The structurization of a set of positive integers and its application to the solution of the Twin Primes problem

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Abstract

One of causes why Twin Primes problem was unsolved over a long period is that pairs of Twin Primes (PTP) are considered separately from other pairs of Twin Numbers (PTN). By purpose of this work is research of connections between different types of PTN. For realization of this purpose by author was developed the "Arithmetic of pairs of Twin Numbers" (APTN). In APTN are defined three types PTN. As shown in APTN all types PTN are connected with each other by relations which represent distribution of prime and composite positive integers less than 2n between them. On the basis of this relations (axioms APTN) are deduced formulas for computation of the number of PTN (NPTN) for each types. In APTN also is defined and computed Average value of the number of pairs are formed from odd prime and composite positive integers $< 2n$. Separately AVNPP for prime and AVNPC for composite positive integers. We also deduced formulas for computation of deviation NPTN from AVNPP and AVNPC. It was shown that if n go to infinity then NPTN go to AVNPC or AVNPP respectively that permit to apply formulas for AVNPP and AVNPC to computation of NPTN. At the end is produced the proof of the Twin Primes problem with help of APTN. It is shown that if n go to infinity then NPTP go to infinity.

Keywords: Twin Primes,The solution of the Twin Primes Problem,The proof of Twin Primes Conjecture.

1 Introduction

A twin prime is a prime number that differs from another prime number by two, for example the twin prime pair (41, 43). Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin. The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime problem, which states There are infinitely many primes p such that p + 2 is also prime.

2 Method

We separate N on n subset which we call by segments. At that any positive integer > 0 is represented in the structured set of positive integers(SSPI) as n = BI + r
Where: I = [n/B] - The number of segment.(integer part of n/B)
K = I + 1 - the number of segments.
(+1) takes into account nul segment.
I = 0,1,2,3,...
K = 1,2,3,...
B - the quantity of elements in segment.
B = 10m; m = 1,2,3,...
r = n-BK - the position of integer into segment.
r = 1,2,3,...(B-1)
n- positive integer > 0 .

For B=10 each segment has the following structure: five even integers, five odd integers in the view of: 10K+1, 10K+3, 10K+5,10K+7,10K+9. But 10K+5 is composite then only four of them can form pairs of Twin Numbers (PTN) : (10K+1; 10K+3) and (10K+7; 10K+9) In depend of that are (10K+r) prime or composite there are three types PTN. One of them is PTP i.e. pairs of Twin Primes. Odd primes and odd composites are distributed between types of PTN by balances which underlied "Arithmetic Pairs of Twin Numbers" (APTN) which permits to compute the number of pairs of each type specifically for Twin Primes(H). Next the equation for the number of the Twin Primes is transformed to H(n) and it is computed lim H(n) which at n work for infinity equals (+infinity). Thus the number of the Twin Primes is infinitely.

3 The arithmetic of pairs of TWIN numbers. (APTN)

3.1 General conception

Definition 3.1 *The structurization of a set of positive integers is its segmentation on K segments. At that any positive integer* > 0 *is represented in the structured set of positive integers(SSPI) as:*

$$n = BI + r \tag{1}$$

Where: I = [n/B] - The number of segment.(integer part of n/B)
K = I + 1 - the number of segments.
(+1) takes into account nul segment.
I = 0,1,2,3,...
K = 1,2,3,...
B - the quantity of elements in segment.
B = 10m; m = 1,2,3,...
r = n-BK - the position of integer into segment.
r = 1,2,3,...(B-1).
n- positive integer > 0.

Definition 3.2 *PTN - pairs of integers that differs from another number by two.*

Definition 3.3 *PTP- pairs of prime integers that differs from another prime number by two,*

Definition 3.4 *At B = 10 each segment has the following structure:*
five even integers in the form of: 10K + 0; 10K + 2;
10K + 4; 10K + 6; 10K + 8;
and five odd integers in the form of 10K + 1; 10K + 3;
10K + 5; 10K + 7; 10K + 9.
.

Definition 3.5 *There are in each segment the following PTN:*
(10K + 1; 10K + 3)and(10K + 3; 10k + 5)and(10k + 5;10K + 7) and(10K + 7; 10K + 9) and (10K + 9; 10(K+1) + 1) which is located in the neighbouring segments.

$$K = [n/10] + 1 \tag{2}$$

Definition 3.6 *A set of pairs Twin Numbers (SPTN) is a set by elements of which are PTN in the form by definition 3.5 imported from each segment. Pairs (10K + 3; 10k = 5)and(10k + 5;10K + 7) are excluded from SPTN since PTN with (10k + 5) which is always composite canno't be PTP and PTN which are included into SPTN can be any type as follows below.*

3.2 The types of PTN into SPTN

Let (x;y) is PTN . In depend of that are x,y prime or composite it can be three types PTN:

Definition 3.7 *There is Type "H"(Twin Primes) if x-odd prime positive integer or "1"*
and y- odd prime positive integer.
If 10K + 1 is odd prime or "1" and 10K + 3; is odd prime we have one pair of Twin primes in segment.
At that if 10K + 7 is odd prime and 10K + 9; is odd prime we have one pair of Twin primes in segment.
At that if 10K + 9 is odd prime and 10(K+1) + 1; is odd prime we have one pair of Twin primes in segment
AT that if 10K + 1 ; 10K + 3 ; 10K + 7 ; 10K + 9 ; are odd primes we have two pairs Twin primes in segment.
AT that if 10K + 1 ; 10K + 3 ; 10K + 7 ; 10K + 9 ;10(K+1) + 1 are odd primes we have three pairs Twin primes in segment.
$|H|$*- the number of PTN of Type "H" (Twin primes).*
$|H|$*- positive integer* $> 0 \; \forall \; n > 0$.

Definition 3.8 *There is Type "Q" if x-odd composite positive integer;*
and y- odd composite positive integer .
At that if x = 10K + 7; y = 10K + 9; are odd composite we have one PTN type "Q" in segment.
At that if x = 10K + 1; y = 10K + 3;are odd composite we have one PTN type "Q" in segment.
At that if x = 10K + 9; y =10(K+1) + 1; are odd composite we have one PTN type "Q"in segment.
At that if x,y = 10K + 1; 10k + 7; 10K + 3; 10K + 9 are odd composite we have two PTN type "Q" in segment.
At that if x,y = 10K + 1; 10k + 7; 10K + 3; 10K + 9
and 10(K+1)+1 are odd composite we have three PTN type "Q" in segment.

