# Clifford Clock and the Complex Spaces of the Qi Men Dun Jia Model



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### Abstract

The concept of the Clifford Clock, which demonstrates the relationships between real, complex, quarternions and octonions, has become established in math physics. Less well-known are the spaces in which the algebras dwell. Space consists of an infinite number of tiny invisible cubes. Under the rubric of the Clifford Clock, these spaces take on significance, since the spaces themselves take on periodic aspects, and are directly related to Supersymmetry and the Super Brauer group of Lie Algebras. In addition, the Clifford Clock of Complex Spaces bears relevance to octagonal cuspid newforms, which are related to the Rhiemann Zeta Function.

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## Introduction

In an earlier paper, the author discussed the concepts of the spinorial and Clifford Clocks. An additional aspect of the Clifford Algebras discussed therein concerns the spaces which the algebras inhabit. In Vedic Physics, space is not considered empty, but instead composed of invisible cubes, stacked side to side, top to bottom throughout the entire universe.

It so happens that the spaces which house the Clifford Algebras of the Clifford Clock form relationships which are isomorphic to the relationships of the Clifford Clock itself. That is to say that the Clifford Clock arrangement of algebras is reflected in the system of complex spaces in which dwell the Clifford Algebras.

In the Qi Men Dun Jia Model, each trigram of the Ba Gua or Eight Trigrams of Chinese metaphysics is housed in a palace, or sometimes a "mansion." For this series we reserve the use of the term mansion for the 28 astrological houses of the Nakshastras. Just as Clifford Algebras reside in complex spaces, with specific properties, the eight trigrams dwell in eight palaces of the Qi Men Dun Jia Cosmic Board.

Each of the palaces has Five Element qualities, and thus exerts an influence on the outcome of analyses in Qi Men Dun Jia. Under the proper Five Element relations, a palace may exert a push or a pull on the symbols housed therein.



The Qi Men Dun Jia Cosmic Board with the eight trigrams in the outer perimeter shell, to mark the palaces and the Earth Pan. The next level shows the eight gates, then Heavenly Stems, then spirits.

In his Week 211, John Baez introduced the Super Brauer algebras as one aspect of the Clifford Clock.

the complex numbers are very aggressive and infectious - tensor anything with a C in it and you get more C's. That's because they're a field in their own right - and that's why they don't live in the Brauer group of the real numbers. They do, however, live in the *super-Brauer* group of the real numbers, which is Z/8 - the Clifford clock itself! Wikipedia explains the Brauer - Wall Group in this way:

In mathematics, the **Brauer–Wall group** or **super Brauer group** or **graded Brauer group** for a field F is a group BW(F) classifying finite-dimensional graded central division algebras over the field. It was first defined by Terry Wall (1964) as a generalization of the Brauer group.

The Brauer group of a field *F* is the set of the similarity classes of finite dimensional central simple algebras over *F* under the operation of tensor product, where two algebras are called similar if the commutants of their simple modules are isomorphic. Every similarity class contains a unique division algebra, so the elements of the Brauer group can also be identified with isomorphism classes of finite dimensional central division algebras. The analogous construction for  $\mathbb{Z}/2\mathbb{Z}$ -graded algebras defines the Brauer–Wall group BW(*F*).<sup>[1]</sup>

- The Brauer group B(F) injects into BW(F) by mapping a CSA *A* to the graded algebra which is *A* in grade zero.
- <u>Wall (1964</u>, theorem 3) showed that there is an exact sequence  $0 \rightarrow B(F) \rightarrow BW(F) \rightarrow Q(F) \rightarrow 0$

where Q(F) is the group of graded quadratic extensions of *F*, defined as an extension of  $\mathbb{Z}/2$  by  $F^*/F^{*2}$  with multiplication  $(e,x)(f,y) = (e + f, (-1)^{ef}xy)$ . The map from W to BW is the **Clifford invariant** defined by mapping an algebra to the pair consisting of its grade and determinant.

