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Semi-fundamental Abundance of the Elements

Abstract and Justification

Fusion is an important process in nature. It the abundance of elements measured throughout the universe. Fusion powers the sun and is important in aging of the stars and other high density, high temperature processes. A binding energy model for atoms is extended in this section to abundance of the elements produced in stellar burning phases. Of particular interest was a model that was largely independent of measured parameters and based on probabilities. Reference 17 presented a temperature history for expansion that was similar to other cosmologies. A short time later (on the order of seconds) the author's R1+R3 model produces temperatures in the range of $1e9K$ from the initial kinetic energy 9.7 MeV [17]. Plasma exists until the temperature drops enough to allow electrons to form orbits around protons. Eventually acoustic and gravitational forces become dominant and accumulation of mass into clusters, galaxies and clusters begins. The concentration process later allows stars to "light up" with fusion when they become dense and hot. This is known in the literature as re-ionization. Stars burn up their hydrogen and follow a well-documented aging cycle that depends on the kinetics of progressive fusion reactions. Literature cites measurements regarding the abundance of the heavy elements [8][7][11] that are produced by these reactions.

The electron, energy and N are related by the relationship $E=e0*\exp(N)$ where the number $N=10.136$ represents the electron since $E=2.025e-5*\exp(10.136)=0.511 \text{ MeV}$, the energy of the electron. In other words $e0/P$ is the electron energy where $e0=2.025e-5 \text{ MeV}$ and $P=1/\exp(10.136)$.

The fundamentals of binding energy appear to be based on the above approach. For example, the probability of a neutron in lithium3 is given by $P=1/\exp(2/3)$. The 2 means there are two types of particles (protons and neutrons) and 3 is the number of neutrons for lithium. Next $N=-\ln(P)=2/3$. Note that in this case N is a number smaller than 1. Following a similar approach in the paragraph above, energy would be modified by P to give the energy release. The value $e0$ is 10.15 MeV for binding energy, the value given above for "kinetic energy in the neutron orbit". Energy release for the neutron contribution to lithium is $10.15/\exp(2/3)=5.21$. In the table below the basic probabilistic approach above is applied to the fundamentals of atomic binding energy. Note that heavy atoms can have over 144 neutrons which give a potential release of 10.01 MeV of atomic binding energy, indicating that the curve is approaching "saturation" at 10.15 MeV .

P neutrons					
neutrons	$P=1/n$	$N=-\ln P$	$E=e0*\exp(N)$		
			$e0=2.025e-5\text{ meV}$		
1.48938E+78	6.71E-79	180			
	P electron				
	3.96E-05	10.136	0.511	Electron	
P energy release			$e0=10.15$		
neutrons	$P=1/\exp(2/n)$	$N=-\ln(P)$	$E=e0/\exp(N)$		
3	0.513	0.667	5.211	Lithium	
144	0.986	0.014	10.010	Plutonium	
			$e0=10.15$		
	$P=2/\text{neutro}$	$N=-P$	$E=e0*\exp(N)$		
3	0.667	-0.667	5.211	Lithium	
144	0.014	-0.014	10.010	Plutonium	

Figure 1 Fundamentals of binding energy

protons	(10.15*EXP(-2/protons))			
p	neutrons (10.15*EXP(-2/neutrons))			
1	1.374	n		1.374
2	3.734	2	3.734	3.734
3	5.211	4	6.156	5.751
4	6.156	5	6.804	6.516
5	6.804	6	7.273	7.060
6	7.273	7	7.627	7.464
7	7.627	8	7.905	7.775
8	7.905	9	8.127	8.023
9	8.127	10	8.310	8.224
10	8.310	11	8.463	8.390
110	9.967	272	10.076	10.044
			(weighted average)	

Figure 2 Example binding energy calculations

The binding energy of the atoms and isotopes is essentially the release of energy above minus the amount of energy retained as the electron is compressed and involved in a re-conversion process from protons to neutrons. The retained energy follows the simple relationship: $E_{\text{retained}} (\text{MeV}) = -0.101/4 * \text{protons}$. This retained energy becomes barrier energy for fusion described below.

Fusion fundamentals

Fusion is based on a proton and electron with kinetic energy from its environment colliding with an existing proton or atom. Since the proton and the existing atom are positively charged they repel one another creating a barrier for fusion. To match the barrier energy (BE), the proton and its associated electron must gain energy from the temperature of the environment but they also must be at high density. This prompts the

properties exchange and nil release (0.511+0.111-0.622) that characterizes the re-conversion process to neutrons for about half of the protons. The barrier energy (BE) is simply the retained energy for the protons (Figure 3).

If the temperature is low only some atoms will achieve the barrier and fuse. Boltzmann's approach to equilibrium kinetics characterizes the process even though it may involve several reaction paths. This probability can be characterized by the expression $P_{\text{barrier}} = \exp(-BE/\text{Environment energy}) = \exp(-BE/(1/5B*T))$.