$|Q|$- the number of PTN of Type "Q".
$|Q|$- positive integer $> 0 \; \forall \; n > 90$ ..

Definition 3.9 *There is Type "L" if x- odd prime positive integer or "1" and y- odd composite positive integer or x- odd composite positive integer and y - odd prime positive integer.*
At that if x = 10K + 1; is odd prime y = 10K + 3; is odd composite we have one PTN type "L" in segment.
At that if x = 10K + 1; is odd composite y = 10K + 3; is odd prime we have one PTN type "L" in segment.
At that if x = 10K + 7; is odd prime y = 10K + 9; is odd composite we have one PTN type "L" in segment.
At that if x = 10K + 7; is odd composite y = 10K + 9; is odd prime we have one PTN type "L" in segment.
At that if x = 10K + 9; is odd prime y = 10(K+1) + 1; is odd composite we have one PTN type "L" in segment.
At that if x = 10K + 9; is odd composite y = 10(K+1) + 1; is odd prime we have one PTN type "L" in segment.
At that if 10K + 1; 10K + 7; are odd primes and 10K + 3; 10K + 9; are odd composites we have two PTN type "L" in segment.
At that if 10K + 1; 10K + 7; are odd primes and 10K + 3; 10K + 9; are odd composites we have two PTN type "L" in segment.
t that if 10K + 1; 10K + 7; are odd composites and 10K + 3; 10K + 9; are odd primes we have two PTN type "L" in segment.
t that if 10K + 1; 10K + 7; are odd primes and 10K + 3; 10K + 9; are odd composites we have two PTN type "L" in segment.
t that if 10K + 1; 10K + 7; 10(K+1) + 1; are odd primes and 10K + 3; 10K + 9; are odd composites we have THree PTN type "L" in segment.
t that if 10K + 1; 10K + 7; 10(K+1) + 1; are odd composites and 10K + 3; 10K + 9; are odd primes we have THree PTN type "L" in segment.
$|L|$ - the number of PTN of Type "L".
$|L|$ - positive integer $> 0 \; \forall \; n > 6$ ..

Definition 3.10 *BPTN in the view of (10K + 9; 10(K+1) + 1) is border PTN which is formed by integers from two PTN:*
in the view of (10K + 7; 10K + 9) and (10(K+1) + 1; 10(K+1) + 3) are located in the neighbouring segments.
If (10K + 9;) and (10(K+1) + 1) are odd pmes then BPTN is type "$|H|$".
IF (10K + 9) is odd prime and (10(K+1) + 1)
is odd composite then BPTN is type "$|L|$" . IF (10K + 9) is odd composite and (10(K+1) + 1) is odd prime then BPTN is type "$|L|$".

If (10K + 9;) and (10(K+1) + 1) are odd composite then BPTN is type "$|Q|$".

3.3 The axioms of APTN

Definition 3.11 *p- the number of noncomposite integers (primes and "1") into SPTN* $< n$
$p = round(\pi(n) + 1)$;
round - round -up to the nearest integer.
$p - positive\ integer\ >\ 1\ \forall\ n\ >\ 1.$

Definition 3.12 $\pi(n) - the\ number\ of\ primes\ <\ n$

Axiom 3.1 *The number of the PTN type H (NPTNH) in SPTN is connected with the number of the binary representations type L (NPTNL) as follows:*

$$2|H| + |L| = p \quad (3)$$

The expression(3) asserts that odd noncomposite positive integers less than n are allotted to types "H","L" in compliance with balance (3).

Remark 3.1 *In these equations is division by '2'. From this it can conclude that equations are correct only for n is even.*
But it is not the case. Below we show that in APTN all equations in which is devision by '2' It always is possibility for any n's and p's parity.

Proposition 3.1 *The equation (3) is true for any n's and p's parity.*

Proof 3.1 *For n is even p is even*
By equation (3) we get: $|H| = (p - |L|)/2$.
By equation (10) $|L| = 4([n/10] + 1) - p - 2|Q|$ *and* $|L|$ *is even*
for n is even p is even then $(p - |L|)$ *is even .*
Hence the division by"2" is possibility then $|H|$ *is integer .*
Thus equation (3) is true for n is even p is even.

Next for n is even ; p is odd
By equation (3) we get: $|H| = (p - |L|)/2$.
By equation (10) $|L| = 4([n/10] + 1) - p - 2|Q|$ *and* $|L|$ *is odd*
for n is even and p is odd then $(p - |L|)$ *is even .*
Hence the division by"2" is possibility then $|H|$

is integer .
Thus equation (3) is true for n is even p is odd.

Next for n is odd or odd prime; p is even
By equation (3) we get: $|H| = (p - |L|)/2$ *.*
By equation (10) $|L| = 4([n/10] + 1) - p - 2|Q|$ *and* $|L|$ *is even*
for n is odd or odd prime and p is even then $(p - |L|)$*is even .*
Hence the division by"2" is possibility then $|H|$
is integer .
Thus equation (3) is true for n is odd or odd prime; p is even.

Next for n is odd or odd prime ; p is odd
By equation (3) we get: $|H| = (p - |L|)/2$ *.*
By equation (10) $|L| = 4([n/10] + 1) - p - 2|Q|$ *and* $|L|$ *is odd*
for n is odd or odd prime and p is odd then $(p - |L|)$*is even .*
Hence the diivision by"2" is possibility then $|H|$ *is integer .*
Thus equation (3) is true for n is odd or odd prime; p is odd.
Finally equation (3) is true for any n's and p's parity. □

Definition 3.13 s_o *- the number of odd composite positive integers into SPTN* $< n$*.*
s_o *- positive integer* $> 0 \ \ \forall\ n > 4$

Axiom 3.2 *The number of the binary representations Type Q (NBRQ) is connected with the number of the binary representations Type "L" (NBRL) as follows:*

$$2|Q| + |L| = s_0 \tag{4}$$

$\forall\ n > 1$ *.*
The expression(4) asserts that odd composite positive integers less than n are allotted to types "Q","L" in compliance with balance (4).