• There is a map from the additive group of the <u>Witt–Grothendieck ring</u> to the Brauer–Wall group obtained by sending a quadratic space to its <u>Clifford algebra</u>. The map factors through the <u>Witt group</u>,<sup>[2]</sup> which has kernel  $I^3$ , where I is the fundamental ideal of W(*F*).<sup>[3]</sup>

#### Examples

• BW(C) is isomorphic to  $\mathbb{Z}/2\mathbb{Z}$ . This is an algebraic aspect of <u>Bott</u> <u>periodicity</u> of period 2 for the unitary group. The 2 super division algebras are C, C[ $\gamma$ ] where  $\gamma$  is an odd element of square 1 commuting with C.

• BW(**R**) is isomorphic to **Z**/8**Z**. This is an algebraic aspect of <u>Bott</u> <u>periodicity</u> of period 8 for the orthogonal group. The 8 super division algebras are **R**, **R**[ $\varepsilon$ ], **C**[ $\varepsilon$ ], **H**[ $\delta$ ], **H**, **H**[ $\varepsilon$ ], **C**[ $\delta$ ], **R**[ $\delta$ ] where  $\delta$  and  $\varepsilon$  are odd elements of square –1 and 1, such that conjugation by them on complex numbers is complex conjugation.

Of interest here especially are the two links to Bott Periodicity and the Z groups. By way of further definitions, we give Wikipedia for Witt:

In mathematics, a **Witt group** of a field, named after <u>Ernst Witt</u>, is an <u>abelian group</u> whose elements are represented by symmetric bilinear forms over the field.

#### Examples

- The Witt ring of C, and indeed any <u>algebraically closed field</u> or <u>quadratically closed field</u>, is  $\mathbb{Z}/2\mathbb{Z}$ .<sup>[18]</sup>
- The Witt ring of **R** is  $\mathbf{Z}$ .<sup>[18]</sup>
- The Witt ring of a finite field  $\mathbf{F}_q$  with q odd is  $\mathbf{Z}/4\mathbf{Z}$  if q is 3 mod 4 and isomorphic to the group ring  $\mathbf{Z}[F^*/F^{*2}]$  if  $q = 1 \mod 4$ .<sup>[19]</sup>

• The Witt ring of a <u>local field</u> with <u>maximal ideal</u> of <u>norm</u> congruent to 1 modulo 4 is isomorphic to the group ring  $(\mathbb{Z}/2\mathbb{Z})[V]$  where V is the <u>Klein 4-group</u>.<sup>[20]</sup>

- The Witt ring of a local field with maximal ideal of norm congruent to 3 modulo 4 it is  $(\mathbb{Z}/4\mathbb{Z})[C_2]$  where  $C_2$  is a cyclic group of order 2.<sup>[20]</sup>
- The Witt ring of  $\mathbf{Q}_2$  is of order 32 and is given by  $\mathbf{Z}_8[s, t]/\langle 2s, 2t, s^2, t^2, st 4 \rangle$ .



John Baez continues his explanation of the Complex Space Clock: But before I explain that, I want to show you what the categories of representations of the Clifford algebras look like:

I've already told you what a super-algebra is. We say it's a "super division algebra" if every nonzero element that's purely even or purely odd is invertible.

That's pretty easy. What are they like?

Well, I don't completely understand all the options yet, so I'll just list the "central" super division algebras over the real numbers, namely those where the elements that super-commute with everything, form a copy of the real numbers. There turn out to be 8, and their beautiful patterns are best displayed in a circular layout:

			R		
	R[e] mod e2 - 1	7	0	1	R[e] mod e2 + 1
C[e] mod ie	e2 - 1, ei + ie 6			2	C[e] mod e2 + 1, ei +
	H[e] mod e2 + 1	5	4	3	H[e] mod e2 - 1
			н		

What does this notation mean? Well, as usual, R, C, and H stand for the reals, complex numbers, and quaternions. In all but two cases, we start with one of those algebras and throw in an odd element "e" satisfying the relations listed: e is either a square root of +1 or of -1, and in the complex cases it anti-commutes with i.