Barrier Energy

The author's binding energy model summarized above results in barrier energy (BE) values for all of the elements.

$$BE \text{ (MeV)} = -0.101/4 * \text{number of protons}$$

Barrier energy is the retained energy from the binding energy model. The results are reproduced below. Since the barrier energy becomes increasingly restrictive there are very few large atoms found in nature and gold is scarce.

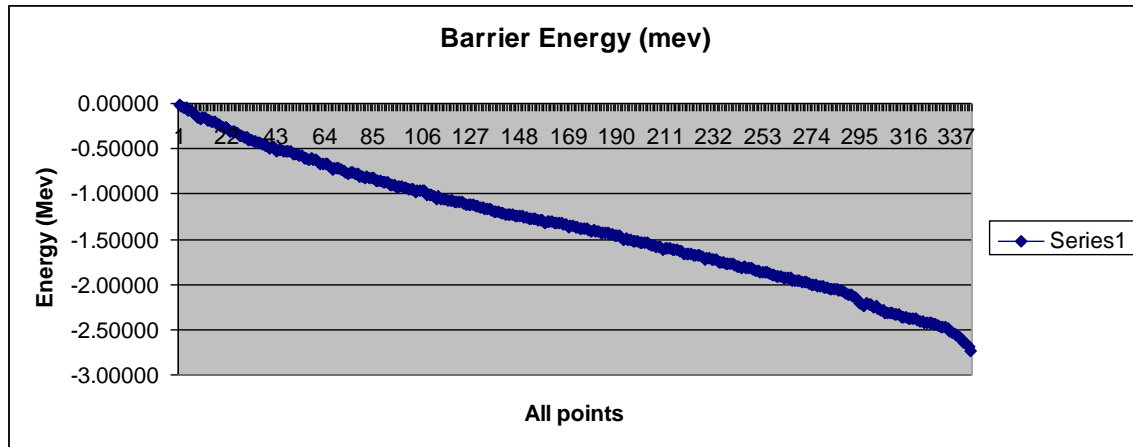


Figure 3 Barrier Energy of Atoms

All points means all atomic numbers for elements and their isotopes.

Fusion probabilities

Specifically energy of the environment is the MeV of the electron that it gets from temperature. Reactions that have lower barrier energies are more frequent and this is the basis for the cosmic abundance of the elements. The proposal is as follows:

$$\text{Probability of fusing/sec} = P_{\text{barrier}} * P_{\text{density ratio}} * \text{reaction rate}$$

$$\text{Number of protons fused} = \text{probability of fusing/sec} * \text{number of protons} * \text{delta time}$$

$$P_{\text{barrier}} = \exp(-.011/(1.5*B*T))$$

Where -0.011 MeV is the barrier energy for the reaction that converts hydrogen to helium/deuterium but each atom has a unique BE.

Boltzmann's constant=8.62e-11 MeV/K

Probability related to density:

At higher density (density ratio) more atoms are present to undergo the reaction. The density ratio is formed from (density/max density). (Maximum density is the initial density associated with time zero radius 7.35e-14 meters and the mass of a proton. For 50% hot matter:

Max Density=(0.5*1.67e-27/(4/3*pi*7.35e-14)=5.02e-11 kg/m³).

Probability of reaction/second:

The number of reactions is also dependent on how fast each reaction can occur (number of reactions per second). The reaction speeds up if an electron can cross a barrier radius and reach the nucleus more quickly. The radius is the degenerate radius 5.29e-11/degeneracy where degeneracy is (density/2699)^{.33}. The value 2699 is the density for protons just separated by the electron orbit 5.29e-11 meters. The velocity is $v=C*(1-(0.511/(0.511+ke))^2)^.5$.

This makes the probability of a reaction in rate/sec:

Reaction Rate=velocity/(2*pi*degenerate radius)/exp(60)

The value exp(60) is the delay "entropy".

Mesons and baryons also have delay "entropies" based on N for their fundamental components.

Solar Example

In the following example, the proposed fusion model is applied to the sun. The core density in the sun is about 1.3e5 kg/m³ [5][6][12] and the temperature is about 1.6e7 K.

Solar example		
Temp	deg K	1.59E+07
Density	kg/m ³	1.28E+05
Dmax	kg/m ³	5.02037E+11
KE temp	1.5*B*T	0.002055756
degeneracy		3.62
Degenerate radius		1.46171E-11
v/c		0.089429948
Barrier		-0.011
Example calculation for above conditions		
rate		P _{barrier}
Probability/sec		exp(-.0139/.005)=.0047
		P _d =(dens/max)
		(1.2e5/5e11) ² =2.55e-7
		Reaction rate R/sec
		v/(2pi*r)/exp(60)=2.56e-9
P _b *P _d *Rate/sec		2.67E-18
Burn time seconds	1/above	
burn time (Byrs)		11.90
sun N		1.1976E+57
fract burning		0.1
burn rate N/sec*me		3.70593E+38
power mev/sec		2.47371E+39

Burn time=1/(P_{barrier}*P_{density}*Reaction rate)

Figure 4 Example calculation for Solar Fusion

Late stage fusion in stars

Following depletion of the hydrogen core of stars, helium, carbon, neon, oxygen, silicon and iron are currently believed to burn in successive stage of the star's life cycle [5][11][12][13]. Based on the proposed fusion kinetics model, very heavy elements can also be produced.

