A New Picture In The Particle under The Frame of Three Dimensional Einstein Theory

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By the analogy with Special Relativity, it shows that there is a fundamental length in three-dimensional space, in which the dimension of time has been a constant, and the remaining coordinates are satisfied with the same mathematical structure as the Lorentz transformation. According to the hypothesis, the energy $\omega$ is corresponding to the $x_i$, and the force $F$ is analogous with coordinate $t$. After proceeding with this, there is a natural way to explain the quark confinement. Finally, this theory is promoted to the curved situation, where the origination of cosmological constant $\Lambda$ is connected with three-dimensional curvature. In addition, it also can construct the relationship between thermodynamics and statistical mechanics (TSM) and three-dimensional Einstein theory. Under the correspondence of three-dimensional cosmology, the new physical meaning of revolutionary factor $R$ can be found.

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I. INTRODUCTION AND MOTIVATION

The principle of confinement, especially for the quark confinement, has various models to explain. Every satisfactory theory of confinement should account for the features of the static quark potential related to the Regge theory, and the problem cannot be solved under the frame of perturbation theory. Thus, in order to find the solution for this problem, some new ideas have been proposed.

One candidate of this configuration are magnetic monopoles and center vortices. G. ’t Hooft [1, 2] proposed that the phase transition, considered as the dual transformation of the Higgs phase, would cause the permanent quark confinement. And A.M. Polyakov [3] held the similar viewpoint. Another approach is ground on the special properties of quantization in Coulomb gauge. In Ref. [4, 5], they argued that, for static sources, the possibility of linearly potential caused by the vacuum expectation value of the Coulomb energy. Another idea is to find the non-perturbatively solution of quark and gluon propagators and vertex functions in an infrared expansion under the frame of a set of Schwinger-Dyson equations. Its recent review can be seen in Ref. [6]. After discovering the amazing AdS-CFT correspondence, J. M. Maldacena [7] firstly considered the relationship between the Wilson loop and AdS-CFT correspondence. However, these viewpoints cannot solve the quark confinement completely. So far, the best tool to illustrate the law of quark confinement and calculate QCD’s scattering process is the lattice QCD [8]. It indicated that we should quantize a gauge field theory on a discrete lattice space in Euclidean space-time. Unfortunately, it would break the Lorentz symmetry.

Our interests in studying the quark confinement are generated from the lattice space and the broken of Lorentz boosting. Assuming that there is a fundamental length in three dimensional space varies from U. Ewz’s idea [9]. Recently, as some new perspectives proposed [10, 11], we use these current developments of QCD to promote the new idea connected with the dark energy under the help of Ref. [12]. Ted Jacobson proves that the Einstein theory can be derived from the thermodynamics [13]. According to Erik Verlinde’s new idea [14], the nature of gravity is just entropy. Thereby, it gives the sign that we can find the relationship between energy $\omega$ and TSM from three dimensional situation in order to obtain its origin of statistic from the different angle.

In order to set the stage, Sec.2 gives that the analogy with the Special Relativity (SR) gives the relevant transformation in there dimensions. Sec.3 presents the promotion in curved space. Finally, in Sec.4, it shows some related discussion and conclusion.

II. THE TRANSFORMATION IN 3-DIMENSIONAL SPACE

Before clarifying the hypothesis, let’s look on the Wilson’s work. In Ref. [8], the lattice is used to define Euclidean space’s coordinates,

$$x_i = n_i a, \quad (1)$$

where the elements are the vector:

$$n_i = (n_1, n_2, ..., n_d). \quad (2)$$

The points (1) are called the lattice sites. The key place is the lattice space $a$ needed when discretizing all the other crucial physical quantity. As we take $a \to 0$, it could get the classical result. In Wilson loop, the scope of the loop is arbitrary. And it can calculate the strong coupling of expansion of $1/g^2$ as calling for $g \to \infty$ under the partition function,

$$Z\left(\frac{N}{g^2}\right) = \int \prod_{x,\mu} dU_{\mu}(x)e^{-\frac{N}{g^2} S[U]}, \quad (3)$$

where $g$ is the coupling constant. And it can explain the quark confinement under the law of the area. In lattice space, the electronic field can have confinement when lattice space $a$ is nonzero. There is something unreasonable needed to be discussed. However, it can be computed correctly in super computer under the frame of the finite lattice space $a$. So the strong

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hint is that there is a existed fundamental length. Meanwhile, there is something could be missed in this lattice space.