So, for example, super division algebra number 1:

R[e] mod e2 + 1

is just the real numbers with an odd element thrown in that satisfies  $e^2 + 1 = 0$ . In other words, it's just the complex numbers made into a superalgebra in such a way that i is odd.

The real reason I've arranged these guys in a circle numbered from 0 to 7 is to remind you of the Clifford algebra clock in "week210", where I discussed the super Brauer group of the real numbers, and said it was Z/8.

Indeed, the central super division algebras form a complete set of representatives for this super Brauer group! In particular, the Clifford algebra Cn is super Morita equivalent to the nth algebra on this circle:

C0 = R	~ R
C1 = C	~ R[e] mod e2 + 1
C2 = H	~ C[e] mod e2 + 1, ei + ie
C3 = H + H	~ H[e] mod e2 - 1
C4 = H(2)	~ H
C5 = C(4)	~ H[e] mod e2 + 1
C6 = R(8)	~ C[e] mod e2 - 1, ei + ie
C7 = R(8) + R(8) - R(8)	~ R[e] mod e2 - 1

where ~ means "super Morita equivalent", and the notation for Clifford algebras was explained last week.

# **Complex Space Clock**

0

1	Real	Vector Spaces 7				
	Split Real Vector Spaces	Complex Vector Spaces				
6	Real Vector Spaces	Quaternionic Vector Spaces	2			
Complex Vector Spaces		Split Quaternionic Vector spaces				
3						
Quaternionic Vector Spaces						
		4				

Wikipedia explains what Morita equivalence means:

In <u>abstract algebra</u>, **Morita equivalence** is a relationship defined between <u>rings</u> that preserves many ring-theoretic properties. It is named after Japanese mathematician <u>Kiiti Morita</u> who defined equivalence and a similar notion of duality in 1958.

#### Definition

Two rings *R* and *S* (associative, with 1) are said to be **Morita equivalent** (or equivalent) if there is an equivalence of the category of (left) modules over *R*, *R-Mod*, and the category of (left) modules over *S*, *S-Mod*. It can be shown that the left module categories *R-Mod* and *S-Mod* are equivalent if and only if the right module categories *Mod-R* and *Mod-S* are equivalent. Further it can be shown that any functor from *R-Mod* to *S-Mod* that yields an

equivalence is automatically additive.

### Examples

Any two isomorphic rings are Morita equivalent.

The ring of *n*-by-*n* matrices with elements in *R*, denoted  $M_n(R)$ , is Morita-equivalent to *R* for any n > 0. Notice that this generalizes the classification of simple artinian rings given by <u>Artin–Wedderburn theory</u>. To see the equivalence, notice that if *M* is a left *R*-module then  $M^n$  is an  $M_n(R)$ -module where the module structure is given by matrix multiplication on the left of column vectors from *M*.

This allows the definition of a functor from the category of left *R*-modules to the category of left  $M_n(R)$ -modules. The inverse functor is defined by realizing that for any  $M_n(R)$ -module there is a left *R*-module *V* and a positive integer *n* such that the  $M_n(R)$ -module is obtained from *V* as described above.

### **Newforms and L Functions**

Somewhere between 2005 and 2009 an interest developed in math physics circles around something called newforms. Despite the interest and the years which have passed since, much of this emerging field remains undefined and unproven, primarily consisting of speculation on the part of some about things which few appear able to prove at present.

Nevertheless, the few bits of solid concepts which have emerged offer tantalizing prospects for the Qi Men Dun Jia Model - or vice - versa. "Octahedral cuspid newforms" seems to suggest eight - sided forms which have emerged recently on the edge of something.

This sounds very much like the Qi Men Dun Jia Model, which posits a substratum that consists of a range of invisible matter (black hole matter) that ranges from the natural logarithm e to a circle at the border with visible matter.

