Two gravitational effects related to rotating masses are described. The first is the decreasing of the gravitational mass when the rotational kinetic energy is increased. In the case of ferromagnetic materials, the effect is strongly increased and the gravitational mass can even become negative. The second is the gravitational shielding effect produced by the decreasing of the gravitational mass of the rotating mass.

**Key words:** Modified theories of gravity, Gravitational Effects of Rotating Masses, Experimental studies of gravity, New topics in Superconductivity.

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### 1. Introduction

In 1918, H. Thirring [1] showed that a rotating mass shell has a weak dragging effect on the inertial frames within it. In today’s literature these results are known as Lense-Thirring effects.

Recently, the Lense-Thirring effect has received new interest because it becomes now possible to directly measure this tiny effect [2]. In the years 1959-1960 it was discovered by G. E. Pugh [3] and Leonard Schiff [4,5] that the mentioned dragging phenomenon leads to another effect - called the Schiff effect - which might be suited for experimental confirmation: The rotation axis of a gyroscope, inside a satellite orbiting the Earth, in a height of 650 km, suffers a precession of 42 milliarcseconds per year, due to the Earth’s rotation [6].

Here, we show new gravitational effects related to rotating gravitational masses, including superconducting masses.

### 2. Theory

From the quantization of gravity it follows that the gravitational mass $m_g$ and the inertial mass $m_i$ are correlated by means of the following factor [7]:

$$
\kappa = \frac{m_g}{m_i} = \left[1 - 2 \left(1 + \left(\frac{\Delta p}{m_i c}\right)^2\right)^{-1}\right]
$$

(1)

where $m_i$ is the rest inertial mass of the particle and $\Delta p$ is the variation in the particle’s kinetic momentum; $c$ is the speed of light.

That equation shows that only for $\Delta p = 0$ the gravitational mass is equal to the inertial mass.

In general, the *momentum* variation $\Delta p$ is expressed by $\Delta p = F \Delta t$ where $F$ is the applied force during a time interval $\Delta t$. Note that there is no restriction concerning the nature of the force $F$, i.e., it can be mechanical, electromagnetic, etc.

For example, we can look on the momentum variation $\Delta p$ as due to absorption or emission of electromagnetic energy by the particle.

In this case $\Delta p$ can be obtained as follows: It is known that the radiation pressure, $dP$, upon an area $dA = dx dy$ of a volume $dV = dx dy dz$ of a particle (the incident radiation normal to the surface $dA$) is equal to the energy $dU$ absorbed (or emitted) per unit volume $(dU/dV)$ i.e.,

$$
dP = \frac{dU}{dV} = \frac{dU}{dxdydz} = \frac{dU}{dAdz}
$$

(2)

Substitution of $dz = v dt$ ($v$ is the speed of radiation) into the equation above gives

$$
dP = \frac{dU}{dV} = \left(\frac{dU}{dAdt}\right) = \frac{dD}{v}
$$

(3)

Since $dPdA = dF$ we can write:

$$
dFdt = \frac{dU}{v}
$$

(4)

However we know that $dF = dp/\sqrt{\Delta t}$, then

$$
dp = \frac{dU}{v}
$$

(5)

From this equation it follows that
\[ \Delta p = U \left( \frac{c}{v} \right) = U \frac{n_r}{c} \]  

(6)

where \( n_r \) is the index of refraction.

Substitution of Eq. (6) into Eq. (1) yields

\[ m_g = \left( 1 - 2 \left[ 1 + \left( \frac{U}{m_0 c^2 n_r} \right)^2 \right] \right) m_0 \]  

(7)

In the case of absorption of a single photon with wavelength \( \lambda \) and frequency \( f \), Eq. (7) becomes

\[
\frac{m_g}{m_0} = \left( 1 - 2 \left[ 1 + \left( \frac{\lambda \omega}{\lambda_0 n_r} \right)^2 \right] \right) = \left( 1 - 2 \left[ 1 + \left( \frac{\lambda \omega}{\lambda_0 n_r} \right)^2 \right] \right) \]  

(8)

where \( \lambda_0 = h/m_0 c \) is the De Broglie wavelength for the particle with rest inertial mass \( m_0 \).