Proposition 3.2 *The equation (4) is true for any n's and p's parity.*

Proof 3.2 *For n is even ; p is even*
By equation (4) we get: $|Q| = (s_0 - |L|)/2$ *.*
By equation (13) $|L| = p - 2|H|$ *and* $|L|$ *is even*
for n is even and p is even .
By equation (6) $s_o = 4([n/10] + 1) - p$ *is even then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$
is integer .

Thus equation (4) is true for n is even ; p is even.
Next for n is even p is odd
By equation (4) we get: $|Q| = (s_0 - |L|)/2$.
By equation (13) $|L| = p - 2|H|$ *and*$|L|$ *is odd*
for n is even ; p is odd .
By equation (6) $s_0 = 4([n/10] + 1) - p$ *is odd then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$
is integer .
Thus equation (4) is true for n is even ; p is odd.

Next for n is odd or odd prime , p is even
By equation (4) we get: $|Q| = (s_0 - |L|)/2$.
By equation (13) $|L| = p - 2|H|$ *and* $|L|$ *is even*
for p is even .
By equation (6) $s_0 = 4([n/10] + 1) - p$ *is even then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$
is integer .
Thus equation (4) is true for n is odd or odd prime ; p is even.
Next for n is odd or odd prime p is odd
By equation (4) we get: $|Q| = (s_0 - |L|)/2$.
By equation (13) $|L| = p - 2|H|$ *and* $|L|$ *is odd*
for p is odd .
By equation (6) $s_0 = 4([n/10] + 1) - p$ *is odd then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$ *is integer .*
Thus equation (4) is true for n is odd or odd prime p is odd.
Finally equation (4) is true for any n's and p's parity. □

Definition 3.14 PTN_1 *is PTN in the view of (10K + 1; 10K + 3) .* PTN_2 *is PTN in the view of (10K + 7; 10K + 9) .*

Remark 3.2 *Equations (3)and(4) are true for* PTN_1, PTN_2 *with regard to BPTN that* $|H|, |L|, |Q|$ *for it are computed indirectly with help of them connections with other PTN specifically in the view of (10K + 7; 10K + 9) and (10(K+1) + 1; 10(K+1) + 3).*

3.4 Axioms of distribution

Definition 3.15 $|H|_1, |H|_2$*- the number of PTN of Type "H" for* PTN_1, PTN_2.

Definition 3.16 $|H|_b$*- the number of PTN of Type "H" for BPTN.*

Definition 3.17 $|L|_1, |L|_2$*- the number of PTN of Type "L" for* PTN_1, PTN_2.

Definition 3.18 $|L|_b$- *the number of PTN of Type "L" for BPTN.*

Definition 3.19

Axiom 3.3 *Odd noncomposite positive integers less than* n *are allotted to PTN1 and PTN2 i.e.* $|H|_1 + |H|_2 = |H|$, $|L|_1 + |L|_2 = |L|$

Definition 3.20 $|Q|_1, |Q|_2$- *the number of PTN of Type "Q" for* PTN_1, PTN_2.

Definition 3.21 $|Q|_b$- *the number of PTN of Type "Q" for BPTN.*

Axiom 3.4 *Odd composite positive integers less than* n *are allotted to* $PTN1$ *and* PTN_2 *i.e.* $|Q|_1 + |Q|_2 = |Q|$

Definition 3.22 $k_1 = |H|_1/|H| - \ coeffic.\ of\ proportionality\ for\ |H|_1$
$k_1\ is\ rational\ <\ 1\ \forall\ n$

Definition 3.23 $k_2 = |H|_2/|H| - \ coeffic.\ of\ proportionality\ for\ |H|_2$
$k_2\ is\ rational\ <\ 1\ \forall n$

Axiom 3.5 $|H|_1 = k_1|H|;\ |H|_2 = k_2|H|$

Definition 3.24 $k_3 = |L|_1/|L| - \ coeffic.\ of\ proportionality\ for|L|_1$
$k_3\ is\ rational\ <\ 1\ \forall n$

Definition 3.25 $k_4\ =\ |L|_2/|L| - \ coeffic.\ of\ proportionality\ for\ |L|_2$
$k_4\ is\ rational\ <\ 1\ \forall n$

Axiom 3.6 $|L|_1 = k_3|L|;\ |L|_2 = k_4|L|$

Definition 3.26 $k_5 = |Q|_1/|Q| - \ coeffic.\ of\ proportionality\ for\ |Q|_1$
$k_5\ is\ rational\ < 1\ \forall n$

Definition 3.27 $k_6 = |Q|_2/|Q| - \ coeffic.\ of\ proportionality\ for\ |Q|_2$
$k_6\ is\ rational\ < 1\ \forall\ n$

Axiom 3.7 $|Q|_1 = k_5|Q|;\ |Q|_2 = k_6|Q|$

Remark 3.3 *Axioms 3.5,3.6,3.7 are based on empirical data.*

3.5 axioms of additivity

Axiom 3.8 $|H| = |H|_0 + |H|_1 + \cdots + |H|_k$
Where : $|H|_k$ - the number of the pairs Twin Primes into each segmtnt.

Taking into account that into nul segment there is one pair of Twin Primes (1;3) then $|H| \ > \ 0 \ \forall \ n \ \ > \ 0$

Axiom 3.9 $|L| = |L|_0 + |L|_1 + \cdots + |L|_k$
Where : $|L|_k$ - the number of the pairs Twin Numbers of type "L" into each segmtnt.

Taking into account that into nul segment there is one pair of Twin Numbersof type "L" (7;9) then $|L| \ > \ 0 \ \forall \ n \ \ > \ 6$

Axiom 3.10 $|Q| = |Q|_0 + |Q|_1 + \cdots + |Q|_k$
Where : $|Q|_k$ - the number of the pairs Twin Numbers of type "Q" into each segment.