Mass accumulation results in a first generation of stars that light the skies at about 500 million years. As stars over a threshold mass age, processes are put into motion that burn hydrogen to helium, helium to carbon, carbon to neon, neon to neon, oxygen to silicon and silicon to iron. There are several sets of data regarding the temperature and density during the life cycle of stars (burns). One set [7] is given below:

Table 10-1. The major stages in the evolution

Burning Stage Temperature Density Time-sec:

(keV) (kg/m³)

Hydrogen 5 5*10⁶ 7*10⁶ yr

Helium 20 7*10⁸ 5*10⁵ yr

Carbon 80 2*10¹¹ 600 yr

Neon 150 4*10¹² 1 yr

Oxygen 200 10¹³ 6 months

Silicon 350 3*10¹³ 1 day

Collapse 600 3*10¹⁵ seconds

Bounce 3000 10¹⁷ milliseconds

Explosive 100-600 varies 0.1-10 seconds

The probability of fusing plotted against atomic number follows from the temperature and barrier energy. This is because the denominator in the following equation (mev) is determined by temperature alone.

Probability of fusing= $\exp(-BE/kT)$.

$k = 1.38 \times 10^{-23} \text{ J/K}$

Burn time, temperature and density information [7] is shown below including the author's barrier energies and fusion calculations for burn time. Each column contains information for a specific burn. The fusion model gives the probability of reactions/second and burn times are calculated from $1/(\text{probability of reactions/sec})$. This produces time for the burn that is converted to years. Densities are listed for comparison.

Comparison of lbl.gov burn times with calculated burn times				
	protons→He	He→Carbon	C→Ne	Ne→O2
Chap10 density				
(kg/M ³)	5.00E+06	7.00E+08	2E+11	4E+12
Density**	1.52E+06	1.85E+09	1.2E+11	7.8E+11
Chap10 k	5	20	80	150
(mev)	0.00187	0.00892	0.03345	0.09915
Chap10 t	3.87E+07	1.55E+08	6.2E+08	1.2E+09
	1.45e7*	1.55E+08	6.2E+08	1.2E+09
V/C elect	0.09	0.18	0.35	0.55
degenera	8.26	88.17	352.68	661.28
Barrier	-0.01360	-0.08	-0.27	-0.49
Energy (mev)				
degen R	6.4E-12	6.0E-13	1.5E-13	8.0E-14
react rate	5.6E-09	1.3E-07	9.6E-07	2.9E-06
years	7.00E+09	5.00E+05	600	1
	1.20E-17	burn rate (n/sec)		
	2.65E+09	predicted burn (yrs)		
	7.00E+09	Chap10 burn (yrs)		
		He→Carbon		
	(n/sec)	6.0401E-14	burn rate (n/sec)	
		5.25E+05	predicted burn (yrs)	
		5.00E+05	Chap10 burn (yrs)	
			C→Ne	
	burn rate (n/sec)		7.1E-11	
	predicted burn (yrs)		447	
	Chap10 burn (yrs)		600	
				Ne→Oxyg
	burn rate (n/sec)			3.2E-08
	predicted burn (yrs)			1.0
	Chap10 burn (yrs)			1
* alternate source for solar temperature				
** density formula 5e-16*T ³ kg/m ³				
http://www.lbl.gov/abc/wallchart/chapters/10/0.html				

Figure 5 Burn Times part 1

Comparison of lbl.gov burn times with calculated burn times			
	O→Si	Si→	fe→
Chap10 density (kg/M ³)	1E+13	3E+13	5.00E+15
Density*	1.9E+12	9.9E+12	6.25E+15
Chap10 k (mev)	200	350	3000.0
Chap10 t	0.10249	0.3115	3.00E+00
	1.5E+09	2.7E+09	2.32E+10
	1.5E+09	2.7E+09	2.32E+10
V/C elect	0.55	0.78	0.99
degenera	881.707	1542.99	1.32E+04
Barrier	-0.55	-0.9	-1.00
Energy (mev)			
degen R	6E-14	3.4E-14	4.00E-15
react rate	3.9E-06	9.6E-06	1.03E-04
years	0.5		
	6.6E-08	burn rate (n/sec)	
	0.48	predicted burn (yrs)	
	0.5	Chap10 burn (yrs)	
		Si→	
burn rate (n/sec)		1E-05	
predicted burn (yrs)		0.0030	
Chap10 burn (yrs)		0.00274	
		burn rate (n/sec)	9.22E-01
		predicted burn (yrs)	3.44E-08
		Chap10 burn (yrs)	3.17E-08

Figure 6 Burn time comparison part 2

Fusion kinetics during star evolution

The example for solar fusion presented above was used for the remainder of the elements with temperatures from figure 5 and figure 6 for each of the supernova “burns”.