From Electrodynamics we know that when an electromagnetic wave with frequency \( f \) and velocity \( c \) incides on a material with relative permittivity \( \varepsilon_r \), relative magnetic permeability \( \mu_r \), and electrical conductivity \( \sigma \), its velocity is reduced to \( v = c/n_r \) where \( n_r \) is the index of refraction of the material, given by [8]

\[ n_r = \frac{c}{v} = \frac{\varepsilon_r \mu_r}{2} \left( \sqrt{1 + \left( \frac{\mu_r \sigma}{\varepsilon_r \omega} \right)^2} + 1 \right) \]  

(9)

If \( \sigma \gg \omega \varepsilon, \omega = 2\pi f \), Eq. (9) reduces to

\[ n_r = \frac{\mu \varepsilon}{4\pi \varepsilon_0 f} \]  

(10)

Thus, the wavelength of the incident radiation (See Fig. 1) becomes

\[ \lambda_{mod} = \frac{v}{f} = \frac{c/f}{n_r} = \frac{\lambda}{n_r} = \frac{4\pi}{\mu \varepsilon \sigma} \]  

(11)

If a lamina with thickness equal to \( \xi \) contains \( n \) atoms/m³, then the number of atoms per area unit is \( n \xi \). Thus, if the electromagnetic radiation with frequency \( f \) incides on an area \( S \) of the lamina it reaches \( nS \xi \) atoms. If it incides on the total area of the lamina, \( S_f \), then the total number of atoms reached by the radiation is \( N = nS_f \xi \). The number of atoms per unit of volume, \( n \), is given by

\[ n = \frac{N_0 \rho}{A} \]  

(12)

where \( N_0 = 6.02 \times 10^{26} \) atoms/kmole is the Avogadro’s number; \( \rho \) is the matter density of the lamina (in kg/m³) and \( A \) is the molar mass(kg/kmole).

When an electromagnetic wave incides on the lamina, it strikes \( N_f \) front atoms, where \( N_f = nS_f/\phi_m \), \( \phi_m \) is the “diameter” of the atom. Thus, the electromagnetic wave incides effectively on an area \( S = N_f S_m \), where \( S_m = \frac{1}{4} \pi \phi_m^2 \) is the cross section area of one atom. After these collisions, it carries out \( n_{\text{collisions}} \) with the other atoms (See Fig.2).
Thus, the total number of collisions in the volume $S \xi$ is
\[
N_{\text{collisions}} = N_f + n_{\text{collisions}} = n_f S_m + (n_f S \xi - n_f S_m) = -n_f S \xi
\]
(13)
The power density, $D$, of the radiation on the lamina can be expressed by
\[
D = \frac{P}{S} = \frac{P}{N_f S_m}
\]
(14)
We can express the total mean number of collisions in each atom, $n_1$, by means of the following equation
\[
n_1 = \frac{n_{\text{total photons}} N_{\text{collisions}}}{N}
\]
(15)
Since in each collision a momentum $h/\lambda$ is transferred to the atom, then the total momentum transferred to the lamina will be
\[
\Delta p = (n_1 N) h/\lambda.
\]
Therefore, in accordance with Eq. (8), we can write that
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{P}{h f^2} \right) n_f S \xi \right] \left( \frac{\lambda}{\lambda} \right) \right] \right] \}
(16)
Substitution of Eq. (17) into Eq. (16) yields
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{P}{h f^2} \right) n_f S \xi \right] \left( \frac{\lambda}{\lambda} \right) \right] \right] \}
(17)
Substitution of $P$ given by Eq. (14) into Eq. (18) gives
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{P}{h f^2} \right) n_f S \xi \right] \left( \frac{\lambda}{\lambda} \right) \right] \right] \}
(19)
Substitution of $N_f \equiv (n_f S_f) \phi_m$ and $S = N_f S_m$ into Eq. (19) results in
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{n_f S \xi \phi_m D}{m_{(0)} c^2} \right) \left( \frac{\lambda}{\lambda} \right) \right] \right] \right] \}
(20)
where $m_{(0)} = \rho_{(i)} V_{(i)}$.