Let’s pull back our discussion. When Wilson discretizes the space in the special shape’s lattice, it also hides the hypothesis, in which the fixed point has its interior structure, just like the lattice sites are used to denote the classical view’s concept of point. Thus, as combining our assumption, the missed part is the internal structure.

Next, we should understand what’s the internal structure of point. In this paper, the new initial point is originated from one of Lie group’s most significant theorem proved by Cartan, it shows that a N-dimensional Lie group’s open subset is a Lie group, and they favor to the same kind of Lie transformation. In there-dimensional space, since one dimension of them is removed, it would automatically break the Lorentz transformation in four dimensional space-time is satisfied with the Lorentz group, such as \( \mathbf{SU}(3) \) and \( \mathbf{SU}(2) \). Thus, it can be seen that the four dimensional space-time is satisfied with the Lorentz transformation. And there is a same mathematical transformation in there-dimensional space. Since one dimension of them is removed, it would automatically break the Lorentz transformation. But which dimension be should delete?

First, Let’s recall the Lorentz transformation in SR :

\[
X_\mu' = \begin{pmatrix} \cosh \lambda & -\sinh \lambda & 0 & 0 \\ -\sinh \lambda & \cosh \lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} X_\mu, \quad (4)
\]

\[
Y_\mu' = \begin{pmatrix} \cosh \lambda & 0 & -\sinh \lambda & 0 \\ 0 & 1 & 0 & 0 \\ -\sinh \lambda & 0 & \cosh \lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} Y_\mu, \quad (5)
\]

\[
Z_\mu' = \begin{pmatrix} \cosh \lambda & 0 & 0 & -\sinh \lambda \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh \lambda & 0 & 0 & \cosh \lambda \end{pmatrix} Z_\mu. \quad (6)
\]

From Eq.(4) to Eq.(6), they denote that the Lorentz transformation is along with the x, y, z axis, respectively. In these group representations of Lorentz transformation, it can be clearly seen that the coordinates \( x_i \) are all dependent on the coordinate \( t \), and it can be considered as the virtual rotation. Moreover, if we accepted that the Lorentz transformation is one kind of rotation(virtual rotation), the coordinates of space without time can also satisfy with this similar Lorentz transformation. It means that the boosted coordinates are not dependent on time.

By imitating the Special Relativity, assuming that there is a fundamental length \( l_0 \). The fundamental length is \( l_0 \), which likes the Wilson’s lattice’s spatial parts. But the meaning of coordinate is not \((x_i, i\epsilon)\), in which they are substituted by \((\omega_1, \omega_2, il_0F)\). The reasons why this coordinate system is used are listed as follows. Firstly, on the analogy with the Special Relativity, e.g. the velocity \( c \)'s dimension can be described like this: \( [c] = \frac{l}{t} \). Thus, the dimension of \( l_0 \) should have the similar formula: \([l_0] = \frac{[l]}{[t]}\). Secondly, for time \( t \) has been ceased, we can consider \((\omega_1, \omega_2, il_0F)\) as the coordinate system. Since \( \omega \) and \( F \) could be independent of dimension of \( t \), e.g. the potential energy and gravity.

In the three-dimensional space, \((\omega_1, \omega_2, il_0F)\) is satisfied with the similar Lorentz transformation:

\[
\omega_1' = \frac{\omega_1 - Fl_0}{\sqrt{1 - l_0^2/F^2}}, \omega_2' = \omega_2, F' = \frac{F - il_0}{\sqrt{1 - l_0^2/F^2}}. \quad (7)
\]

Meanwhile, two significant expressions can be derived:

\[
\omega = \omega_0 \sqrt{1 - l_0^2/F^2}, F = \frac{F_0}{\sqrt{1 - l_0^2/F^2}}. \quad (8)
\]

In this section, it can be clearly seen that, in little scale of space, where the force is increased as the distance raised. It does not set \( g \rightarrow \infty \) in order to present quark confinement under the large area of the loop. Therefore, it can provide a natural explaining way of asymptotic freedom under this hypothesis. As for \( F_0 \), it is the value of force as the relative fundamental length \( l_0 = 0 \), and the \( F \) is not the directly observable quantity. As for its relation with Regge theory, we will analyze it when we obtain the energy line element corresponding with the \( ds^2 \) in the Special Relativity. \( \omega \) is a manifestation of energy. It describes the energy value of space under little scale of three dimensional space.

