PROTIUM AND ANTIPROTUIM IN
RIEMANNIAN DUAL 4D SPACE-TIME

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Abstract
In this preliminary paper, we apply the Riemannian dual (fractional quantum Hall superfluidic) space-time topology and the six-coloring Gribov vacuum to protium and antiprotium. The results suggest that it may be possible to generalize this framework to all atomic elements. Therefore, this subject warrants further scrutiny, collaboration, refinement, and investigation.

Keywords: Hydrogen; Holographic principle; Inopin Holographic Ring; Quark confinement; Baryon-antibaryon duality; Topological deformation.
1 Introduction

V. N. Gribov is often considered to be one of the founders of modern particle physics [1]. Gribov’s pioneering developments in the realm of quark confinement helped forge our understanding of QCD as the microscopic dynamics of hadrons [1, 2, 3]. During his twenty-year venture, Gribov constructed an elegant physical explanation of asymptotic freedom, demonstrated the inconsistency of the conventional gluon field perturbative treatment, established that color confinement is determined by the existence of practically massless quarks, and formulated QCD as a quantum field theory comprising both perturbative and non-perturbative phenomena, where quark confinement is achieved through light quark supercritical binding [1].

Recently, in [4], Gribov’s quark confinement theory [2, 3] was extended with an analytic quark confinement proof in the Riemannian dual (fractional quantum Hall superfluidic) space-time topology. In particular, Gribov’s vacuum [2, 3] was upgraded in [4] to include a total of 18 zones for quasi-particles and Laughlin excitations [5], where quarks and leptons are “split” into spinons, holons, and orbitons [6] of fractional statistics [7] for the spontaneous breaking of several simultaneous superfluidic symmetries with long range order [4]. The dual space-time topology and upgraded Gribov vacuum were initially assembled in [4] by isometrically embedding the Inopin Holographic Ring $T$ (as a modified topological Riemannian sphere) in a 1D Riemann surface $X$, where the Time Zone $T \subset X$ (of luminal quasi-particles) is simultaneously dual to both the Micro Space Zone $X_- \subset X$ (of superluminal quasi-particles) and the Macro Space Zone $X_+ \subset X$ (of sub-luminal or luminal quasi-particles) sub-spaces—the interconnection of two distinct distance scales that are topologically equipped with order parameter fields for deformations [4]. The three $q\bar{q}$ pairs (for a baryon-antibaryon pair), namely $q_r\bar{q}_r$, $q_g\bar{q}_g$, and $q_b\bar{q}_b$, are arranged along the six-coloring (three-coloring and three-anticoloring) hexagonal kagome antiferromagnet with triangular chirality and superbound to the vacuum as coupled oscillators in accordance to the CPT-Theorem and the relevant antisymmetries [4]—so $q_r, \bar{q}_r, q_g, \bar{q}_g, q_b, \bar{q}_b \in T$, where the $q_r, q_g, q_b \in T$ are non-Abelian color-electric-magnetic quark monopoles and $\bar{q}_r, \bar{q}_g, \bar{q}_b \in T$ are the corresponding non-Abelian anticolor-electric-magnetic antiquark antimonopoles [4]. Then, with the identification of triplex numbers with triplex multiplication in
[8], the dual space-time topology, vacuum, order parameters, and baryon-antibaryon antisymmetric wavefunction (with tensor notation) for the developing dual 3D space-time $X$ in [4] were systematically upgraded to $Y$ for a complete dual 4D space-time, where $X \subset Y$ so $X_- \subset Y_-$ and $X_+ \subset Y_+$ for the dual interconnected spatial 3-branes, such that Inopin’s $T$ becomes a topological Riemannian 2-sphere, and exists within the dual confinement horizons—see Figure 1. The confined baryon-antibaryon pair forms an $SU(2)$ gauged Bose-Einstein condensate and is directly connected to 1D, 2D, and 3D Skyrmions with “massive ’Higgs-like’ scalar amplitude-excitations” and “massless Nambu-Goldstone pseudo-scalar phase-excitations” in [4], and additionally with the new “massless Nambu-Goldstone pseudo-scalar inclination-excitations” in [8].