In the case in which the area $S_f$ is just the area of the cross-section of the lamina $S_d$, we obtain from Eq. (20), considering that $m_{(0)} = \rho_{(i)} S \xi$, the following expression
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{n_f S \xi \phi_m D}{m_{(0)} c^2} \right) \left( \frac{\lambda}{\lambda} \right) \right] \right] \right] \}
(21)
If the electrical conductivity of the lamina, $\sigma_{(i)}$, is such that $\sigma_{(i)} \gg \omega e$, then Eq. (9) reduces to
\[
n_{r} = \frac{\mu \sigma}{4 \pi \varepsilon_0 j}
\]
(22)
Substitution of Eq. (22) into Eq. (21) gives
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{n_f S \xi \phi_m D}{m_{(0)} c^2} \right) \left( \frac{\lambda}{\lambda} \right) \right] \right] \right] \}
(23)
This is therefore the expression of correlation between gravitational mass and inertial mass in the particular case of incident radiation on ordinary matter (non-coherent matter) at rest.

If the body is also rotating, with an angular speed $\omega$ around its central axis, then it acquires an additional energy equal to its rotational energy $E_k = \frac{1}{2} I \omega^2$. Since this is an increase in the internal energy of the body, and this energy is basically electromagnetic, we can assume that $E_k$, such as $U$, corresponds to an amount of electromagnetic energy absorbed by the body. Thus, we can consider $E_k$ as an increase $\Delta U = E_k$ in the electromagnetic energy $U$ absorbed by the body. Consequently, in this case, we must replace $U$ in Eq. (7) for $U + \Delta U$, i.e.,
\[
\frac{m_{g(0)}}{m_{(0)}} = \begin{bmatrix} 1 - 2 \left[ \left( \frac{U + \Delta U}{m_{(0)} c^2} \right) \right] \right] \right] \}
(24)
Note that the variable $U$ can refer to both the electromagnetic energy of a radiation as the electromagnetic energy of the electromagnetic field due to an electric current through the rotating gravitational mass.

Thus, Eq. (24) can be rewritten as
Note that $E_k$ is not an amplitude of a wave such as $U_m$, $U = U_m \sin 2\pi f t$. Therefore, $E_k$ and $\sin 2\pi f t$ are independent parameters. Consequently, there is no sense to talk about average values for $E_k \sin 2\pi f t$, such as in the case of $U_m \sin 2\pi f t$, where the average value for $U^2$ is equal to $\frac{1}{2} U_m^2$ because $U$ varies sinusoidaly ($U_m$ is the maximum value for $U$).

Then, if $U_m << E_k$, the Eq. (25) reduces to

$$m_g \approx \left\{ 1 - 2 \left[ 1 + \frac{\mu_0 \sigma_0 f^2 \sin^2 2\pi f t}{16 \pi^2 \sigma_0^2 c^2} - 1 \right] \right\} m_0 \tag{26}$$

For $\sigma >> \omega \epsilon$, Eq.(9) shows that and

$$n_r = \sqrt{\frac{\mu_0 \sigma_0}{4 \pi f^2}}$$

In this case, Eq. (26) gives

$$m_g \approx \left\{ 1 - 2 \left[ 1 + \frac{\mu_0 \sigma_0 f^2 \sin^2 2\pi f t}{16 \pi^2 \sigma_0^2 c^2} - 1 \right] \right\} m_0 \tag{27}$$

Note that the effect of the electromagnetic field applied upon the mass is highly relevant, because in the absence of this radiation the index of refraction, present in Eq. (26), becomes equal to 1. Under these circumstances, the possibility of reducing the gravitational mass is null. On the other hand, the equation above shows that, in practice, the decreasing of the gravitational mass can become relevant in the particular case of ferromagnetic materials subjected to electromagnetic fields with extremely low frequencies (ELF).