From the illustration of this paragraph, it can be clearly seen that the confinement phenomenon is the intrinsic property of the three dimensional space when time has been a constant. It not only explains the asymptotic freedom, but also can provide a effective explanation for the quark confinement. And it is agreed with the principles of aesthetics—simple. The existed fundamental length \( l_0 \) is not the same as the lattice space \( a \). In Eq.(1), the \( a \) contains four components \((a_1, a_0)\), \( a_i \) and \( a_0 \) are usually not the same. We assume \( l_0 \) is \( a_i \). But in Wilson’s work, it shows the confinement mechanism is long-time scale. However, we conclude that the three-dimensional Lorentz transformation caused the confinement. From Ref. [15], it can be obtained the expressions between the lattice space \( a \) and coupling constant:

\[
K = \frac{1}{a^2} \ln(2Ng^2). \quad (9)
\]

Where \( K \) is the energy(constant) of the string per unit length. It likes that the string tension prevents the quark and antiquark from isolation. \( N \) is the number of colors, i.e. \( SU(3) \). And some useful information can be extracted between \( l \) and coupling constant \( g \). And the lattice spatial parts could be substituted by \( l \). Thus we can obtain that:

\[
K = \frac{1}{l^2 + a_0^2} \ln(2Ng^2). \quad (10)
\]

It should be remarked that the \( k^2 \) is increasing as the \( l^2 \) is raising when the \( a_0 \) is fixed. Combined with Eq.(7), then we can get:

\[
e^{K(\frac{1}{l^2} - \frac{1}{a_0^2})} = 2Ng^2; e^{K(\frac{1}{l^2} - \frac{1}{a_0^2})} = 2Ng^2. \quad (11)
\]
Now, we have established the relationship between $F$, $\omega$, and the coupling constant $g$.

Now that the coordinates are $(\omega_i, \eta_0 F)$, it is very natural to find the similar line element on the analogy with the Special Relativity. Thereby, the line element can be obtained:

$$dE^2 = dF^2 l_0^2 - da_1^2 - da_2^2. \quad (12)$$

Here, we will give the response that the $dE^2$ is related with the Regge theory. From its definition, it can be clearly seen that the $E^2$ is a constant due to the cease of time. And it is an observable quantity due to the existence of particle’s mass. The $K$ can be defined by $K \propto dE/dr(R \leq 2l_0)$. On the one hand, it can apply for the potential rises linearly. On the other hand, it can avoid the unreasonable place where distance between the static quark and the anti-quark can become infinite. Note that the Eq.(12) is a strong restriction of its range of the application. It can be only used for the internal structure of one particle.

The next step is that we should obtain physical correspondence of $dE^2$. In Einstein’s Energy-momentum formula, it can be clearly seen that:

$$m_0^2 c^4 = W^2 - p^2 c^2. \quad (13)$$

From the hypothesis, the variable of time has become a constant. Thus, both momentum $p^2$ and general energy have been a constant. When the 4 dimensions of space-time reduced into 3 dimensions, it can be deducted that:

$$E^2 = p^2 c^2, \quad (14)$$

where $p^2$ is Einstein’s momentum formula, $E = \kappa m_0 c^2$, where $\kappa = \gamma^2$ and $\gamma = \frac{1}{\sqrt{1 - v^2}}$. However, the force and energy’s phenomenon cannot be observed directly due to the time $t$ has been ceased. Now let’s analyze the confinement in QED as lattice space $a \neq 0$. Here, the confinement is caused by the internal structure of one particle, but the QED is not of the property of asymptotic freedom. It means that the electron and photon do not possess any internal structure. It agrees with the current research viewpoint that these particles are all elementary particles. Thus, it can be concluded that the lattice space is not arbitrary. Indeed it has the concrete physical meaning, namely, it is the internal structure mentioned in this section.

