It is shown that neutral currents map 1-1 to charged currents and that charge conservation implies mass conservation. This has consequences for quantum theory, quantum field theory, and cosmology, which are explored.

1 Introduction

[2] began with Maxwell’s equations in its covariant form

\[ \square A^\mu = j^\mu, (0 \leq \mu \leq 3), \]

where the \( A^\mu, j^\mu \) are functions of time \( t = x^0 \) and space coordinates \( x^1, x^2, x^3 \), and \( c \equiv 1 \) is understood. Because \( A = (A^0, \cdots, A^3) \) is a 4-vector, it was deduced that

\[ J := j^0 \gamma^0 + \cdots + j^3 \gamma^3 \]

must be Lorentz invariant, where \( \gamma^\mu, (0 \leq \mu \leq 3) \), are the Dirac matrices, given by

\[
\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \\
\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \text{and} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.
\]

This then led to the rule that a stationary charge density \( \rho_0(t, x) \gamma_0 \) of either electrons or positrons transforms under a boost \( \Lambda_v \) to a velocity \( v \in \mathbb{R}^3 \) according to:

\[ \Lambda_v(\rho_0 \gamma_0) = \rho_0(\gamma_0 + \gamma \cdot v)^{-1}. \]
The matrix \( C := -\gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) was then identified as the charge inversion, therefore, with \( \rho_0 \) being a real or complex valued function, \( \rho_0 \gamma_0 \) is representing a stationary charge density, and the additional (velocity-dependent) flux part \( \rho_0 (\gamma_0 + \gamma \cdot v)^{-1} - \rho_0 \gamma_0 = \rho_0 \sum_{n \in \mathbb{N}} (-1)^n \gamma_0 (\gamma \cdot v)^n \) is neutral, i.e. invariant w.r.t. \( C \).

2 \hspace{1em} \textbf{The Neutral Particles}

As charge inversion, \( C \) interchanges particles with antiparticles. We can therefore identify the projection \( \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \) with the projection onto the electron particles and \( \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \) with the projection onto the positrons.

Hence, for every charge density \( \rho \gamma_0 \) there is a unique neutral density \( \rho 1_4 = \rho \left( \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right) \), and incidentally it is seen that the result of adding a stationary density distribution \( f \) of electrons to a stationary density distribution \( g \) of positrons is \( \begin{pmatrix} f & 0 \\ 0 & g \end{pmatrix} \), and it is not the difference of the two.

Let’s see how neutral particles transform under boosts \( \Lambda_v \): Because \( J := j^0 \gamma^0 + \cdots + j^3 \gamma^3 \) is Lorentz invariant, \( \gamma_0 J \) also is Lorentz invariant. Therefore a neutral particle density \( \rho_0 1_4 \) maps under \( \Lambda_v \) to

\[
\Lambda_v (\rho_0 1_4) = \rho_0 \left( 1_4 + \gamma_0 (\gamma \cdot v) \right)^{-1}.
\]

Again, because the above is symmetric w.r.t the charge inversion \( C \), the neutral charge density \( \rho_0 1_4 \) is added a neutral amount of charge density under a boosts \( \Lambda_v \).

In all, charges are up to unitary transformations the elements of the vector space of \( 4 \times 4 \)-matrices spanned by boosts of the diagonal matrices \( \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \) with \( a, b \in \mathbb{C} \).

Their natural norm then is what is called their mass. Every charge and every composite of the charges unequal zero therefore always has a strictly positive mass. Mass is to the charges what the radius is to the Euclidean polar coordinate system. Energy is positively proportional to mass, and therefore, energy will also always be non-negative.

