A formula that generates a type of pairs of Poulet numbers

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Abstract. Starting from the observation that the number $13^2 + 81 \times 13 + 3 \times 13 \times 41$ is a Poulet number (2821), and the number $41^2 + 81 \times 41 + 3 \times 13 \times 41$ is a Poulet number too (6601), and following my interest for the number 30, I found a formula that generates such pairs of Poulet numbers like (2821,6601).

Observation: The formula $p^2 + 81 \times p + 3 \times p \times q$, where $p$ is a prime of the form $30 \times k + 13$ and $q$ is a prime of the form $30 \times k + 41$ (case I), or, vice versa, $p$ is a prime of the form $30 \times k + 41$ and $q$ is a prime of the form $30 \times k + 13$ (case II), generates Poulet numbers.

Examples:

: for $(p,q) = (13,41)$, we got 2821, a Poulet number;
: for $(p,q) = (41,13)$, we got 6601, a Poulet number;

: for $(p,q) = (43,71)$, we got 14491, a Poulet number;
: for $(p,q) = (71,43)$, we got 19951, a Poulet number.

Conjecture 1: There is an infinity of Poulet numbers of the form $p^2 + 81 \times p + 3 \times p \times q$, where $p$ is a prime of the form $30 \times k + 13$ and $q$ is a prime of the form $30 \times k + 41$, where $k$ is an integer, $k \geq 0$.

Conjecture 2: There is an infinity of Poulet numbers of the form $p^2 + 81 \times p + 3 \times p \times q$, where $p$ is a prime of the form $30 \times k + 41$ and $q$ is a prime of the form $30 \times k + 13$, where $k$ is an integer, $k \geq 0$.

Conjecture 3: If the number $p^2 + 81 \times p + 3 \times p \times q$, where $p$ is a prime of the form $30 \times k + 13$ and $q$ is a prime of the form $30 \times k + 41$, is a Poulet number, then the number $p^2 + 81 \times p + 3 \times p \times q$, where $p$ is a prime of the form $30 \times k + 41$ and $q$ is a prime of the form $30 \times k + 13$ is a Poulet number too ($k$ is an integer, $k \geq 0$).
Note: The differences between the two numbers that form such a pair might also have interesting properties; in the examples above, we have 6601 - 2821 = 3780 and 19951 - 14491 = 5460. Note that 5460 - 3780 = 1680 = 41^2 - 1.

Note: There are many Poulet numbers that can be written as p^2 + 81p + 3p*q, where p,q primes, but it’s not satisfied the reciprocity from the formula above.