### III. IN CURVED SITUATION

Finishing the discussion of flat space, it is naturally promoted into the curved situation. The first question in this situation is what is the principle of equivalence in the three-dimensional situation. Like the accelerating field is equivalent to the gravitational field? Let $d^2 \omega_i/dF^2$ be the equivalent part which is analogous with GR’s gravitational field in this paper. $d^2 \omega_i/dF^2 = dl/dF$, but what is $dl/dF$. There is no classical correspondence in current theory. So let $dl/dF$ denote linear intensity per unit of force presenting a new physical quantity. Next, it should find what the $dl/dF$ is equivalent to. Exactly speaking, which field makes the flat space curved. The detailed response will be given in the last part of this section.

In this section, let’s see the mathematical expression of the so-called principle of equivalence. Due to the analogy with Relativity system. It is natural for us to use the tensor analysis to study the principle. According to this Relativity system, there is a geodesic line for force and energy in the language of tensor:

$$\frac{d^2 F}{d^2 \omega} = 0, \quad (15)$$

where $dF_0^2 = -\eta_{ij} dE^a dE^b$, $F_0$ is the minimum value of force under the scale of $l_0$. In appropriate coordinates of energy, it also can be derived from the principle of the least action:

$$\int dE = \int (dE dE^{1/2}) \quad \delta \int (dE) = 0 (i, j = 1, 2, 3). \quad (16)$$

Finally, the formula can be derived like this:

$$\frac{d^2 e^i}{dF_0^2} + \Gamma^{i}_{jk} \frac{de^j}{dF_0} \frac{de^k}{dF_0} = 0, \quad (17)$$

where $e^i$ denotes the coordinate of energy-force in three dimensions. It is natural for us to explore the so-called weak approximation. But Ref. [8] has told us that it would break the Newtonian law in there-dimensional Einstein equation. Let’s give a brief analysis. In three-dimensional space, the curve curvature can be expressed by:

$$R_{\alpha \beta \gamma \delta} = e_i (g_{i\alpha} T_{j\beta} - g_{i\beta} T_{j\alpha} - g_{i\gamma} T_{\beta j} + g_{i\delta} T_{\gamma j} + T_{j\delta} \eta_{i\alpha} - T_{j\alpha} \eta_{i\delta}). \quad (18)$$

It is clear that the curvature is completely determined by the local matter distribution. In addition, in empty space this expression can be simplified into:

$$R_{\alpha \beta \gamma \delta} = 0. \quad (19)$$

In other words, the effects of the three dimensional Einstein equation can not propagate outside of the matter. So, Eq.(18) just has the local property. From Ref. [8], it can be clearly seen that:

$$\nabla^2 h_{00} = -2n - 3 \quad (n - n - 2 \tau_{00}). \quad (20)$$

where $n$ is the dimension of space, and $h_{00}$ is from the small perturbation $g_{ij} = \eta_{ij} + h_{ij}$. Firstly, let’s recall that the four dimensional approximation of Einstein’s equation. It is the so-called Newtonian approximation:

$$\frac{d^2 x}{dt^2} = \frac{1}{2} \frac{d^2 \omega}{dF^2} \nabla h_{00} \quad (21)$$

$$\frac{d^2 \omega}{dF^2} = \frac{1}{2} \frac{d^2 t}{dF^2} = 0. \quad (22)$$
And it can be derived like this:
\[
d^2x \frac{dt^2}{d\tau^2} = \frac{1}{2} \nabla h_{00}.
\] (23)

Corresponding with Newtonian result:
\[
d^2x \frac{dt^2}{d\tau^2} = -\nabla \phi,
\] (24)

where \( \phi = \frac{GM}{r} \). Thus, Eq. (20) can be easily concluded that it is two dimensional Laplace equation in this paper. Also its analytic solutions can be obtained(The meaning of the coordinates are changed):
\[
h_{00} = -\frac{\log \omega}{2\pi}.
\] (25)

Before clarifying the relationship between this formula and TSM. Let’s revisit the illustration above corresponded with Wilson’s work. It has also been connected with TSM. From Eq. (3), it is on the analogy with the Boltzmann distribution, where the entropy has micro statistic’s explanation under this distribution. Just like the original definition of the entropy,
\[
S = k \log(\Omega),
\] (26)

we should find the correspondence of \( \Omega \) in this paper. The \( \Omega \) has the clear content in original definition, the micro-state, including \( N, a_i, w_i, e_i \) and \( Z \), denoting the total amount of particles, the amount of per energy level, degeneracy, energy level and partition function, respectively. Now let’s find the corresponded micro-state here.