Using barrier energy from the binding energy model, the fusion model and temperature listed above, abundance can be calculated for each element. Using abundance data from reference 8 it is fairly easy to determine the burn that formed each element. Once the source is determined the elements can be plotted and the slope determined. The slope is unique for each burn because the denominator in the following equation (MeV) is determined by temperature alone.

$$P_{\text{barrier}} = \exp(BE / (1.5 * B * T))$$

Density was calculated as follows and compare favorably.

$$\text{Density (kg/m}^3\text{)} = 5e-16 * (T^3) \text{ where T is degrees K}$$

Abundance calculations

Equation for abundance calculations:

$$\text{Abundance fraction} = P_{\text{barrier}} * P_{\text{density}} * \text{Reaction rate} * \text{fraction} * \text{burn time}$$

Where:

$P_{\text{barrier}} = \exp(-BE/ke)$ barrier energy from binding energy model

Reaction rate = $v / (2 * \pi * r * \text{degenerate}) / \exp(60)$

$P_{\text{density}} = \text{density} / 5.03e11$

Fraction = fraction available in the core to burn

Time = burn time in seconds

$$\text{Burn time} = 1 / (P_{\text{barrier}} * P_{\text{density}} * \text{Reaction rate})$$

Figure 7 Abundance example

The table below contains the constants for each burn in the vertical column. The only unique thing about each element is its barrier energy.

Carbon Abundance Example		
Temp	deg K	1.55E+08
Density	kg/m ³	1.20E+08
Dmax	kg/m ³	5.02E+11
KE temp	1.5*B*T	0.020
degeneracy		35.43
Degenerate radius (DR)		1.49E-12
v/c		0.27
Barrier		-0.15
Example calculation for above conditions		
	Pbarrier	
	$\exp(-.152/.02) = 5e-4$	
	P density	
	$(\text{dens}/\text{crit}) = 2.3e-4$	
	P rate/sec	
	$v / (2\pi * r) / \exp(60) = 7.6e-8$	
fract at temp		8.00E-03
Pb*Pd*Rate/sec		7.25E-17
time (seconds)		1.58E+13
calculated carbon abundai		0.0011
	data	0.005

Figure 8 Carbon abundance example

The calculation above was carried out for each element. The resulting abundance data was grouped by burn (this required selecting which atoms were produced in the burn but a pattern was fairly clear because each burn has a unique slope) and plotted on a semi-log plot for all points (atomic numbers including isotopes). A statistical fit was determined (the line below) from the abundance groupings. The burn lines were produced from the barrier energies, densities and fusion kinetics. The main burn slopes are shown below.

