

QUESTIONS AND ANSWERS

Contributions to this section, both Questions and Answers, are welcomed. Please submit four copies to the editorial office. Please include a *title* for each submission, include name and address at the end, and put references in the standard format used in the *American Journal of Physics*. For further suggestions, sample Questions and Answers, and requested form for both Questions and Answers, see Robert H. Romer, "Editorial: 'Questions and Answers,' a new section of the *American Journal of Physics*," *Am. J. Phys.* **62** (6), 487–489 (1994).

Questions at any level and on any appropriate AJP topic, including the "quick and curious" question, are encouraged.

Question #78. A question about the Maxwell relations in thermodynamics

Various mnemonics exist for the Maxwell relations connecting thermodynamic partial derivatives for a gas. Perhaps the most compact is the Jacobian identity

$$\frac{\partial(p, V)}{\partial(T, S)} = 1.$$

[Here p is the pressure, V the volume, T the temperature, and S the entropy.] In preparing lectures over the years, we have independently noticed that this formula has a simple physical explanation not mentioned in the standard textbooks. Consider a loop in the p, V plane, describing a quasi-static cycle, and the corresponding loop in the T, S plane. The area of the former is the work done; that of the latter is the heat absorbed. By the first law, these areas must be equal. An area preserving map requires that the Jacobian of the transformation between coordinates be unity.

Can any reader tell us where in the literature this pleasing and apparently little known way of looking at Maxwell's relations can be found?

Vinay Ambegaokar and N. David Mermin
Laboratory of Atomic and Solid State Physics
Cornell University
Ithaca, New York 14853

Question #79. Does plane wave not carry a spin?

As is generally known, a circularly polarized plane wave with infinite extent can have no angular momentum.¹ Only a quasiplane wave of finite transverse extent carries an angular momentum whose direction is along the direction of propagation. This angular momentum is provided by an outer region of the wave within which the amplitudes of the electric \mathbf{E} and magnetic \mathbf{B} fields are decreasing. These fields have components parallel to the wave vector there, and the energy flow has components perpendicular to the wave vector. This angular momentum is the spin of the wave.² Within an inner region the \mathbf{E} and \mathbf{B} fields are perpendicular to the wave vector and the energy flow is parallel to the wave vector.³

Now suppose that such a quasiplane wave is absorbed by a round flat target which is divided concentrically into outer and inner parts. According to previous reasoning, the inner part of the target will not perceive a torque. Nevertheless R. Feynman⁴ clearly showed how a circularly polarized plane wave transfers a torque to an absorbing medium. What is true? And if R. Feynman is right, how can one express the torque in terms of ponderomotive forces?

I have not found an answer in J. M. Jauch *et al.*, *The Theory of Photons and Electrons*, 2nd ed. (Springer, New York, 1976), or in L. Allen *et al.*, "The Orbital Angular

Momentum of Light," in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1999). For example, L. Allen *et al.* write only, "The ratio [of spin angular momentum to energy] changes from place to place. The problem of the way the polarization of the beam depends on its finite extent has been the subject of detailed examination (Simmonds and Guttman [1970])" (p. 300).

¹W. Heitler, *The Quantum Theory of Radiation* (Clarendon, Oxford, 1954), p. 401.

²H. C. Ohanian, "What is spin?," *Am. J. Phys.* **54**, 500–505 (1986).

³J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), p. 201.

⁴R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, London, 1965), Vol. 3, p. 17–10.

R. I. Khrapko
Moscow Aviation Institute
4 Volokalamskoe Shosse
125871, Moscow, Russia

Question #80. Relating scalar and pseudoscalar quantities in electromagnetism

In Maxwell's equations without magnetic monopoles, either the electric field, \mathbf{E} , or the magnetic field, \mathbf{B} , is a pseudovector. According to Jackson,¹ it is an experimental fact that electric charge is invariant under Galilean and Lorentz transformations and rotations. It is then a choice, albeit natural, to take electric charge to be also invariant under spatial inversion (and even under time reversal). Given this choice, this means that the electric field is a vector and the magnetic field is a pseudovector.