First, the \( dE^2 \) is used to take the place of \( ds^2 \), all of the physical quantity described here is concerned with the energy. In Eq. (3), we could see that the coupling of \( g^2 \) is on analogy with the temperature \( T \). The essence of \( g^2 \) is about energy. Let’s revisit the definition of the coupling constant \( g \), e.g. in the most simple case of quantum field theory, the scalar field equation:
\[
(\partial^a \partial^\mu + m^2)\phi = 0.
\] (27)

From the renormalization theory, \( m \) is one kind of the coupling constant appeared in the field including the same properties. Another example is in interacting theory, such as:
\[
L_{\text{int}} = -e\bar{\psi} \gamma^\mu \psi A_{\mu},
\] (28)

the electron \( e \) is also a coupling constant. The coupling constant is the manifestation of the observable physical quantity. Meanwhile, the nature of field is energy. So, it is quite natural to use the Eq. (3) as the similar Boltzmann partition function, and the \( g^2 \) is considered as the role of temperature in TSM. Here, let’s rewrite Eq. (3) under three-dimensional situation:
\[
Z\left( \frac{N}{g^2} \right) = \int \prod_{x,i} dU_{ij}(E_i) e^{-\frac{g^2}{\beta} S[U]}(i = 1, 2, 3),
\] (29)

where \( S[U] = \sum_{\mu} 1 - \frac{1}{N} \text{tr} U(\partial p) \)[indicating the energy level], and the more detail of \( U(\partial p) \) can be found at Ref. [15].

Note that \( Z_0(\frac{N}{g^2}) \) is the partition function of a pure lattice gauge theory. It presents that the coupling constant is just a observed formulation, and the gauge theory is one kind of energy. By the analogy, it can be concluded that the gauge field has given us a statistic background, where \( N \) is corresponded with the amount of the total particles, actually the \( N = 3 \). However, there are lots of potential particles can be created from nothing(vacuum). Thus, it can supply for a perceptual explanation for the expansion of \( N \). It can be considered as the potential particles which are not really created from the vacuum, but why there are just three particles in the nature, maybe it is the God’s will. And the degeneracy is the \( \int \prod dU(E_i) \). After these so-called micro-states are obtained, using these to construct the entropy under the frame of pure gauge field:
\[
S = \frac{1}{N} \Omega, \tag{30}
\]

where \( \frac{1}{N} \) is corresponded with \( k \) from the analog: \( \beta = \frac{N}{g^2} \rightarrow \frac{1}{T} \). While every corresponded state is found, it is time to establish the entropy here:
\[
dS = N_i \frac{1}{N} \int d\ln Z - \beta \frac{\partial \ln Z}{\partial \beta}, \tag{31}
\]

where \( N_i \) denotes the total potential particles, and \( Z = Z\left( \frac{N}{g^2} \right) \). Here, I want to emphasize that \( N_i \) is not necessary to equal \( N \), since \( N \) is just part of the total particles of \( N_i \), from deeper level, the pure gauge field is just one manifestation of the energy in this paper. Next, it should compute Eq. (31). Firstly, Let’s calculate \( d\ln Z \). When it is proceed with this, the \( dU \) must be understood. Here, the Haar measure is used in a special case of \( SU(2) \):
\[
dU = \frac{1}{\pi^2} \prod_{\mu=4}^3 da_\mu \delta^{(4)\mu}(a^2_\mu - 1), \tag{32}
\]

since \( \det U = a^2_\mu \), where the \( U = a_4 I + i a_\sigma \hat{\sigma} \). Now we can calculate \( d\ln Z \):
\[
d\ln Z = \frac{dZ}{Z} = d \int \prod_{E,i} dU_{ij}(E_i) e^{-\frac{g^2}{\beta} S[U]}(i = 1, 2, 3)
\]
\[
= \frac{1}{\pi^2} \prod_{E,i} \delta^{(4)}(a^2_\mu - 1) \exp(-\beta S[U]). \tag{33}
\]