Taking into account that into segment 9 there is one pair of Twin Numbers type "Q" (91;93) then $|Q| \ > \ 0 \ \forall \ n \ > \ 90$

3.6 The computation of F, s_0, p

Proposition 3.3

$$F = 2([n/10] + 1) \tag{5}$$

Proof 3.3 *By definition 3.2 SPTN is formed by the way of import of two PTN from each segment hance F = 2K Taking into account (2) we get: F = 2([n/10] + 1)* □

Proposition 3.4

$$s_o = 4([n/10] + 1) - p \tag{6}$$

Proof 3.4 *The general number of odd positive integers in SPTN $(s_g) s_g = 2F$. Taking into account () F =2K then $s_g = 4K$ for computation s_0 it needs subtract p from s_g . Then $s_o = 4K - p$ .Taking into account (2)* $s_o = 4([n/10] + 1) - p$ □

Definition 3.28 $p_1 - the\ first\ approximation\ of\ p.$
$p_2 - the\ second\ approximation\ of\ p.$
$p_3 - the\ third\ approximation\ of\ p.$

Proposition 3.5

$$p_1 = round(n/lnn) \tag{7}$$

$$p_2 = round(n/lnn + n/ln^2n) \tag{8}$$

$$p_3 = round(n/lnn + n/ln^2n + 2n/ln^3n) \tag{9}$$

Proof 3.5 *As everybody knows* [*1*] *that the number of the primes less than* $2n$ *is expressed as follows:*
$\pi(n) = (n/lnn)\int_0^1(1 - (lny/lnn) + (ln^2y/ln^2n) + \ldots)dy$
We are limited to three of the first terms of the series.
Integrating in parts then we get:
$\pi(n) = (n/lnn)(1 + 1/lnn + 2/ln^2n)$
Whence taking into account definition (4.1) that p is (primes + 1) then we get :
$p = (n/lnn + n/ln^2n + 2n/ln^3n + 1)$
But taking into account that "2" is not odd prime and that by def. 4.1 p is the number of odd prime < 2n then
$p = (n/lnn + n/ln^2n + 2n/ln^3n + 1 - 1)$ *Finally we get:* $p = (n/lnn + n/ln^2n + 2n/ln^3n)$ *Whence we get:*
$p_1 = round(n/lnn);$
$p_2 = round(n/lnn + n/ln^2n);$
$p_3 = round(n/lnn + n/ln^2n + 2n/ln^3n)$ *;* □

3.7 The computation of $|H|, |L|, |Q|$

Proposition 3.6

$$|L| = 4([n/10] + 1) - p - 2|Q| \tag{10}$$

Proof 3.6 *By axiom 3.2 we get:*
$|L| = s_0 - 2|Q|$ *Taking into account (6) then we get:* $|L| = 4([n/10] + 1) - p - 2|Q|$ □

Remark 3.4 *From equation (10) it follows that* ($p + |L|$) *is even that it is possibility if* p $|L|$ *have equaL parity. Then* ($p - |L|$) *is also even .*

Proposition 3.7

$$|H| = |Q| - 2([n/10] + 1) + p \tag{11}$$

Proof 3.7 *Next subtracting (3) from (4) we get:*
$2|Q| - 2|H| = s_o - p$ *Whence we get:* $|H| = (2|Q| - s_0 + p)/2$ *Taking into account (6) then we get:* $|H| = |Q| - 2([n/10] + 1) + p$ □

Proposition 3.8

$$|Q| = (4([n/10] + 1) - p - |L|)/2 \tag{12}$$

Proof 3.8 *By axiom 3.2 we get:* $|Q| = (s_0 - |L|)/2$ *Taking into account (6) then we get:* $|Q| = (4([n/10] + 1) - p - |L|)/2$ *.*
By remark (3.2) $p + |L|$ *is even. Hence the division by"2" is possibility then* $|Q|$ *is integer .*
Thus equation (12) is true for any n □

Proposition 3.9

$$|L| = p - 2|H| \tag{13}$$

Proof 3.9 *From axiom 3.1 we get:* $|L| = p - 2|H|$ □

Proposition 3.10

$$|H| = (p - |L|)/2 \tag{14}$$

Proof 3.10 *From axiom 3.1 we get:* $|H| = (p-|L|)/2|$ *. By remark (3.2)* $p - |L|$ *is even. Hence the division by"2" is possibility then* $|H|$ *is integer .*
Thus equation (14) is true for any n □ *.*

Proposition 3.11

$$|Q| = 2([n/10] + 1) - p + H \tag{15}$$

Proof 3.11 *Subtract (3) from (4) then we get:* $|Q| - |H| = (s_0 - p)/2$ *; Whence* $|Q| = (s_0 - p + 2|H|)/2$ *Taking into account (6) then we get:* $|Q| = 2([n/10] + 1) - p + H$ □

Proposition 3.12

$$|Q| - |H| = 2([n/10] + 1) - p \tag{16}$$

Proof 3.12 *Subtracting equation (3) from equation(4) then we get:* $2|Q| - 2|H| = s_0 - p$ *Whence we get:* $|Q| - |H| = (s_0 - p)/2 =$ *. Taking into account equation (6) we get:*
$|Q| - |H| = ((4([n/10] + 1) - p - p)/2 = 2([n/10] + 1) - p)$ *.* □

Remark 3.5 *It was controlled of APTN for some values of n. For each value of n it was computed by direct computation the following:* $p, s_o, |Q|, |L|, |H|, F$ *for values* $< n$ *. Next we compare data direct computations with the same parameters which are computed by equations of APTN and we get full coincidence. The full text of the control of APTN see subsection (10.2) of appendices.*

Proposition 3.13 4 The limited values of possible range of $|Q|, |L|, |H|$

Definition 4.1 *$|H|_b$ -lower limit of possible range of $|H|$.*

Axiom 4.1

$$|H|_b = 0 \tag{17}$$

Definition 4.2 *$|L|_c$ -upper limit of possible range of $|L|$.*

Proposition 4.1

$$|L|_c = p \tag{18}$$

Proof 4.1 *Substituting lower limit of $|H|$ by (34) to (7) then we get upper limit for $|L|$: $|L|_c = p$* □

Definition 4.3 *$|Q|_b$ - lower limit of possible range of $|Q|$.*
$|Q|_b$- positive integer $> 0 \ \ \forall\ n\ >\ 319$
see subsection 10.1 (numerical solution $|Q|_b = 0$)

