Moutaoikil-Ripà’s conjecture on Prime Numbers:
∀ p₀ ≥ 7, p₀ = 2 ⋅ p₁ + p₂

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Abstract

An original result about prime numbers and unproved conjectures. In this paper I will show that, if the Goldbach conjecture is true, any prime number greater than 5 can be expressed as the sum of a prime and the double of another (different) prime. A computational analysis shows that the conjecture is true (at least) for every prime below 7465626013.

1. Introduction


The statement is as follows:

Moutaoikil-Ripà’s Conjecture. For every prime number p₀ ≥ 7, we have that p₀ = 2 ⋅ p₁ + p₂ (where p₁ and p₂ are both primes and p₁ ≠ p₂).

2. “Proof” (assuming Goldbach’s conjecture as true)

Let us consider a base 10 scenario and let us assume Goldbach’s conjecture as true, we can see that we have just two cases to analyze (p₁ = 2 or p₁ > 2).
In fact, $p_0 = 2 \cdot n + 1 \rightarrow 2 \cdot p_1 + p_2$ is odd only if $p_2$ is odd ($p_1$ is the only prime which can be 2).

Let us assume $p_1 \neq 2$, we will show that, assuming Goldbach’s conjecture as true, the new conjecture is true as well (for any $p_0 \geq 11$). It is trivial that, if $p_0 = 7$, there is only one possible solution (but there is one!) and it is $7 := 2 \cdot 2 + 3$.

We have the following constraints:

\[
\begin{cases}
  p_1 \neq p_2 \\
p_1 \neq 2 \\
p_2 \neq 2 \\
p_1, p_2 \text{ are prime}
\end{cases}
\rightarrow \min[p_1+p_2] = 3+5=8 \rightarrow \min[2 \cdot n] = 8 \rightarrow \min[n] = 4.
\]

Thus $p_0 - p_1 \geq 8 \rightarrow \min[p_0]$ such that $p_1 = 2 \rightarrow \min[p_0] = 11$ (because $11 = 8 + 3$).

$p_0 = 2 \cdot p_1 + p_2 \rightarrow p_0 - p_1 = p_1 + p_2$. But Goldbach said that $2 \cdot n = p_1 + p_2$, where (in our case) $n$ is an element of $\mathbb{N} \setminus \{0, 1, 2, 3\}$ and $p_1 + p_2 \geq 8$.

The new relation we have to “prove” is easy now: $p_0 - p_1$ is the difference between two odd primes $\rightarrow$ it is even ($p_0 - p_1 \geq 8$), as we have already shown… and, on the other side of the “$=$”, there is the sum of two distinct primes? Goldbach? Yes, but we need to point out that we are searching for $p_1 \neq p_2$ solutions only.

But we can see that, for every value of $n$ we are considering ($n \geq 4$), the partition number of $2 \cdot n$ is $\geq 2$ (so we have at least one solution of the form $p_1 \neq p_2$). (Q.E.D.)

3. Computational analysis

Emanuele Dalmasso wrote a specific program to test the new conjecture for “small” values of $p_0$: the test has shown that the conjecture is right for any $7 \leq p_0 \leq 746562601$ ($746562601 = 2 \cdot 7 + 746562587$).

$p_0$ can be written in many different ways (for $11 \leq p_0$): you can see this just looking at the figure below (the number of ways such that $p_0 = 2 \cdot p_1 + p_2$ is shown on the vertical axis).