Figure 3 shows a schematic diagram of a Mumetal disk ($\mu_r = 105,000 \text{ at } 100 \text{ gauss}$; $\sigma = 2.1 \times 10^6 \text{ S.m}^{-1}$) with radius $R = 0.10 m$ ($f = \sqrt{\frac{c}{m_0 R^2}}$) rotating with an angular velocity $\omega = 209 \times 10^6 \text{ rad/s} \sim (200,000 \text{ rpm})$. Thus, if an ELF radiation or an electrical current with extremely low frequency e.g., $f = 0.1 \text{ Hz}$ is applied on the Mumetal disk, then according to Eq.(27), the gravitational mass of the disk will oscillate between $m_g = m_{0g}$ and

$$m_g \approx -0.96 m_{0g} \tag{28}$$

It has been shown that there is an additional effect - Gravitational Shielding effect - produced by a substance whose gravitational mass was reduced or made negative [9]. The effect extends beyond substance (gravitational shielding) , up to a certain distance from it (along the central axis of gravitational shielding). This effect shows that in this region the gravity acceleration, $g_1$, is reduced at the same proportion, i.e., $g_1 = \chi_1 g$ where $\chi_1 = m_g/m_{0g}$ and $g$ is the gravity acceleration before the gravitational shielding. Here, according to Eq.(28), we have $-0.96 \leq \chi_1 \leq 1$. Thus, the gravity acceleration above the Mumetal disk will vary in the range $-0.96 g \leq g_1 \leq g$ since the gravity before (below) the gravitational shielding is $g$.

Let us now consider the case in which the rotating mass is a superconducting material.

The most famous characteristic of superconductivity is zero resistance. However, the superconductors are not the same as a perfect conductor. The observed surface resistance, $R_s$, of most superconductors to alternating currents shows that the resistivity can be extremely small at the internal region close to the surface of the superconductor. The thickness of this region is known as London penetration depth, $\lambda_{L}$ [10]. According to BCS theory $R_s = (1/\lambda_{L}) (\sigma/\sigma_n)^2$ where $\sigma$ is the normal-state conductivity and $\sigma_n$ is the conductivity of the mentioned region, which is given by [11]:

$$\sigma_n = \frac{1}{2 \pi \mu \lambda_{L}^2 f} \tag{29}$$

It is important to note that Eq. (16) refers to the case of ordinary matter (non-coherent matter). In the case of superconductors the radiation is absorbed by the Cooper-pairs fluid (coherent part of the superconductors) and there is no scattering of the incident radiation. Consequently, $N_{collisions} = 1$ (the total number of collisions). Therefore, in the case of superconductors Eq. (16) reduces to

$$\frac{m_g (f)}{m_{0g}} = \left\{ 1 - 2 \left[ 1 + \frac{n_{total \ photons} \frac{h f}{m_0 c^2} n_r}{n_r^2} - 1 \right] \right\} \tag{30}$$
which is exactly the equation (7). Thus, we conclude that Eq. (7) is general for all types of matter (coherent and non-coherent).

Since \( U = U_m \sin 2\pi ft \), the average value for \( U^2 \) is equal to \( \frac{1}{2} U_m^2 \) because \( U \) varies sinusoidaly (\( U_m \) is the maximum value for \( U \)). On the other hand, \( U_{rms} = U_m/\sqrt{2} \). Consequently we can change \( U^2 \) by \( U_{rms}^2 \) in the Eq. (7).

Alternatively, we may put this equation as a function of the radiation power density, \( D_{rms} \), since \( U_{rms} = VD_{rms}/\nu \) (See Eq. (3)). Thus, we obtain

\[
m_g = \left( 1 - \frac{\rho}{V} \right) \left( 1 + \frac{D_{rms}^2}{\rho c^2} \right) - 1 \]

where \( \rho = m_0/V \).