Proposition 4.2

$$|Q|_b = 2([n/10] + 1) - p \tag{19}$$

$n > 1$

Proof 4.2 *Substituting upper limit of $|L|$ by equation (37) to equation (8) then we get*
lower limit for $|Q|$: $|Q|_b = (S_o - p)/2$.
Substituting S_0 by equation (6), then we get:
$|Q|_b = (S_o - p)/2.|Q|_b = (4([n/10]+1)-2p)/2—Q—_b = 2([n/10]+1)-p.$ □

Proposition 4.3

$$|Q| - |H| = |Q|_b \tag{20}$$

$\forall\ n\ >\ 1$

Proof 4.3 *By equation (16) we have: $|Q| - |H| = (2([n/10] + 1) - p)$ By equation (19) we have: $|Q|_b = (2([n/10] + 1) - p)$. Whence we get: $|Q| - |H| = |Q|_b$.* □

5 Average value of the number of binary pairs are formed from odd composite positive integers $< n$

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Definition 5.1 *$S -$ ordered set of positive integers $< n$*
$s -$ element of S
$|S| = s_o -$ power of S
$s_i -$ vary over all s
$s_j -$ vary over all s

Definition 5.2 *$V = \{v_k | v_k \in N, v_k = (s_i; s_j)\}$*
is a set by elements of which are every possible binary pairs of odd composite integers $< n$. (each with all the rest)
$|V|$ - the power of set V.

Definition 5.3 *$W = \{w | w \in N, w = k, 0 < k \leq n\}$*
is a set of positive integers $< n$
$|W| -$ the power of set $W |W| = n$.

Definition 5.4 *$|Q|_m$ - mean quantity of binary pair $v_k = (s_i \; s_j)$ which can be formed of odd composite positive integers $< n$ and which are mapped into W by surjective mapping :*
$f : V \Rightarrow W$

$$|Q|_m = |V|/|W| \tag{21}$$

i.e. uniform mapping regardless of real.
$|Q|_m$ $-$ positive rational number > 0

$\forall \; n \; > \; 2.$

Proposition 5.1

$$|V| = s_o^2 \tag{22}$$

Proof 5.1 *The number of every possible binary pairs in the view of $v_k = (s_i; s_j)$ are formed of odd composite positive integers $< n$ is equal the power of Cartesian product: $S \; X \; S$ [2] [3]. Then:*
$|V| = s_o \cdot s_o = s_o^2 \;\; \square$

Proposition 5.2

$$|Q|_m = s_o^2/n \tag{23}$$

Proof 5.2 *By equations (21 and 22) we have :* $|Q|_m = |V|/|W| = s_o^2/2n$ □

Proposition 5.3

$$|Q|_m = (4([n/10]+1) - p)^2/n \tag{24}$$

$For\ n\ >\ 1$

Proof 5.3 *Substituting* s_o *by equation (6) to equation (23) then we get :*
$|Q|_m = (4([n/10]+1) - p)^2/2n;$
For $n > 1$ □

6 Average value of the number of binary pairs are formed from odd noncomposite positive integers $< n$

Definition 6.1 P *- ordered set of odd noncomposite positive integers* $< 2n$
p *- elements of* P
$|P| = p$ *- power of* P
p_i*- vary over all* p
p_j *- vary over all* p

Definition 6.2 $T = \{t_k | t_k \in N, t_k = (p_i; p_j)\}$
is a set by elements of which are every possible binary pairs of odd noncomposite integers $< n$*. (each with all the rest)*
$|T|$ *- the power of set* T*.*

Definition 6.3 $|H|_m$ *is mean quantity of binary pairs* $(p_i; p_j)$ *which can be formed of odd noncomposite positive integers* $< n$ *and which are mapped into* W *by surjective mapping :*
$f:\ T \Rightarrow\ W$

$$|H|_m = |T|/|W| \tag{25}$$

i.e. uniform mapping regardless of real.
$|H|_m$*-positive rational number* $> 0\ \ \forall n > 2.$

Proposition 6.1

$$|T| = p^2 \tag{26}$$

Proof 6.1 *The number of every possible binary pairs are formed of odd noncomposite positive integers* $<\ n$ *in the view of* $(p_i; p_j)$ *is equal the power of Cartesian product* $P\ X\ P$ *[2] [3]* .
Then $|T| = p \cdot p = p^2$ □

Proposition 6.2

$$|H|_m = p^2/n \tag{27}$$

$\forall\ n > 1$

Proof 6.2 *By equations (25 and 26) we have :* $|H|_m = p^2/2n\ \forall\ n > 1$ □

6.1 The relationship between n and p

Proposition 6.3

$$2([n/10] + 1) > p \tag{28}$$

$\forall\ n\ >\ 319$

Proof 6.3 *We need to prove that:*
$2([n/10] + 1) > p$. *Let* $f = 2([n/10] + 1) - p$. *We transform its to f(n) Substituting for p its the first order approximation by equation (20) and replacing [n/10]* $\approx n/10$ *we get:*
$f(n) = 2n/10 + 2 - n/lnn$
Next we computation the derivative :
$f'(n) = (2ln^2n/10 + lnn - 1)/ln^2n$
Whence $f'(n) > 0\ \forall\ n\ > 2$ *Then* $f(n)$ *increase* $\forall\ n\ >\ 2$.
Next we can see from nummerical solution (subsection 10.1) that $|Q|_b\ >\ 0\ for\ n\ >\ 319$.
Thus $2([n/10] + 1) > p\ \forall\ n\ >\ 319$. □

6.2 The properties of $|Q|_b$

Proposition 6.4 $|Q|_b$ *is positive integer* $> 0 \; \forall \; n > 319$

Proof 6.4 *By equation (19) we have* $|Q|_b = 2([n/10] + 1) - p$ *.By profosition (6.4)* $|Q|_b > 0 \; \forall \; n > 319 \; \square$

Proposition 6.5 $|Q| > |Q|_b$;

Proof 6.5 *By axiom* (3.8) $|H| > 0 \; \forall \; n > 0$ *then by equation (20)* $|Q| > |Q|_b \; \square$