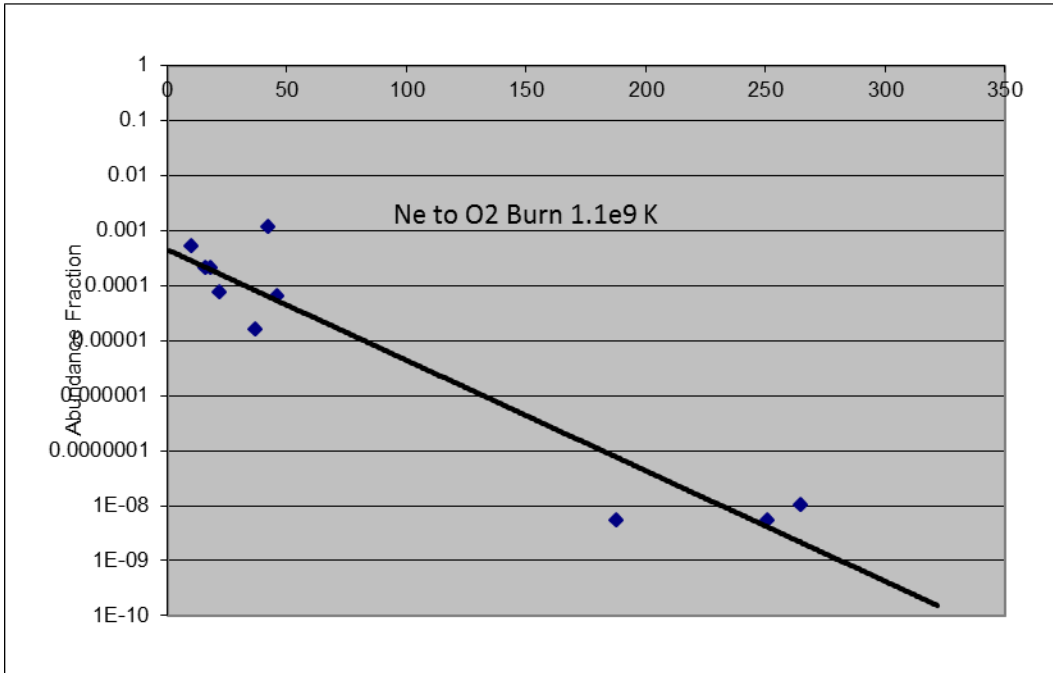


Figure 9 Neon burn

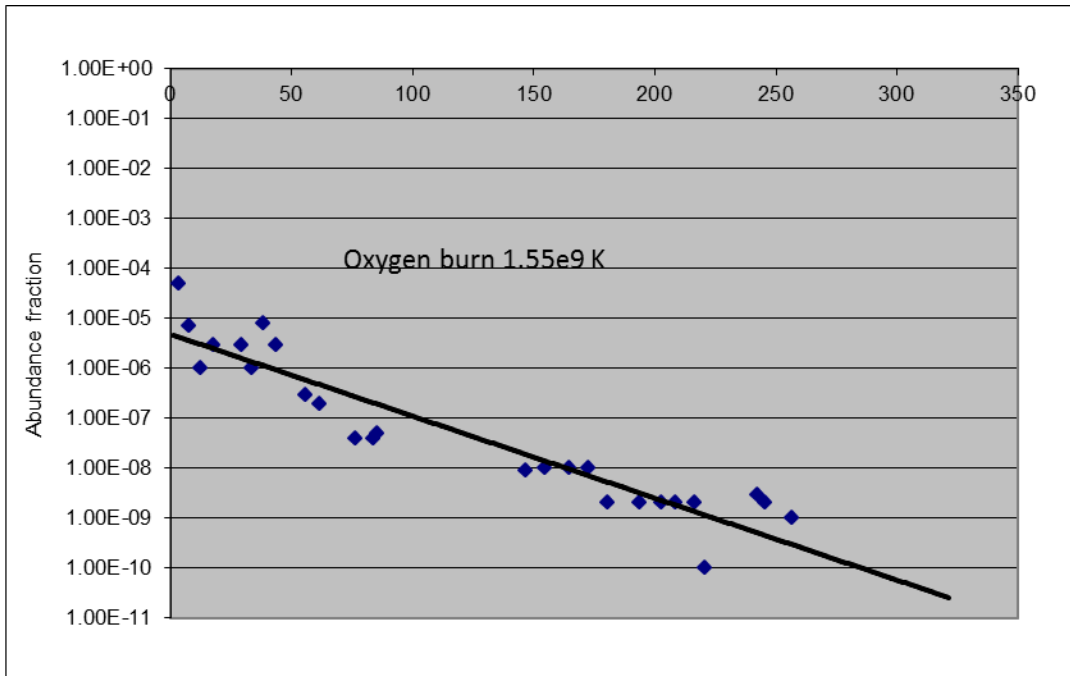


Figure 10 Oxygen burn slope

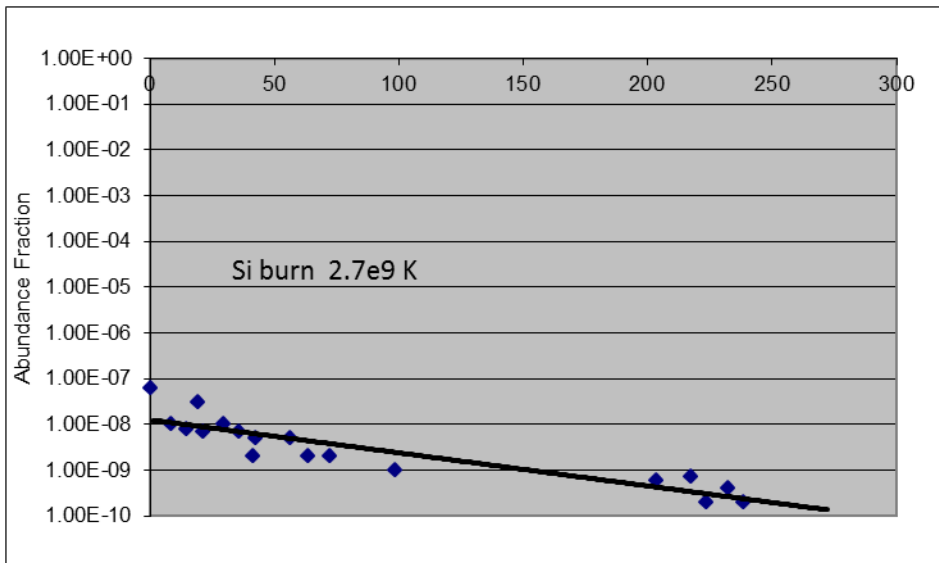


Figure 11 Silicon burn

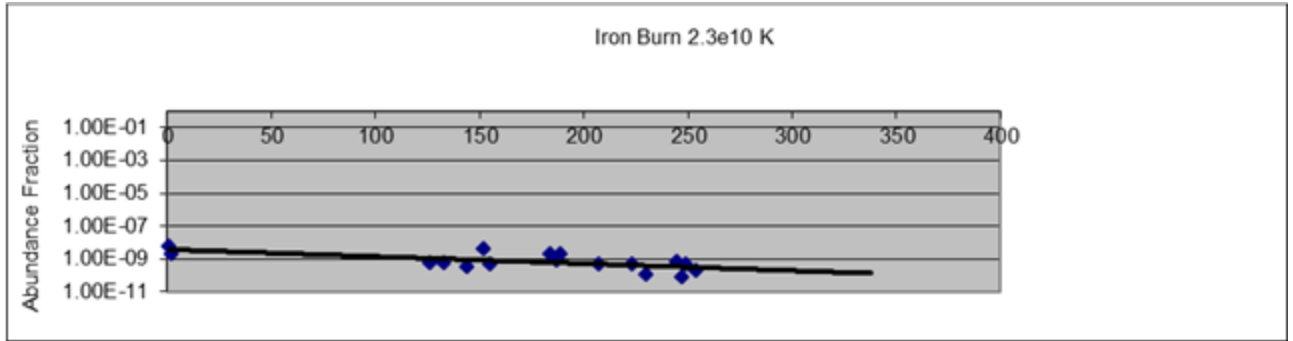


Figure 12 Iron burn slope