Now it is known that hydrogen is the most abundant element in the universe, which constitutes approximately 3/4 of the baryonic mass in the universe [9]. Moreover, the most common isotope of hydrogen is protium, which comprises one proton, no neutrons, and one electron [10]. Reversely, antiprotium comprises one antiproton, no neutrons, and one positron [10]. Thus, in this introductory paper, we are interested in applying the work of [4, 8] to protium (and antiprotium) because of its simple structure and natural profusion. Hence, in Section 2, we propose an alternative and preliminary model of the protium-antiprotium pair with the quark-antiquark confinement, baryon-antibaryon duality, dual space-time topology, and upgraded Gribov vacuum of [4, 8]. We finalize our paper in Section 3 with a brief recapitulation of the results, where we consider some potential implications and suggest future modes of research.

2 Application

First, we visually consider the six-coloring Gribov vacuum depicted in Figures 4–5 of [4] and find it necessary to label the vacuum’s 18 zones. Thus, for the zones of $Y_- \subset Y$, $T \subset Y$, and $Y_+ \subset Y$, we define

$$Y_- \equiv Y_{-r} \cup Y_{-g} \cup Y_{-b} \cup Y_{-\bar{r}} \cup Y_{-\bar{g}} \cup Y_{-\bar{b}},$$  \hfill (1)

$$T \equiv T_r \cup T_g \cup T_b \cup T_{\bar{r}} \cup T_{\bar{g}} \cup T_{\bar{b}},$$  \hfill (2)

and

$$Y_+ \equiv Y_{+r} \cup Y_{+g} \cup Y_{+b} \cup Y_{+\bar{r}} \cup Y_{+\bar{g}} \cup Y_{+\bar{b}},$$  \hfill (3)

Fig. 1: The 2D Position-Point Space $X$ is a dual 3D space-time “slice” of the 3D Position-Point Space $Y$ for the dual 4D space-time, where $X \subset Y$ and $Y$ share the $y_\text{R}$-axis and $y_\text{I}$-axis, such that $Y$ contains the additional $y_\text{Z}$-axis (not shown) [8]. The Time Zone $T$ is simultaneously dual to the two 3-branes $Y_-$ and $Y_+$, where the quarks and/or antiquarks are confined to $T$ [4, 8]. Thus, $T$ serves as the proton and/or antiproton shell and exists within the dual confinement horizons [4].
respectively, with the corresponding topological partition constraints

\[ \emptyset = Y_r \cap Y_g \cap Y_b \cap Y_{\bar{r}} \cap Y_{\bar{g}} \cap Y_{\bar{b}}, \]  

(4)

\[ \emptyset = T_r \cap T_g \cap T_b \cap T_{\bar{r}} \cap T_{\bar{g}} \cap T_{\bar{b}}, \]  

(5)

and

\[ \emptyset = Y_{+r} \cap Y_{+g} \cap Y_{+b} \cap Y_{+\bar{r}} \cap Y_{+\bar{g}} \cap Y_{+\bar{b}}, \]  

(6)

with the effective volume equivalence constraints

\[
\begin{align*}
\text{f}_{\text{vol}}(Y_{-r}) &= \text{f}_{\text{vol}}(Y_{-g}) = \text{f}_{\text{vol}}(Y_{-b}) = \text{f}_{\text{vol}}(Y_{-\bar{r}}) = \text{f}_{\text{vol}}(Y_{-\bar{g}}) = \text{f}_{\text{vol}}(Y_{-\bar{b}}), \\
\text{f}_{\text{vol}}(T_r) &= \text{f}_{\text{vol}}(T_g) = \text{f}_{\text{vol}}(T_b) = \text{f}_{\text{vol}}(T_{\bar{r}}) = \text{f}_{\text{vol}}(T_{\bar{g}}) = \text{f}_{\text{vol}}(T_{\bar{b}}), \\
\text{f}_{\text{vol}}(Y_{+r}) &= \text{f}_{\text{vol}}(Y_{+g}) = \text{f}_{\text{vol}}(Y_{+b}) = \text{f}_{\text{vol}}(Y_{+\bar{r}}) = \text{f}_{\text{vol}}(Y_{+\bar{g}}) = \text{f}_{\text{vol}}(Y_{+\bar{b}}), 
\end{align*}
\]  

(7)

where the function \( \text{f}_{\text{vol}}(P) \) returns the volume of the partition argument \( P \)—see Figure 2.