7 The deviation of $|Q|, |H| \; from \; |Q|_m, |H|_m$

Definition 7.1 *The deviation of* $|Q| \; from \; |Q|_m$ *is:*
$\Delta|Q| = |Q|_m - |Q|$
$\Delta|Q| > 0 \; if \; |Q|_m > |Q|$
$\Delta|Q| < 0 \; if \; |Q|_m < |Q|$

Definition 7.2 *The deviation of* $|H| \; from \; |H|_m$ *is:*
$\Delta|H| = |H|_m - |H|$
$\Delta|H| > 0 \; if \; |H|_m > |H|$
$\Delta|H| < 0 \; if \; |H|_m < |H|$

8 The computation of the real values of $|Q|, |H|$

8.1 The relative accuracy of computation of $|Q|, |H|$

Definition 8.1 *The relative accuracy of computation of* $|Q|$ *as follows below :*

$$\delta_Q = (\Delta|Q|/|Q|_m)100\% \tag{29}$$

Definition 8.2 *The relative accuracy of computation of* $|H|$ *as follows below:*

$$\delta_H = (\Delta|H|/|H|_m)100\% \tag{30}$$

8.2 The estimation of δ_Q

Proposition 8.1

$$100((4([n/10]+1)-p)^2-n(2([n/10]+1)-p))/(4([n/10]+1)-p)^2 > \delta_Q \quad (31)$$

Proof 8.1 *By definition (7.1) we have* $|Q| = |Q|_m - \Delta|Q|$.
By definition. (8.1) we have:
$\Delta|Q| = (\delta_Q|Q|_m)/100$
then we get : $|Q| = |Q|_m - (\delta_Q|Q|_m)/100$.
By proposition (6.5) $(|Q|_m - (\delta_Q|Q|_m)/100) > |Q|_b$.
Whence it follows that $(100(|Q|_m - |Q|_b)/|Q|_m)) > \delta_Q$.
Taking into account equations (46 and 38).
Then we get:
$100(4([n/10]+1)-p)^2)-n(2([n/10]+1)-p)/(4([n/10]+1)-p)^2) > \delta_Q$.
□

8.3 The dependence of $\delta_Q(n)$

Theorem 8.1 *If* $n \to \infty$ *then* $\delta_Q \to -1/4$.

Proof 8.2 *We equate* δ_Q *with its estimation by equation(31)*
then $\delta_Q = 100((4([n/10]+1)-p)^2-n(2([n/10]+1)-p))/(4([n/10]+1)-p)^2. We replace [n/10] with n/10 then we replace p with its first order approximation by equation (7) and we get:$
$\delta_Q = ((16n^2ln^2n+320nln^2n+1600ln^2n-80n^2lnn-800nlnn+100n^2-20n^2ln^2n-200nln^2n+100n^2lnn)/100nln^2n)/((16n^2ln^2n+320nln^2n+1600ln^2n-80n^2lnn-800nlnn+100n^2)/100nln^2n)$
$\delta_Q = (16n^2ln^2n+320nln^2n+1600ln^2n-80n^2lnn-800nlnn+100n^2-20n^2ln^2n-200nln^2n+100n^2lnn)/(16n^2ln^2n+320nln^2n+1600ln^2n-80n^2lnn-800nlnn+100n^2-80n^2lnn-800nlnn+100n^2)$
$\delta_Q = n^2ln^2n(16+320/n+1600/n^2-80/lnn-800/nlnn+100/ln^2n-20-200/n+100/lnn)/n^2ln^2n(16+320/n+1600/n^2-80/lnn-800/nlnn+100/ln^2n)$
Whence follows that if $n \to \infty$ *then* $\delta_Q \to -1/4$. □

8.4 The character of dependence of $|Q|(n)$

Theorem 8.2 *If* $n \to \infty$ *then* $|Q| \to |Q|_m$.

Proof 8.3 *By Theorem (8.1)* $\delta_Q \to -1/4$ *if* $n \to \infty$
then by Defenition 8.1 $\Delta|Q| = -|Q|_m/400$.
Whence by Definition 7.1 we have: $|Q| = |Q|_m - \Delta|Q| = |Q|_m - (-|Q|_m/400 = 1,0025|Q|_m \approx |Q|_m$.
If $n \to \infty$*then* $|Q| \to |Q|_m$. □

8.5 The formulas for computation of the real values of $|Q|, |H|$

$$|Q| = round((4([n/10]+1)-p)^2/n) \tag{32}$$

Where: $p = round(n/lnn + n/ln^2n)$
The computed value of $|Q|$ *it can be used for computation of* $|H|$ *by equations (11):*
$|H| = |Q| - 2([n/10]+1) + p$

9 The solution of Twin Primes Problem

Theorem 9.1 *If* $n \to +\infty$ *then the number of the pairs Twin Primes* $|H| \to +\infty$

Proof 9.1 *For the proof of it we must to prove that* $|H|(n) \to +\infty$ for $n \to +\infty$.
For it we transform equation (11) for $|H|$ *to* $|H|n)$ *replacing by theorem (8.2)*
$|Q|$ *with* $|Q|_m$*, [n/10] with n/10, by equation (7) p with n/lnn then we get for values of* $n \geq 500000$.
$|H|(n) = |Q|_m - 2(n/10+1) + n/lnn$.
$|H|(n) = (4n/10 + 4 - n/lnn)^2)/n - 2(n/10+1) + n/lnn$
$|H|(n) = ((16n^2ln^2n + 320nln^2n + 1600ln^2n - 80n^2lnn - 800nlnn + 100n^2)/100nln^2n) - 2n/10 - 2 + n/lnn =$
$\lim n \to \infty H(n) =$
$= \lim n \to \infty\ (16n^2ln^2n)/100nln^2n = \lim n \to \infty\ 0,16n\ +$
$+\lim n \to \infty\ (320nln^2)/100nln^2n = \lim n \to \infty\ 3,2\ +$
$+\lim n \to \infty\ (1600ln^2n)/100nln^2n = \lim n \to \infty\ 16/n$ -
- $\lim n \to \infty\ (80n^2lnn)/100nln^2n = \lim n \to \infty\ 0,8n/lnn$ -
-$\lim n \to \infty\ (800nlnn)/100nln^2n = \lim n \to \infty\ 8/lnn\ +$
$+\lim n \to \infty\ (100n^2)/100nln^2n = \lim n \to \infty\ n/ln^2n$ -
-$\lim n \to \infty\ (2n/10)/100nln^2n = \lim n \to \infty\ 0,2n\ +$
$+\lim n \to \infty\ n/lnn$