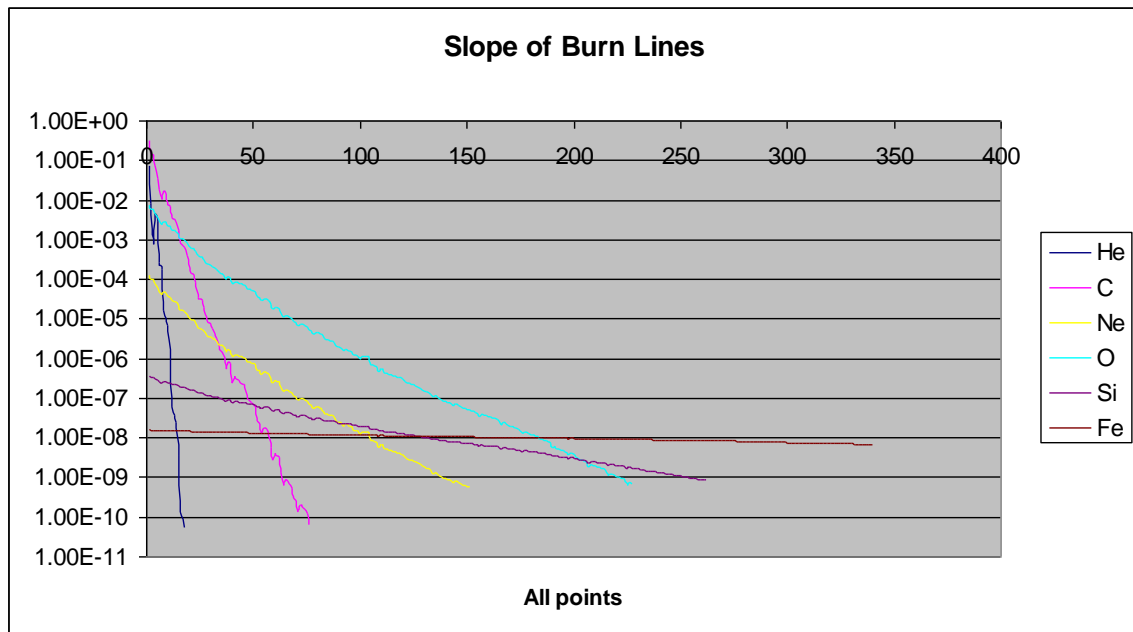


Figure 13 Slope of Burn Lines

Abundance of the elements

Abundance data [8] is presented below on the vertical axis of a semi-log plot. The horizontal axis is all the atomic numbers and their isotopes.