Following [4], \( T \), is between the “outer confinement horizon” (i.e. the event horizon in \( Y_{+} \)) and the “inner confinement horizon” (i.e. the cauchy horizon in \( Y_{-} \)) so \( q_r, q_{\bar{r}}, q_g, q_{\bar{g}}, q_b, q_{\bar{b}} \in T \) experience the dual confinement scenario—recall Figure 1. Moreover, \( T \) is a Mott insulator [4], thus if its temperature permits it to operate as a conductor, then we can populate the zones of eq. (2) with quasi-particles for a toroidal vortex of current that winds, twists, and deforms. We recall that \( T \), which acquires a Berry phase, is simultaneously dual to both \( Y_{-} \) and \( Y_{+} \) 3-branes [4, 8], so any such toroidal vortex of \( T \) is dual to the six-coloring supercurrent vortices that circulate the corresponding thin flux tubes in \( Y_{-} \) and \( Y_{+} \)—an interconnected vacuum model for QCD and QED. See Table 1 for a list of the 18 zones of the six-coloring Gribov vacuum.

At this point, we’ve established and labeled the 18 zones of the six-coloring Gribov vacuum in Riemannian dual 4D space-time [4, 8]. Thus, we are ready to introduce the protium atom into the three-coloring “matter zones” of the vacuum. Hence, let there exist a protium atom in \( Y \). The protium’s proton is centered at the origin-point \( O \in Y_{-} \) with the amplitude-radius \( \epsilon_{\text{proton}} = 2M_{\text{proton}} \) and the effective mass \( M_{\text{proton}} = 1 \text{ GeV} \), while
Fig. 2: A depiction of the (empty) six-coloring Gribov vacuum in the Riemannian dual space-time topology [4, 8]. The 18 zones of this vacuum in $Y$ can be populated with point-particles and/or quasi-particles of fractional statistics for the protium-antiprotium pair. The superluminal Micro Space Zone $Y_r \subset Y$, the luminal Time Zone $T \subset Y$, and the luminal (or optionally sub-luminal) Macro Space Zone $Y_+ \subset Y$ are partitioned into $Y_{-r}, Y_{-g}, Y_{-b}, Y_{-r}, Y_{-g}, Y_{-b} \subset Y_-, T_r, T_g, T_b, T_r, T_g, T_b \subset T$, and $Y_{+r}, Y_{+g}, Y_{+b}, Y_{+r}, Y_{+g}, Y_{+b} \subset Y_+$, respectively. Note: This is a dual 3D space-time “slice” of the dual 4D space-time.
Table 1: A summary of the 18 zones comprising the six-coloring Gribov vacuum in the Riemannian dual space-time topology with the Inopin Holographic Ring [4, 8]. The zones can be populated with point-particles and/or quasi-particles. $T$ is a Mott insulator, so it may be an insulator or conductor, depending on temperature.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Contains</th>
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<tbody>
<tr>
<td>Red Micro Space Zone</td>
<td>$Y_{-r} \subset Y_-$</td>
<td>Red-magnetic supercurrent vortex; Thin red-electric flux tube</td>
</tr>
<tr>
<td>Green Micro Space Zone</td>
<td>$Y_{-g} \subset Y_-$</td>
<td>Green-magnetic supercurrent vortex; Thin green-electric flux tube</td>
</tr>
<tr>
<td>Blue Micro Space Zone</td>
<td>$Y_{-b} \subset Y_-$</td>
<td>Blue-magnetic supercurrent vortex; Thin blue-electric flux tube</td>
</tr>
<tr>
<td>Antired Micro Space Zone</td>
<td>$Y_{-\bar{r}} \subset Y_-$</td>
<td>Antired-magnetic supercurrent vortex; Thin antired-electric flux tube</td>
</tr>
<tr>
<td>Antigreen Micro Space Zone</td>
<td>$Y_{-\bar{g}} \subset Y_-$</td>
<td>Antigreen-magnetic supercurrent vortex; Thin antigreen-electric flux tube</td>
</tr>
<tr>
<td>Antiblue Micro Space Zone</td>
<td>$Y_{-\bar{b}} \subset Y_-$</td>
<td>Antiblue magnetic supercurrent vortex; Thin antiblue-electric flux tube</td>
</tr>
<tr>
<td>Red Time Zone</td>
<td>$T_r \subset T$</td>
<td>Red toroidal current (temperature dependent)</td>
</tr>
<tr>
<td>Green Time Zone</td>
<td>$T_g \subset T$</td>
<td>Mott insulator ring green segment; Blue toroidal current (temperature dependent)</td>
</tr>
<tr>
<td>Blue Time Zone</td>
<td>$T_b \subset T$</td>
<td>Mott insulator ring blue segment; Antired toroidal current (temperature dependent)</td>
</tr>
<tr>
<td>Antired Time Zone</td>
<td>$T_{\bar{r}} \subset T$</td>
<td>Mott insulator ring antired segment; Antigreen toroidal current (temperature dependent)</td>
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<tr>
<td>Antigreen Time Zone</td>
<td>$T_{\bar{g}} \subset T$</td>
<td>Mott insulator ring antigreen segment; Antiblue toroidal current (temperature dependent)</td>
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<tr>
<td>Antiblue Time Zone</td>
<td>$T_{\bar{b}} \subset T$</td>
<td>Mott insulator ring antiblue segment; Thin antiblue-electric flux tube</td>
</tr>
<tr>
<td>Red Macro Space Zone</td>
<td>$Y_{+r} \subset Y_+$</td>
<td>Red-electric supercurrent vortex; Thin red-magnetic flux tube</td>
</tr>
<tr>
<td>Green Macro Space Zone</td>
<td>$Y_{+g} \subset Y_+$</td>
<td>Green-electric supercurrent vortex; Thin green-magnetic flux tube</td>
</tr>
<tr>
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<tr>
<td>Antired Macro Space Zone</td>
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</table>