$\lim n \to \infty H(n) =$
$= (\lim n \to \infty\ ((-0,04n) + \lim n \to \infty\ (n/ln^2 n))\ =\ \lim n \to \infty\ (-0.04n + 0,5n) =\ \lim n \to \infty\ (0,46n)\ = +\infty\ .\ +$
$+\lim n \to \infty\ (0,2n/lnn(\ = \lim n \to \infty\ (0,2n)\ =\ +\infty\ +$
$+\lim n \to \infty\ (16/n)\ = 0 -$
$-\lim n \to \infty\ (8/lnn)\ = 0.$
Thus $|H|(n) \to\ +\infty$ *for* $n\ \to\ +\infty$. □

Corollary 9.1 *The number of Twin Primes is infinitely.*

Proof 9.2 *By theorem (9.1)* $|H| \to\ +\infty$ *for* $n\ \to\ +\infty$.
Then by axioms (3.5) $|H|_1 \to\ +\infty$ *for* $n\ \to\ +\infty$.
And by axioms (3.5) $|H|_2 \to\ +\infty$ *for* $n\ \to\ +\infty$.
By definition (3.10) and remark (3.1) $|H|_b \to\ +\infty$ *since* $|H|_1$ *and* $|H|_2\ \to\ +\infty$.
Thus the number of Twin Primes is infinitely. □

10 The appendices

10.1 The numerical solution of $|Q|_b = 0$ *in the range* $1\ <\ n\ <\ 350$

$n = 280; p = 58; |Q|_b = 0$ *by (19).*
$n = 281; p = 59; |Q|_b = -1$ *by (19).*
$n = 282; p = 59; |Q|_b = -1$ *by (19).*
$n = 283; p = 60; |Q|_b = -2$ *by (19).*
$n = 284; p = 60; |Q|_b = -2$ *by (19).*
$n = 285; p = 60; |Q|_b = -2$ *by (19).*
$n = 286; p = 60; |Q|_b = -2$ *by (19).*
$n = 287; p = 60; |Q|_b = -2$ *by (19).*
$n = 288; p = 60; |Q|_b = 0$ *by (19).*
$n = 289; p = 60; |Q|_b = -2$ *by (19).*
$n = 290; p = 60; |Q|_b = 0$ *by (19).*
$n = 291; p = 60; |Q|_b = 0$ *by (19).*
$n = 292; p = 60; |Q|_b = 0$ *by (19).*
$n = 293; p = 61; |Q|_b = -1$ *by (19).*
$n = 294; p = 61; |Q|_b = -1$ *by (19).*
$n = 295; p = 61; |Q|_b = -1$ *by (19).*
$n = 296; p = 61; |Q|_b = -1$ *by (19).*

$n = 297; p = 61; |Q|_b = -1$ *by (19).*
$n = 298; p = 61; |Q|_b = -1$ *by (19).*
$n = 299; p = 61; |Q|_b = -1$ *by (19).*
$n = 300; p = 61; |Q|_b = 1$ *by (19).*
$n = 301; p = 61; |Q|_b = 1$ *by (19).*
$n = 302; p = 61; |Q|_b = 1$ *by (19).*
$n = 303; p = 61; |Q|_b = 1$ *by (19).*
$n = 304p = 61; |Q|_b = 1$ *by (19).*
$n = 305; p = 61; |Q|_b = 1$ *by (19).*
$n = 306; p = 61|Q|_b = 1$ *by (19).*
$n = 307; p = 62; |Q|_b = 0$ *by (19).*
$n = 308; p = 62; |Q|_b = 0$ *by (19)*
$n = 309; p = 62; |Q|_b = 0$ *by (19).*
$n = 310; p = 62; |Q|_b = 2$ *by (19).*
$n = 311; p = 62; |Q|_b = 2$ *by (19).*
$n = 312; p = 63; |Q|_b = 1$ *by (19).*
$n = 313; p = 63; |Q|_b = 1$ *by (19).*
$n = 314; p = 64; |Q|_b = 0$ *by (19).*
$n = 315; p = 64; |Q|_b = 0$ *by (19).*
$n = 316; p = 64; |Q|_b = 0$ *by (19).*
$n = 317; p = 64; |Q|_b = 0$ *by (19)*
. $n = 318; p = 65; |Q|_b = -1$ *by (19).*
$n = 319; p = 65; |Q|_b = -1$ *by (19).*
$n = 320; p = 65; |Q|_b = 1$ *by (19).*
$n = 321; p = 65; |Q|_b = 1$ *by (19).*
$n = 322; p = 65; Q|_b = 1$ *by (19).*
$n = 323; p = 65; |Q|_b = 1$ *by (19).*
$n = 324; p = 65; |Q|_b = 1$ *by (19).*
$n = 325; p = 65; |Q|_b = 1$ *by (19).*
$n = 326; p = 65; |Q|_b = 1$ *by (19).*
$n = 327; p = 65; |Q|_b = 1$ *by (19).*
$n = 328; p = 65; |Q|_b = 1$ *by (19).*
$n = 329; p = 65; |Q|_b = 1$ *by (19).*
$n = 330; p = 65; |Q|_b = 3$ *by (19).*
$n = 331; p = 65; |Q|_b = 3$ *by (19).*
$n = 332; p = 66; |Q|_b = 2$ *by (19).*
$n = 333; p = 66; |Q|_b = 2$ *by (19).*
$n = 334; p = 66; |Q|_b = 2$ *by (19).*
$n = 335; p = 66; |Q|_b = 2$ *by (19).*
$n = 336; p = 66; |Q|_b = 2$ *by (19).*
$n = 337; p = 66; |Q|_b = 2$ *by (19).*