3 \hspace{1em} \textbf{Mass Conservation}

In [4] it was shown that the Lorentz gauge condition is equivalent to the conservation of the rest charge under Lorentz transformations. It means that the rest charge within a given volume of space is a Lorentz invariant. One might think that this was well-understood. But wait:

Multiplying the charge conservation equation

\[
\partial j^0 / \partial x^0 + \cdots + \partial j^3 / \partial x^3 = 0
\]

by \( \gamma_0 \) gives the conservation law for neutral composites, and then along with all charges and all composites, the (rest) mass is to be conserved either. That is,
the energy is to be conserved within the particles themselves. That implies a.o.
that in a overhead collision of a particle with its antiparticle the total mass of
the two particles stays constant at all times. And it means that in the process
of atomic fission, the rest mass of the chunk of atoms has to stay constant at all
times. (Note that the rest mass of a compound system is the mass at compound
system’s rest; it is not however the sum of the rest masses of its composites, but
also contains internal potential and kinetical energy between the composites.)
But now, since there is no energy left within the systems of particles, energy
conservation forbids an energy transfer from a particle to its electromagnetic
field, a concept that was first discussed by A. Einstein in [1]. And it was just
this paper which laid the foundation for quantum theory and its Kopenhagen
interpretation.

So, the inconspicuous, well-known charge conservation challenges the funda-
mental concept of fields within quantum theory. Accepting charge conservation,
let us ask what would need to change:

4 Consequences

First, with charge being conserved, the particles under themselves define a com-
plete, self-contained theory (right hand side of Maxwell’s equations). Then the
fields (the left hand side of Maxwell’s equation) must be an equivalent, dual
theory by itself, and we are back to the mechanical concept of fields, by which
a field equivalently defines the particles’ state through a field action, and vice
versa (see e.g. [2]). So, rather than to say that the charged particles are suppy-
ling power to the electromagnetic field, one would conceive work being done on
the charged particles which accelerates these particles. At each instance of local
time, the particle states get radiated gratis to the outside, and outside particles
can then react with the fields, leading to a backwards radiation. With that we’d
loose two quantum mechanical principles: first, we’d loose the undeterminacy
principle, because particle and field will be equivalent, rather than complemen-
tary. And secondly, due to the lack of interaction of a particle with its own
field, bare charge infinities are missing, which means that calculations would
start with the renormalized quantities from scratch. Other than that, however,
the mathematical quantum mechanical relations still apply, as was shown in [3].

For quantum field theory, the notion of a vacuum state will have to be re-
placed by a state of n particle-antiparticle pairs, where such pairs should behave
like neutrinos, so, neglecting the gravitational interaction, should move freely.
That ”sea” of particle-antiparticles has a well-defined, finite and positive (rest)
mass/energy proportional to its number. Other than that, the mathematical
aparatus of that theory will still supply.

What will change considerably, however, is standard cosmology: Again, the
vacuum state as the source of a big bang will have to be replaced by an amount
of particle-antiparticles matching the energy of the universe (subjected to the
force of gravitation). The size of that ball of that matter is to be huge, probably
several light years of diamater, and not of the size of a golf ball. That makes
the difference, as this will avoid two serious problems of the standard theory:

The first problem has often been discussed before. It is that the standard model demands the big bang to be started at highest temperature in a state of thermal equilibrium. That sets the start condition to maximal temperature and maximal entropy. So, as the universe expands, temperature will drop, but the entropy will stay maximal. In order to achieve a state transition, a fluctuation (impurification) will be needed, but at maximal entropy the system is in the equilibrium state, which is the state of maximal fluctuation at the given temperature, so no additional fluctuation is possible.

The second problem is that we appear to be sited with our galaxy in a privileged location: whereas our galaxy is observed to be relatively stable, the speed of galaxies is accelerating radially towards the horizon, so that outer, far away galaxies are encountering a (perhaps virtual) force which must lead to a distorted, different view than we have. In other words, we appear to be living near the center of an exploding universe. That however would be highly improbable, unless our galaxy was given the bliss of a late birth. In other words, the big bang would be demanded to have had occurred within a macroscopic period of time of perhaps days or years, but not within a nano second.

5 Conclusion

Basing big bang on a huge bulk of matter of particle-antiparticles would overcome both, the entropy problem (entropy would be maximal only locally) and the big bang duration problem (big bang could have even happened in stages over some macroscopic time period).

References