For \( \sigma >> \omega \epsilon \), Eq.(9) shows that and \( n_r = \sqrt{\mu \epsilon c^2 / 4 \pi f} \). In this case, equation above becomes

\[
m_g = \left( 1 - \frac{\mu \sigma D_{rms}^2}{4 \pi c f} \right) - 1 \]

Now consider a superconducting disk (YBCO) on the Earth’s atmosphere. It is known that the Schumann resonances [12] are global electromagnetic resonances (a set of spectrum peaks in the extremely low frequency ELF), excited by lightning discharges in the spherical resonant cavity formed by the Earth’s surface and the inner edge of the ionosphere (60km from the Earth’s surface). The Earth–ionosphere waveguide behaves like a resonator at ELF frequencies and amplifies the spectral signals from lightning at the resonance frequencies. In the normal mode descriptions of Schumann resonances, the fundamental mode \( (n=1) \) is a standing wave in the Earth–ionosphere cavity with a wavelength equal to the circumference of the Earth. This lowest-frequency (and highest-intensity) mode of the Schumann resonance occurs at a frequency \( f_1 = 7.83 \text{Hz} \) [13].

It was experimentally observed that ELF radiation escapes from the Earth–ionosphere waveguide and reaches the Van Allen belts [14-17]. In the ionospheric spherical cavity, the ELF radiation power density, \( D \), is related to the energy density inside the cavity, \( W \), by means of the well-known expression:

\[
D = \frac{c}{4} W
\]

where \( c \) is the speed of light, and \( W = \frac{1}{2} \epsilon_0 E^2 \). The electric field \( E \), is given by

\[
E = \frac{q}{4\pi \epsilon_0 r_0^3}
\]

where \( q = 500,000 \text{C} \) [16] and \( r_0 = 6.37 \times 10^6 \text{m} \). Therefore, we get

\[
E = 1107 W/m, \\
W = 5.4 \times 10^8 J/m^3, \\
D_{rms} = 4.1 W/m^2.
\]

In the case of YBCO, \( \lambda = 140 \text{nm} \) [18,19]. Then, substitution of this value into Eq.(29) gives

\[
\mu \sigma_s = \frac{1}{2 \pi \lambda^2 f} = 8.12 \times 10^{-2}
\]

The variable \( \sigma \) in Eq. (32) is \( \sigma_s \), and the density of the YBCO is \( \rho = 6300 kg/m^3 \) [20,21]. Thus, we can rewrite Eq. (32) as follows

\[
m_g = \left( 1 - \frac{0.34 D_{rms}^2}{f^2} \right) - 1
\]

Since the superconducting disk is inside the Earth’s atmosphere then it is subjected to Schumann resonances. Thus, the values of \( f \) and \( D_{rms} \) are given respectively by \( f = f_1 = 7.83 \text{Hz} \) and \( D_m = 4.1 W/m^2 \) (Eq. (35)). Therefore the value of \( \chi \) given by Eq. (37) is

\[
\chi = \frac{m_g}{m_{0g}} = 0.9995
\]

Since the weight of the disk is \( \chi m_{0g} \) then \( m_{0g} - \chi m_{0g} = 5 \times 10^{-4} m_{0g} \) is the decrease in the weight of the disk. Therefore the disk 0.05% of its weight (without any rotation). Due to the Gravitational Shielding effect, these variations are the same for a sample above the disk.
When the disk acquires an angular velocity \( \omega \), then the additional value \( \chi_a \), due to the rotation, can be obtained making \( U_m = 0 \) in Eq. (25), i.e.,

\[
\chi_a \approx \left\{ 1 - 2 \left[ \frac{1}{1 + \frac{\mu \sigma \omega^2 I^2 \sin^2 2\gamma}{16\pi \sigma m_{\text{YBCO}}^2 \epsilon^2}} - 1 \right] \right\} (39)
\]

In the Podkletnov experiment, the YBCO disk is a rectangular toroid with radius \( R_{\text{outer}} = 275 \text{mm}, R_{\text{inner}} = 80 \text{mm} \), 10 mm-thickness, with an angular velocity \( \omega = 523.6 \text{rad/s} \) (5,000 rpm) [22,23]. Considering these values and the value of \( \mu \sigma \), given by Eq. (36), then Eq. (39) shows that \( \chi_a \), in this case, is given by

\[
\chi_a \approx -1
\]