the protium’s electron is a point-particle in a circular orbit centered at $O \in Y_-$ with the amplitude-radius $\epsilon_{electron}$ and the effective mass $m_{electron} = 0.5 \times 10^{-4}$ GeV. Here, $T$ serves as a spherical shell for the proton, where its quarks $q_r, q_g, q_b \in T$ are confined to $T_r, T_g, T_b \subset T$, respectively. Thus,
to encode the protium atom, we populate the 9 pertinent effective mass zones \(Y_{-r}, Y_{-g}, Y_{-b} \subset Y_{-}, T_{r}, T_{g}, T_{b} \subset T\), and \(Y_{+r}, Y_{+g}, Y_{+b} \in Y_{+}\) with point-particles and quasi-particles. In each Micro Space Zone \(Y_{-r}, Y_{-g}, Y_{-b} \subset Y_{-}\), there is a thin color-electric flux tube circulated by a corresponding color-magnetic supercurrent vortex. Moreover, in each Macro Space Zone \(Y_{+r}, Y_{+g}, Y_{+b} \subset Y_{+}\), there is a thin color-magnetic flux tube circulated by a corresponding color-electric supercurrent vortex for the protium’s electron. Additionally, along the Time Zone \(T\), there is toroidal vortex of current that spans all \(T\)’s zones and is simultaneously dual to \(Y_{-}\) and \(Y_{+}\). See Figure 3 for a complete depiction of the protium atom and recall Table 1.

Next, we are ready to introduce the antiprotium atom, which, in accordance to the antisymmetry constraints of \([4, 8]\), is dual, opposite, reverse, and inverse to the protium atom. Thus, we are ready to introduce the antiprotium atom into the three-anticoloring “antimatter zones” of the upgraded Gribov vacuum. Hence, let there exist an antiprotium atom in \(Y\) (which is paired with the protium atom in Figure 3). The antiprotium’s antiproton is centered at \(O \in Y_{-}\) with the amplitude-radius \(\epsilon_{\text{antiproton}} = 2M_{\text{antiproton}} = 2M_{\text{proton}} = \epsilon_{\text{proton}}\) and the effective antimass \(M_{\text{antiproton}} = M_{\text{proton}} = 1\ \text{GeV}\), while the antiprotium’s positron is a point-particle in a circular orbit centered at \(O \in Y_{-}\) with the amplitude-radius \(\epsilon_{\text{positron}} = \epsilon_{\text{electron}}\) and the effective antimass \(m_{\text{positron}} = m_{\text{electron}} = 0.5 \times 10^{-4}\ \text{GeV}\). Here, \(T\) serves as a spherical shell for the antiproton, where its antiquarks \(\bar{q}_{r}, \bar{q}_{g}, \bar{q}_{b} \in T\) are confined to \(T_{\bar{r}}, T_{\bar{g}}, T_{\bar{b}} \subset T\), respectively. Thus, to encode the antiprotium atom, we populate the 9 pertinent effective antimass zones \(Y_{-\bar{r}}, Y_{-\bar{g}}, Y_{-\bar{b}} \subset Y_{-}, T_{\bar{r}}, T_{\bar{g}}, T_{\bar{b}} \subset T\), and \(Y_{+\bar{r}}, Y_{+\bar{g}}, Y_{+\bar{b}} \in Y_{+}\) with point-particles and quasi-particles. In each Micro Space Zone \(Y_{-\bar{r}}, Y_{-\bar{g}}, Y_{-\bar{b}} \subset Y_{-}\), there is a thin anticolor-magnetic flux tube circulated by a corresponding anticolor-magnetic supercurrent vortex. Moreover, in each Macro Space Zone \(Y_{+\bar{r}}, Y_{+\bar{g}}, Y_{+\bar{b}} \in Y_{+}\), there is a thin anticolor-magnetic flux tube circulated by a corresponding anticolor-electric supercurrent vortex for the antiprotium’s positron. Additionally, along the Time Zone \(T\), there is toroidal vortex of current that spans all \(T\)’s zones and is simultaneously dual to \(Y_{-}\) and \(Y_{+}\). See Figure 4 for a complete depiction of the antiprotium atom and recall Table 1.