In the second line, the index of \( i, E \) is different. \( i \) indicates that \( U \) has three components in three dimensions, and the index \( E \) denotes that \( E \) can be divided into many parts like the definition of Feynman integration. Next, the \( \beta \frac{\partial \ln Z}{\partial \beta} \) must be computed:
\[
\beta \frac{\partial \ln Z}{\partial \beta} = -\beta S[U] = -\frac{N}{g^2} S[U]. \tag{34}
\]

Where \( S[U] \) is the action of the \( U \).
Also we can find the entropy's definition of Clausius,
\[
\frac{dQ}{T} = dS.
\]
(35)
The temperature can be corresponded with the \( g^2 \), but what is the correspondence of \( dQ \) in this paper. Recall that our definition of \( dE^2 = dF^2 \ln^2 T - d\omega^2 \), that is just the energy. The internal energy of matter is defined by this:
\[
dU =TdS + PdV,
\]
(36)
where \( T \) is the temperature. \( P \) is the pressure intensity. By analogy, \( dE^2 \) could be corresponded with internal energy, and \( dF \) is analogous with \( PdV \). The remaining part \( \omega \) is the correspondence of \( Q \). We use the flat spatial situation to substitute the curve situation. Since in three-dimensional space, although there is mass existed, its value is quite small. From Eq.(15), the curvature is determined by the local metric, it could be considered as the flat situation. Also, it should be noted that the general background is three dimensional Einstein theory, and the background of TSM is just the energy transitional form under this general frame. So it means that the \( dQ \) is from \( \omega \) under this general background. Thus, it can establish the relationship between \( \omega \) and the corresponded entropy. Let’s combine with Eq.(33-36). Finally, the formula is deduced:
\[
\frac{d\omega}{g^2} = N_i \left( \frac{1}{\pi^2} \prod_{\mu} \frac{\theta(W - 1)}{Z} \exp(-\beta S[U]) - \frac{Z}{g^2} \frac{N}{S[U]} \right).
\]
(37)
Next, we should further study the properties which are analogous with Friedmann cosmology. Firstly, let’s adopt the Robertson-Walker line element in curvature coordinates:
\[
dE^2 = -\rho^2 dF^2 - R^2(F)\omega^2 dx^2_{ij} \quad (\alpha, \beta = 1, 2).
\]
(38)
Note that \( [\omega(F)] = [\text{work}] \). In Einstein’s hydrodynamics, \( \rho \) and \( \mu \) denote the pressure intensity and intrinsic energy intensity. Here, it keeps the same mathematical structure and its similar physical correspondence. So the three-dimensional hydrodynamics’ equation can be obtained:
\[
T_{ij} = (\mu + p)U_iU_j + P\delta_{ij} (i, j = 1, 2, 3).
\]
(39)
Also, the equation of the force-energy conservation and the equation of state can be derived:
\[
R^2 + k = \frac{1}{3} \kappa R^2 \mu.
\]
\[
\rho R^2 = \frac{d}{dF} [R^2 (\rho + \mu)],
\]
(40)
where \( \dot{R} \) and \( \dot{\rho} \) are the differentiation with respect to \( F \). With the analyze of these equations, as Ref. [12], it can be briefly concluded this:
\[
R^2(F_0) \geq K \mu(F_0) \ln R.
\]
(41)
Taking the logarithm in both sides of this equation, which is given by:
\[
R_m = \exp \left( \frac{R^2(F_0)}{K \mu(F_0)} \right) \geq R.
\]
(42)
Where the \( R \) should have the maximum value. Recall that the flat situation in three dimensions, it tells us that \( \omega \) should get the maximum as \( l = 0 \) and \([R(F)] = \omega = [\text{work}] \). When it meets with this situation, the matter in scale of \( l_0 \) should expand. Thus, it agrees with the reality instead of the universe gets the maximum radius in three dimensions.