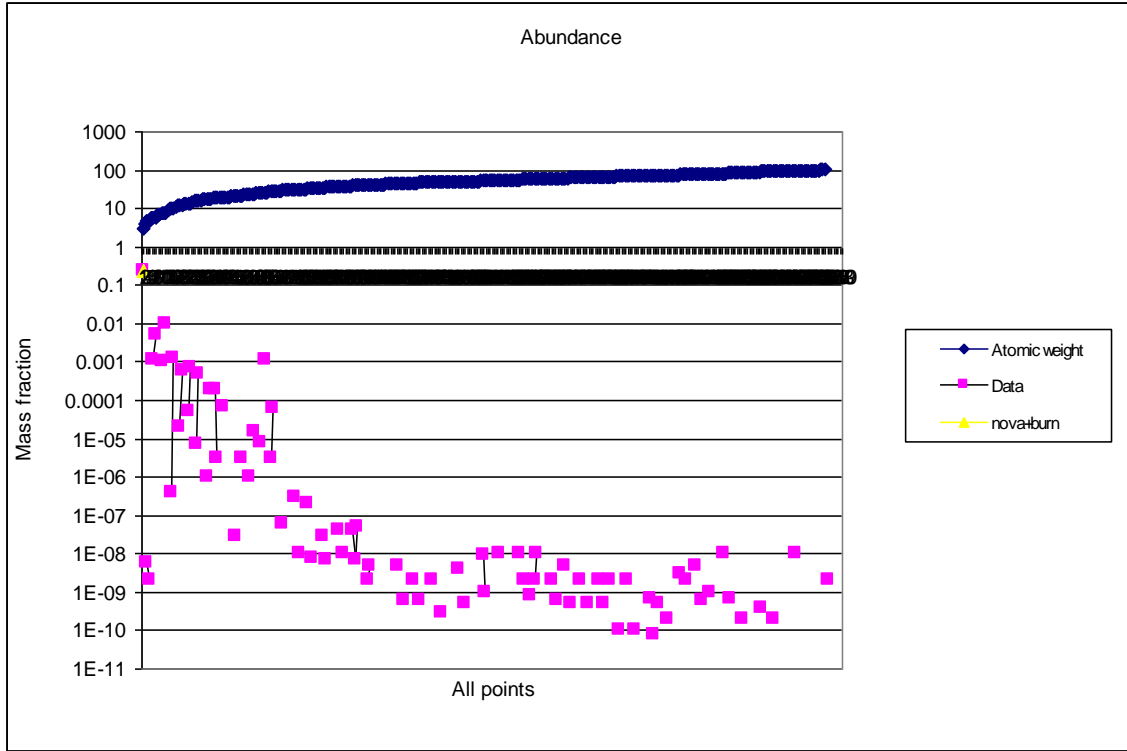


Figure 14 Abundance Data

The burn lines (the points marked in yellow that determine the burn line) above are superimposed on the raw data below to produce a comparison of data and calculated abundances. The ratios for each data set below compared with its temperature line are on the order of 1 or 2 standard deviations.

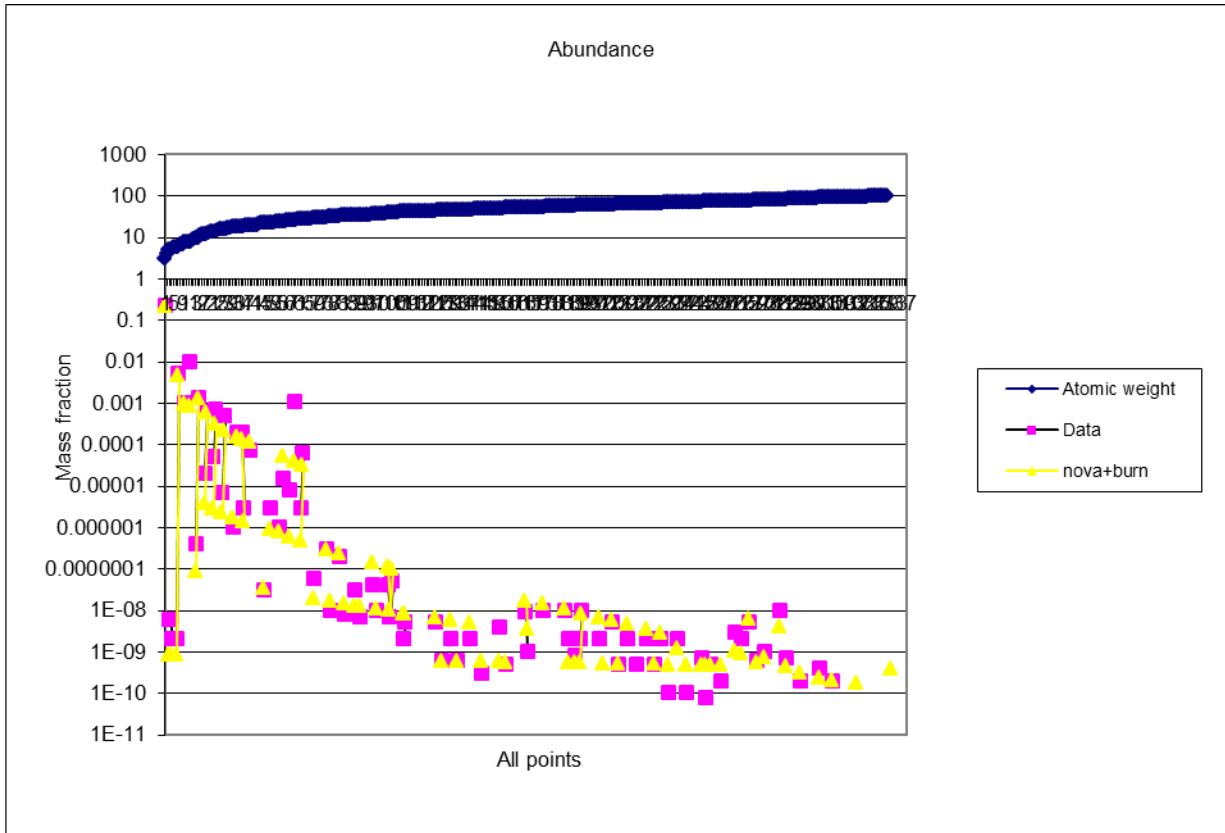


Figure 15 Comparison of Abundance Data with Abundance Calculations

As indicated above, the model is semi-empirical since the vertical position of the line is also dependent on the fraction of the burning element that is subjected to high temperature. This will remain empirical because the history of all supernovae cannot be known.