In many texts on electromagnetism, inevitably some discussion is given as to how Maxwell's equations change in the presence of magnetic monopoles.² This is often justified by first noting that their inclusion gives a pleasing symmetry to Maxwell's equations, and second that Dirac showed that the presence of one magnetic monopole ensures that electric charge is quantized. Dirac's quantization condition relates the sizes of the electric and magnetic charges, e and g_m , respectively, as

$$\frac{g_m e}{\hbar c} = \frac{n}{2}, \quad (1)$$

where n is an integer, $n = 0, \pm 1, \pm 2, \dots$

However, according to Arfken and Weber,³ it is not reasonable to relate scalars and pseudoscalars, since this would distinguish between left- and right-handed reference frames.

Arfken and Weber also note that there are processes, such as beta decay, which do distinguish between the handedness of a reference frame, and polar and axial vector interactions

add together here. Are there any similar consequences with the introduction of magnetic monopoles in electromagnetism?

Of course, if we apply the inversion operator to (1), we can write $g_m e / \hbar c = m/2$ with $m = 0, \pm 1, \pm 2, \dots$, so is it only when we consider a specific integer that the objection arises? Is (1) telling us something about particles and antiparticles?

Given that (even with magnetic monopoles) \mathbf{E} and \mathbf{B} have well-defined transformation properties under reflection, is it admissible, as some authors do,⁴ to introduce duality transformations that mix the fields:

$$\begin{aligned} \mathbf{E}' &= \mathbf{E} \cos(\alpha) + c\mathbf{B} \sin(\alpha), \\ c\mathbf{B}' &= -\mathbf{E} \sin(\alpha) + \mathbf{B} \cos(\alpha)? \end{aligned} \quad (2)$$

Presumably, the angle α is a pseudoscalar.

As an aside, when authors introduce the magnetic scalar potential, should they really refer to it as the magnetic *pseudoscalar* potential?

ACKNOWLEDGMENT

The author would like to thank Professor C. Michael (Liverpool University) for making available the facilities of D.A.M.P.T.

¹J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., pp. 249–250.

²See, for example, R. H. Good, *Classical Electromagnetism* (Saunders College Publishing, Orlando, 1999), pp. 125–128; D. J. Griffiths, *Introduction to Electrodynamics* (Prentice–Hall, New York, 1999), 3rd ed., pp. 327–328; J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., pp. 251–253; P. Lorrain, D. R. Corson, and F. Lorrain, *Electromagnetic Fields and Waves* (Freeman, New York, 1988), 3rd ed., p. 327; J. Vanderlinde, *Classical Electromagnetic Theory* (Wiley, New York, 1993), Sec. 2.1, pp. 86–89.

³G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists* (Academic Press, Inc., San Diego, 1995), 4th ed., pp. 135–140.

⁴See, for example, D. J. Griffiths, *Introduction to Electrodynamics* (Prentice–Hall, New York, 1999), 3rd ed., Problem 7.60, p. 342; J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., p. 252; J. Vanderlinde, *Classical Electromagnetic Theory* (Wiley, New York, 1993), Sec. 2.1, p. 89.

J. P. McTavish
School of Engineering
Liverpool John Moores University
Byrom Street
Liverpool
L3 3AF, United Kingdom
Electronic mail: j.p.mctavish@livjm.ac.uk

OPPIE'S LIMITATIONS

The general wanted Oppenheimer anyway. ‘‘He’s a genius,’’ Groves told an interviewer off the record immediately after the war. ‘‘A real genius. While Lawrence is very bright he’s not a genius, just a good hard worker. Why, Oppenheimer knows about everything. He can talk to you about anything you bring up. Well, not exactly. I guess there are a few things he doesn’t know about. He doesn’t know anything about sports.’’

Richard Rhodes, *The Making of the Atomic Bomb* (Simon & Schuster, New York, 1986), pp. 448–449.