As a conclusion, the three-dimensional curved space is determined by the dark energy. Firstly, it cannot be observed directly. Secondly, as Ref. [10, 11] mentioned, it could originate from the ghost particle(also could not be seen), and the starting point of the hypothesis is to solve the quark confinement, which can be repaired the missing part of the Wilson’s lattice QCD theory. So I conclude intuitively that the curved part is the dark energy due to the ghost particle, even it offers us a clue to analyze and detect the so-called ghost particle. From the Einstein’s most serious mistake in his life’s research, the cosmological constant model is the most simple and agreed with experimental data. Thus, the next question: where is the cosmological constant from? Here, the given response is the three-dimensional spatial curvature, if there was little matter existed in the three-dimensional space. It should be the curvature are totally determined by the local matter distribution, and the curvature is nearly flat due to the little mass of realistic matter. These mean that the cosmological constant is quite small agreed with the observed value.

IV. CONCLUSION AND DISCUSSION

In this paper, the new application of the three-dimensional Einstein theory is found, which is a new picture of the internal structure of one particle. And the energy scale has the same scope with quarks.

Firstly, from research to the Wilson’s masterpiece [8], it can supply for an explanation of quark confinement due to the existence of the lattice space \( a \). Especially for the strong coupling situation, the confinement is caused by the large-area law. However, the scope of the lattice space \( a \) is not determined and brought some properties of arbitrary. By the analogy with the Special Relativity, the fundamental length is given. Under this hypothesis, it can break the Lorentz transformation since we adopt this coordinate-transformation:
\[
X'_i = \left( \begin{array}{ccc}
ch\lambda & -sh\lambda & 0 \\
sh\lambda & ch\lambda & 0 \\
0 & 0 & 1
\end{array} \right) X_i
\]
(43)
\[
Y'_i = \left( \begin{array}{ccc}
ch\lambda & 0 & -sh\lambda \\
0 & 1 & 0 \\
-sh\lambda & 0 & ch\lambda
\end{array} \right) Y_i
\]
(44)
\[
Z'_i = \left( \begin{array}{ccc}
1 & 0 & 0 \\
0 & ch\lambda & -sh\lambda \\
0 & -sh\lambda & ch\lambda
\end{array} \right) Z_i
\]
(45)
These transformations are independent with \( t \). But they have the same mathematical structure as the Lorentz transformation. After proceeding with this, from Eq.(8), it can be clearly
seen that the force will be infinite when the distance is \( l_0 \), which is a new way of exploring the confinement. Meanwhile, it seems like that there is still a ‘motion’ when the time stopped. In section III, it is giving the relationship between TSM and the energy of \( \omega \), illustrating the micro-state is random. And Wilson’s theory can be clarified the essence of the renormalization group. Thereby, it can further study the nature of the renormalization group under the frame of three-dimensional Einstein theory.

Next, it is quite natural to promote it into the curved situation. But what is the principle of equivalence in three-dimensional space? The \( \frac{d^2\omega}{dF^2} = \frac{dl}{dF} \) is a new physical quantity. With the help of [10, 11], it is showing that the dark energy’s value is quite probably originated from the ghost particle in QCD. Recalling the end of part III, it is concluding that the curvature is the origination of the cosmological constant \( \Lambda \). In addition, due to the un-observation and strangeness of the dark energy, it is confirmed that the \( \frac{d^2\omega}{dF^2} = \frac{dl}{dF} \) is corresponded with dark energy. In other words, the background is dark energy. By the analogy with three-dimensional cosmology, the revolutionary factor \( R \)’s physical meaning has completely changed. From Eq.(42), we can see it clearly when the \( R(|R|)=|\text{work}| \) has the maximum value, it must reduce. It means that while \( \omega \) has reached the maximum value, the value of \( l \) is zero, and the minimum of \( l \) should expand.

Finally, let’s put the initial point into the deeper level, from the level of the symmetry, the GR, QED, QCD, etc. All of them have their own gauge symmetry, just like C.N.Yang said, “the symmetry dominates the physics”. The starting point of this paper is that a \( N \)-dimensional Lie group’s open subset is also a Lie group, and all of the symmetry can be described by the Lie group. Minkowski’s space is a special case of curved space, and the global symmetry is a special case of local symmetry. The next promotion is that the local symmetry is a special case of one point’s symmetry. If the local symmetry is a special case, the point should have the internal structure. So, we can promote the so-called principle of equivalence and the gauge symmetry. Then put them into one symmetry. At last, there is a principle(conjecture), that the \( N \) dimensional symmetry is determined by the \( N-1 \) dimensional symmetry. But what is the mathematical structure is also our future work.

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