Elements produced by each burn:

He >Carbo C >Ne	Ne >Oxyg O >Si	Si >	Fe >		
C	Ne	O	Na	F	B
	Mg	N	Al	Sc	Be
	Si	S	P	Cu	Rh
		Ar	Cl	Ga	Ag
		Ca	K	As	In
		Cr	Ti	Se	Sn
		Ni	V	Br	Sb
		Fe	Mn	Rb	La
			Co	Y	Ce
			Zn	Nb	Pr
			Ge	Mo	Eu
			Kr	Ru	Ho
			Sr	Pd	Tm
			Zr	Cd	Hf
			Xe	Te	Ta
			Ba	I	W
			Nd	Au	Re
			Sm	Bi	
			Dy	Rn	
			Er	Th	
			Hg	U	

Figure 16 Elements produced by burns

Primordial helium fusion

Early work by Andrei Sakharov indicated that high initial temperatures would fuse elements from primordial nucleons and “freeze” their abundances at the observed levels during expansion and cooling. Current literature cites measurements indicating that 25% of primordial matter consists of helium. This process is cited in the literature as evidence of a “big bang”. Radiation pressure prevents gravitational accumulation until radiation is attenuated by expansion, a condition known as equality. Current literature cites measurements indicating that 25% of primordial matter consists of Helium 4. Fusion of neutrons to deuterium occurs in the first few minutes and this is quickly converted to He4. This is quite a different process than stellar fusion because neutrons are readily available and an electron does not have to receive energy from its environment. This is the subject of reference 17.

Summary

1. A fusion model was developed that relies on barrier energies from reference [2] to characterize fusion rates of stars, temperatures, densities and element abundances. The model determines which elements are formed from each of the burning phases and determines temperature from the slope of the abundance line. The model is dependent on observed burn times of supernovae and abundance data. It uses many fundamentals but, in the end, is semi-fundamental since the fraction of mass exposed to high temperature is highly complex.
2. Supernovae are the source of heavy elements. This means that the sum of the measured abundances for elements heavier than helium is the fraction that was at some point in the core of stars.
3. The abundance of elements that burn in supernovae is the sum of the amount produced minus the burned amount. For example, Carbon was produced in copious amounts during the Helium burn but was consumed as C burned to produce Ne, Na, etc.

Appendix

Temperature fundamentals

(A short review of known relationships is presented below as a reference.)

The relationship between the external kinetic energy of a particle and temperature is $T = k_e/B/1.5$, where B is a conversion constant (Boltzmann's constant = 8.62×10^{-11} meV/N/K). Interaction between particles is based on the electron resisting changes in its orbit when one particle hits another. When the electron settles into its orbit and the overall volume continues to expand, space develops between the gas molecules and hydrogen becomes diatomic. The interaction between particles becomes statistical and the ideal gas law applies. Pressure is now determined by the equation $P = 287.05 \text{ nt/kg/K} \cdot \text{density} \cdot \text{temperature}$, where density is kg/m^3 and temperature is degrees K. The constant of proportionality (287.05) is called R and is a published value that changes slightly with temperature. Of course temperature can change independently and this changes the energy.

When the electron is forced to position itself at less than 5.29×10^{-11} meters, the electron kinetic energy is degenerate. For high temperatures the electron kinetic energy from the electron quad (0.1114 meV) is scaled down by $(\text{dens}/\text{dens}_0)^{.33}$. If the hydrogen proton and electron were ideal and disassociated, the gas constant would be $8.32 \text{ J/mol/K} \cdot 1000 \text{ mols/kg mols} = 8300 \text{ J/K/kg} = 8.6 \times 10^{-11} = \text{Boltzmann's constant}$. The constant volume specific heat ideally would be $5/3 \cdot 8300$ and the constant pressure specific heat should be about $3/2 \cdot 8300 \text{ (J/K/kg)}$. It is known that the equation of state for hydrogen plasma is quite complicated and that extra heat is retained [10]. The specific heat [10] gives a

value of 2.3×10^{-13} for a much lower temperature of 6000K and it is known that specific heat increases with temperature).

The degenerate condition ends when electrons are just packed so their orbits are in contact. For this condition the density is 2697 kg/m^3 . Our atmosphere is at a much lower density (1.27 kg/m^3) and the electrons are separated. We observe a temperature that is consistent with the kinetic energy divided by mass and cp (specific heat).

The temperature associated with this kinetic energy state is about $8.6 \times 10^8 \text{ K}$.

simple.xls cell ae175			
Compression Thermodyna		8300.00	8300.00
		0.11143395	→
		base	Expanded
degeneracy	(dens/dens0)^	4407.01807	4407.01807
Radius	compressed	1.2006E-14	1.2006E-14
Volume V	M^3	7.249E-42	7.249E-42
dens-	kg/M^3	2.307E+14	2.307E+14
KE electron	(n E0*r/r=E0(d/d)	1.114E-01	1.114E-01
pressure=R	rh R=287.05	1.652E+27	1.651E+27
P*V	mev	7.473E-02	7.470E-02
Ke at max burn			9.24E-04
Ke =1.5BT		1.115E-01	1.115E-01
Temperature= ke elect/cp/93		8.625E+08	8.620E+08
cp (mev/mev/K)		1.38E-13	1.38E-13

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