 \quad (15)$$
**Tau lepton mass**

Formula (14) can be used to determine the mass Tau lepton with the same accuracy as the other two leptons. We obtain $m_{\tau}/m_{\tau}=0.52805989288087$. Here are some taken in account values more precisely than in [2] ($\mu=1836.15267245005$, $\hat{\alpha}=137.03599907361$).

In Table 1, we count the mass of Tau lepton from previously set values of physical constants which are much easier to determine experimentally.

<table>
<thead>
<tr>
<th>$\hat{\alpha}$</th>
<th>137.03599907361</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1836.15267245005</td>
</tr>
<tr>
<td>$m_{e}$</td>
<td>9.1093829075E-31</td>
</tr>
<tr>
<td>$m_{\mu}$</td>
<td>1.8835314739E-28</td>
</tr>
</tbody>
</table>

$A=m_{e}+m_{\mu}$

$z_{p}=(\mu/\hat{\alpha}+1)/(\mu/\hat{\alpha}+2)+1$ 1.935060944

$\zeta=2\pi\hat{\alpha}^{2}z_{p}^{3}/\mu$ 1.146691714861000

$B=sq(m_{e}/\zeta)+sq(m_{\mu})$ 1.461547544451E-14

$t=log(2\pi,2)$ 2.6514961295

$cy=exp(2\pi)$ 535.4916555248

$g=3\pi/4+3t/4-z_{p}/4=202.314227683011$

$K=2/3+1/(4\pi g/3)=0.667846674703530$

$x=(m_{e}+m_{\mu}+m_{\tau})/(\sqrt{m_{e}}+\sqrt{m_{\mu}}+\sqrt{m_{\tau}}^{2}-1/[2\pi(cy/4+t/2-z_{p}/6)]=0.6666666667$

$m_{\tau}=((2KB/(1-K)+sq((-2KB/(1-K))^{2}-4g(A-KB^{2}/(1-K))))/2)^{2}3.167484975E-27$

**Table 1 Calculating the mass of Tau lepton**

Or if we simplified:

$$m_{\tau} = \left[ \frac{KB + \sqrt{KB^{2} + A(K - 1)}}{1 - K} \right]^{2} = 3.16748497(28) \times 10^{-27} \text{ kg} \quad (16)$$

The analysis of Tau lepton mass throughout history CODATA, with parallel use of the procedure in Table 1 can test the validity of formula (16). If we write Koide formula so that the two lepton masses are constants, and one unknown, which is required, is obtained by quadratic equations. The root of the mass of Tau lepton ($\sqrt{m_{\tau}}$) is a variable, and then you get two solutions. It is interesting relationship between these two solutions. Research and discussion of these relationships, would be interesting and useful for all three leptons.

**The accuracy of the formula - Analysis of Hugh Matlock's**

Notice that CODATA did not publish a mass_tau until 1998. The value that formula (16) could predict in 1969 appears to be more precise than the current 2010 CODATA value for mass_tau.
Table 2 Comparison of Tau lepton mass determined by two methods

<table>
<thead>
<tr>
<th>Year</th>
<th>CODATA tau *e-27 kg</th>
<th>Zivlak tau *e-27 kg</th>
<th>CODATA error</th>
<th>Zivlak error</th>
<th>C Error / Z Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>0</td>
<td>3.167550(113)</td>
<td>1</td>
<td>1.27E-04</td>
<td>7857.805</td>
</tr>
<tr>
<td>1973</td>
<td>0</td>
<td>3.167542(33)</td>
<td>1</td>
<td>1.02E-04</td>
<td>9812.244</td>
</tr>
<tr>
<td>1986</td>
<td>0</td>
<td>3.1674871(33)</td>
<td>1</td>
<td>9.26E-05</td>
<td>10800.26</td>
</tr>
<tr>
<td>1998</td>
<td>3.16788(52)</td>
<td>3.16748435(47)</td>
<td>2.56E-04</td>
<td>9.17E-05</td>
<td>2.78674</td>
</tr>
<tr>
<td>2002</td>
<td>3.16777(52)</td>
<td>3.16748485(97)</td>
<td>2.56E-04</td>
<td>9.19E-05</td>
<td>2.78383</td>
</tr>
<tr>
<td>2006</td>
<td>3.16777(52)</td>
<td>3.16748468(32)</td>
<td>2.56E-04</td>
<td>9.17E-05</td>
<td>2.790086</td>
</tr>
<tr>
<td>2010</td>
<td>3.167470(290)</td>
<td>3.167484977(281)</td>
<td>1.83E-04</td>
<td>9.16E-05</td>
<td>1.998053</td>
</tr>
</tbody>
</table>

Conclusion

We started from the assumption that in the Koide formula are missed pieces, those should be attached to the dual nature of electron and the mass of proton, the result to be an ideal 2/3. We have come to such a solution, provided that the formula still participate and mathematical constants and the fine structure constant. Is, Koide formula with corrections, speculative?

- Defining mass of virtual wave lepton we introduced in formula corpuscular/wave duality of electron nature.
- The assumption that the material world, expressed through a proton to have an impact on the relationship between the three charged leptons is rational;
- The result confirms this assumption;
- obtained the formula (14) and (16), which show that the Koide formula with corrections, is a function of the charged particles, which is more rational than that is a function of only three negatively charged particles;
- Improved Koide formula is less speculative than the original Koide formula.

Previously, should be further explained in other rational approaches. Physical interpretation of the results demands a new article with more specific details. One such approach is the use of Boscovich force curve [5], [6].

The advantages of using formula (16) are clear from the analysis presented in Table 2. The results obtained by (16) are considerably more accurate than those obtained by CODATA methods.

Novi Sad, august 2013

References:

3. Branko Zivlak - Calculate Universe 1, viXra: 1303.0209
4. Branko Zivlak - Calculate Universe 2, viXra: 1304.00051