Hence, at this point, we’ve given a preliminary demonstration on how a
Fig. 3: The three-coloring Gribov vacuum in the Riemannian dual space-time topology [4, 8] for the protium atom. In this depiction, $T$, which is a Mott insulator, is operating as an insulator so $T$'s zones are not currently populated with toroidal current quasi-particles due to temperature.
Fig. 4: The three-anticoloring Gribov vacuum in the Riemannian dual space-time topology [4, 8] for the antiprotium atom. In this depiction, $T$, which is a Mott insulator, is operating as an insulator so $T$’s zones are not currently populated with toroidal current quasi-particles due to temperature.
protium atom and its corresponding antiprotium atom can be modeled with
the six-coloring Gribov vacuum in Riemannian dual 4D space-time [4, 8].

3 Conclusion
In this preliminary paper, we introduced the protium and antiprotium
atoms to the six-coloring Gribov vacuum in Riemannian dual 4D space-time
of [4, 8]. We began by summarizing the pertinent aspects of V. N. Gribov’s
20-year-long research project on his theory of quark confinement, which
helped forge our understanding of QCD as the microscopic dynamics of
hadrons [1, 2, 3]. Next, we exemplified the analytic quark confinement and
baryon-antibaryon duality proof of [4, 8] by explaining how its core topol-
ogy and vacuum are fundamentally based on, and inspired by, Gribov’s work
[1, 2, 3]. Additionally, we summarized key concepts such as the six-coloring
kagome antiferromagnet and triangular chirality, the Inopin Holographic
Ring and space-time duality, the non-Abelian color-electric-magnetic \(q\bar{q}\)
dipoles, the correlated order parameter configurations, the quasi-particle
fractional statistics and excitations, and the Skyrmion connection [4, 8],
where all of these formulations may be applied to a model of protium and
antiprotium. It is important to investigate alternative models of protium for
two reasons: first, because it is universally abundant and second, because
it is structurally simple.

Next, in Section 2, we prepared the six-coloring Gribov vacuum [4]
by partitioning and labeling its 18 zones for point-particles and/or quasi-
particles—recall Figure 2. We explained how the vacuum zones are inter-
connected through the dualities and antisymmetries of [4, 8], while indi-
cating the types of thin flux tubes and supercurrent vortices that populate
the 3-brane Space Zones, and current toroidal vortex that populates the
Time Zone (depending on temperature)—recall Table 1. Subsequently, we
modeled the protium and antiprotium atoms with the six-coloring Gribov
vacuum in the dual 4D space-time [4, 8] with systematic explanations com-
bined with visual depictions—recall Figures 3–4. The results suggest that
it may be possible to generalize this framework to all atomic elements, but
this framework is still rudimentary and should therefore be subject to se-
rious scientific scrutiny, collaboration, refinement, and investigation. Also,
 it may be beneficial to match the predictions of this theory with real-time
astronomical data and conduct high-energy physics experiments to prove (or disprove) pertinence to Santilli’s Hadronic Mechanics [11, 12, 13, 14].

References