$n = 338; p = 67; |Q|_b = 1$ *by (19).*
$n = 339; p = 67; |Q|_b = 1$ *by (19).*
$n = 340; p = 67; |Q|_b = 3$ *by (19).*
$n = 341; p = 67; |Q|_b = 3$ *by (19).*
$n = 342; p = 67; |Q|_b = 3$ *by (19).*
$n = 343; p = 67; |Q|_b = 3$ *by (19).*
$n = 344; p = 67; |Q|_b = 3$ *by (19).*
$n = 345; p = 67; |Q|_b = 3$ *by (19).*
$n = 346; p = 67; |Q|_b = 3$ *by (19).*
$n = 347; p = 67; |Q|_b = 3$ *by (19).*
$n = 348; p = 68; |Q|_b = 2$ *by (19).*
$n = 349; p = 68; |Q|_b = 2$ *by (19).*
$n = 350; p = 69; |Q|_b = 3$ *by (19).*

10.2 The control of APTN

The computation of $|Q|,\ |L|,\ |H|,\ s_o,\ p$*, for* $n < 310$
Construction SPTN
$n < 310$
** - prime number*
SPTN $n < 310$
N segment PTN1 type PTN2 type
0. 1 3* H 7* 9 L*
1. 11 13* H 17* 19* H*
2. 21 23 L 27 29* L*
3. 31 33 L 37* 39 L*
4. 41 43* H 47* 49 L*
5. 51 53 L 57 59* L*
6. 61 63 L 67* 69 L*
7. 71 73* H 77 79* L*
8. 81 83 L 87 89* L*
9. 91 93 Q 97 99 L*
10. 101 103* H 107* 109* H*
11. 111 113 L 117 119 Q*
12. 121 123 Q 127 129 L*
13. 131 133 L 137* 139* H*
14. 141 143 Q 147 149 L*
15, 151 153 L 157* 159 L*
16. 161 163 L 167* 169 L*
17. 171 173 L 177 179* L*

18. 181 183 L 187 189 Q*
19. 191 193* H 197* 199* H*
20. 201 203 Q 207 209 Q
21. 211 213 L 217 219 Q*
22. 221 223 L 227* 229* H*
23. 231 233 L 237 239* L*
24. 241 243 L 247 249 Q*
25. 251 253 L 257* 259 L*
26. 261 263 L 267 269* L*
27. 271 273 L 277* 279 L*
28. 281 283* H 287 289 Q*
29. 291 293 L 297 299 Q*
30. 301 303 Q 307 309 L*

Data of direct computations for $n < 310$
$p = 62,\ s_o = 62,\ |Q| = 12,\ |L| = 38,\ |H| = 12$.
The computations with equations of APTN
at that $p, s_o, |Q|, |L|, |H|,$ *are took from the data of direct computations.*
$s_o = 4([n/10] + 1) - p = 4([309/10] + 1) - 62 = 62$ *by equation (6)*
$|L| = 4([n/10]+1) - p - 2|Q| = 4([309/10]+1) - 62 - 24 = 38$ *by equation (10)*
$|H| = |Q| - 2([n/10]+1) + p = 12 - 2([309/10]+1) + 62 = 12$ *by equation (11)*
$|Q| = (4([n/10] + 1) - p - |L|)/2 = (4([309/10] + 1) - 62 - 38)/2 = 12$ *by equation (12)*
$|Q| - |H| = 2([n/10] + 1) - p = 2([309/10] + 1) - 62 = 0$ *by equation (16)*
$p = 2|H| + |L| = 24 + 38 = 62;$ *by equation (3)*
$s_o = 2|Q| + |L| = 24 + 38 = 62$ *by equation (4)*
The compare of data of direct computations with computed values shows full coincidence of results.
The computation of $|Q|,\ |L|,\ |H|,\ s_o,\ p,$ *for* $n < 320$
Construction SPTN
$n < 320$
** - prime number*
SPTN $n < 320$
N segment PTN1 type PTN2 type
0. 1 3* H 7* 9 L*
1. 11 13* H 17* 19* H*
2. 21 23 L 27 29* L*
3. 31 33 L 37* 39 L*
4. 41 43* H 47* 49 L*

5. 51 53 L 57 59* L*
6. 61 63 L 67* 69 L*
7. 71 73* H 77 79* L*
8. 81 83 L 87 89* L*
9. 91 93 Q 97 99 L*
10. 101 103* H 107* 109* H*
11. 111 113 L 117 119 Q*
12. 121 123 Q 127 129 L*
13. 131 133 L 137* 139* H*
14. 141 143 Q 147 149 L*
15, 151 153 L 157* 159 L*
16. 161 163 L 167* 169 L*
17. 171 173 L 177 179* L*
18. 181 183 L 187 189 Q*
19. 191 193* H 197* 199* H*
20. 201 203 Q 207 209 Q
21. 211 213 L 217 219 Q*
22. 221 223 L 227* 229* H*
23. 231 233 L 237 239* L*
24. 241 243 L 247 249 Q*
25. 251 253 L 257* 259 L*
26. 261 263 L 267 269* L*
27. 271 273 L 277* 279 L*
28. 281 283* H 287 289 Q*
29. 291 293 L 297 299 Q*
30. 301 303 Q 307 309 L*
31 311 313* H 317* 319 L*

Data of direct computations for $n < 320$
$p = 65,\ s_o = 63,\ |Q| = 12,\ |L| = 39,\ |H| = 13$.
The computations with equations of APTN
at that $p, s_o, |Q|, |L|, |H|$, *are took from the data of direct computations.*
$s_o = 4([n/10]+1) - p = 4([319/10]+1) - 62 = 63$ *by equation (6)*
$|L| = 4([n/10]+1) - p - 2|Q| = 4([319/10]+1) - 65 - 24 = 39$ *by equation (10)*
$|H| = |Q| - 2([n/10]+1) + p = 12 - 2([319/10]+1) + 65 = 13$ *by equation (11)*
$|Q| = (4([n/10]+1) - p - |L|)/2 = (4([319/10]+1) - 65 - 39)/2 = 12$ *by equation (12)*
$|Q| - |H| = 2([n/10]+1) - p = 2([319/10]+1) - p = -1$ *by equation (16)*
$p = 2|H| + |L| = 26 + 39 = 65;$ *by equation (3)*

$s_o = 2|Q| + |L| = 24 + 39 = 63$ *by equation (4)*
The compare of data of direct computations with computed values shows full coincidence of results.

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