Note that this value corresponds to the region of the disk with thickness \( \lambda_L \). Thus, we can write that

\[
m_{\text{g(disk)}} = m_{\text{\text{YBCO} (disk)}} + \chi_a m_{\text{\text{YBCO} (disk)}} = m_{\text{\text{YBCO} (disk)}} + \chi_a \left( \frac{m_{\text{\text{YBCO} (disk)}}}{m_{\text{\text{YBCO} (disk)}}} \right) m_{\text{\text{YBCO} (disk)}} (40)
\]

Thus, we get

\[
\frac{m_{\text{\text{YBCO} (disk)}}}{m_{\text{\text{YBCO} (disk)}}} = \frac{V_{\text{\text{YBCO} (disk)}}}{V_{\text{\text{YBCO} (disk)}}} = \frac{2\lambda_L}{10 \text{mm}} = 200 \lambda_L (41)
\]

Substitution of Eq. (41) into Eq. (40) gives

\[
m_{\text{g(disk)}} = m_{\text{\text{YBCO} (disk)}} + 200 \lambda_L \chi_a m_{\text{\text{YBCO} (disk)}} = m_{\text{\text{YBCO} (disk)}} - 2.9 \times 10^{-3} m_{\text{\text{YBCO} (disk)}} (42)
\]

In this case the disk loses more 2.9 \times 10^{-3}\% of its weight due to its rotation. This corresponds to a decrease of about 5.8\% on the initial value of 0.05\% that the disk loses without any rotation. Due to the Gravitational Shielding effect, a sample above the disk will have its weight decreased of the same percentage (5.8\% on the initial value of 0.05\% that the sample loses).

Thus, when \( \sin \omega = 0 \) Eq. (39) shows that \( \chi_a = 1 \), i.e., the decreases of gravitational mass vanish, this corresponds to an increase in the weight of the sample of about 5.8\% on the initial value of 0.05\% that the sample lost more a portion due to the increase of the weight of the air column above the sample. Due to the gravitational shielding effect, the gravity acting on the air column above the sample (height~12 \( R_{\text{outer}} \)) is reduced in the same proportion that is reduced the gravitational mass of the disk (gravitational shielding). Thus, there is also an increase in the weight of the sample of 5.8\% on the weight of the air column above the sample. Considering that 5.8\% on the weight of the air column is equivalent to \( x\% \) on the initial value of 0.05\% that the sample lost, i.e.,

\[
5.8\% m_{\text{g (air)}} = x\% \text{of} 0.05\% m_{\text{g (sample)}} (43)
\]

Thus, we get

\[
x\% = \frac{5.8\% m_{\text{g (air)}}}{0.05\% m_{\text{g (sample)}}} = \frac{5.8\% \rho_{\text{air}} V_{\text{air}}}{0.05\% \rho_{\text{sample}} V_{\text{sample}}} \leq 2\%
\]

Since \( \rho_{\text{air}} = 1.2 \text{kg/m}^3, \rho_{\text{sample}} = 1400 \text{kg/m}^3 \) and \( V_{\text{air}}/V_{\text{sample}} \approx 12 R_{\text{outer}}/150 \text{mm} = 80 \text{mm} \). Under these circumstances, the weight of the sample becomes unstable with fluctuations from \(-5.8\%\) to \(+7.8\%\) of the initial value of 0.05\%, i.e., a total increase of about 7.8\% on the initial value of 0.05\%.

Consequently, the weight of the sample oscillates in the range

\[
0.047\% \text{ to } 0.053\%
\]

of its weight.

In the Podkletnov’s experiment the findings were \(-2.5\%\) to \(+5.5\%\) of the initial value of 0.05\% [22,23]. This means that the total variation of the weight of the sample oscillates in the range

\[
0.048\% \text{ to } 0.052\%
\]

of its weight.

Note that, according to Eq. (39) and Eq. (42), for \( \omega = 2.09 \times 10^4 \text{ rad/s} \) (200,000 rpm) the sample weight decrease can reach about 17\%. This very smaller than the 96\% in the case of the Mumetal disk (Eq. (28)).
Fig. 3 – Schematic diagram of an experimental set-up to measure the decrease of the gravitational mass of the Mumetal disk and the gravitational shielding effect produced by the rotating disk.
References