From the opposite side of the border with visible matter, emerging matter forms along the outline of the Clifford Clock and the Complex Spaces Clock described herein, before meeting with the alternative group A5 and 60 Stellated Icosahedra, as the aspects of Time, Frequency and the Five Elements are stamped into all forms of emerging visible matter.

#### In "AVERAGE NUMBER OF OCTAHEDRAL NEWFORMS OF PRIME LEVEL" by MANJUL BHARGAVA AND EKNATH GHATE, we find:

#### 1. INTRODUCTION

This paper is concerned with counting holomorphic cuspidal newforms of weight 1. To each such form f, Deligne and Serre associate an odd irreducible Galois representation  $\rho_f: G_{\mathbb{Q}} \to \operatorname{GL}_2(\mathbb{C})$ , whose image in  $\operatorname{PGL}_2(\mathbb{C})$  is, by a standard classification, either

- a dihedral group, or,
- $A_4, S_4$  or  $A_5$ .

In the last three cases the form f is said to be of tetrahedral, octahedral, or icosahedral type, respectively. Extensive numerical computations [Fre94] suggest that these forms occur extremely rarely, and they have been traditionally labelled as *exotic*. Forms of prime level have in particular long held a special place in the literature [Ser77], [Duk95]. It is a standard conjecture that

**Conjecture 1.1.** For any  $\epsilon > 0$ , the number of exotic newforms of prime level N is  $O(N^{\epsilon})$ , where the implied constant depends only on  $\epsilon$ .

#### Furthermore, Bhargava and Ghate write:

#### It is known that any exotic weight one form of prime level must be either octahedral or icosahedral.

This last statement supports the Qi Men Dun Jia Model in that the present author argues that matter emerges from the substratum (black hole) form either in octagonal or in icosahedral form and no other, and that A5 plays a key role in organizing emerging matter, along with the 60 Stellated Icosahedra.

In addition, this paper by Bhargava and Ghate concerns series of prime numbers. In the same way, the Qi Men Dun Jia is concerned with the series of Fibonacci Numbers, some of which have common denominators as the prime numbers, eg, those for the icosahedra.

Therefore, the work that has been done on the concept of newforms to date wholly supports the Qi Men Dun Jia Model. In a future paper, the author will discuss additional relations between newforms and the Qi Men Dun Jia model, including the Riemann Zeta Function. The Riemann zeta-function:

$$\zeta(s) = \frac{1}{k^s} \sum_{m=1}^k \zeta\left(s, \frac{m}{k}\right).$$

Riemann zeta function From Wikipedia,



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Riemann zeta function  $\zeta(s)$  in the complex plane. The color of a point s encodes the value of  $\zeta(s)$ : colors close to black denote values close to zero, while hue encodes the value's argument. The white spot at s = 1 is the pole of the zeta function; the black spots on the negative real axis and on the critical line Re(s) = 1/2 are its zeros. Values with arguments close to zero including positive reals on the real half-line are presented in red.

### The Riemann zeta function or Euler-Riemann zeta function,

 $\zeta(s)$ , is a function of a complex variable *s* that analytically continues the  $\sum_{n=1}^{\infty} \frac{1}{n^s}$ , which converges when the real part of *s* is greater than 1. More general representations of  $\zeta(s)$  for all *s* are given below. The Riemann zeta function plays a pivotal role in analytic number theory and has applications in physics, probability theory, and applied statistics.

# Conclusion

In this paper we have reproduced the Complex Space Clock as described by John Baez, as a counterpart to the Clifford Clock or the Spinorial Clock. The Complex Space Clock provides the dwellings in which the Clifford Algebras live, much as archetypes live in the collective consciousness of a group of humans.

In the same way, the Eight Trigrams of Chinese metaphysics inhabit the eight of nine palaces in the 3 x 3 Magic Square of the Qi Men Dun Jia Cosmic Board, which further possess Five Element qualities which interact with the gates and other symbols within the Qi Men Dun Jia palaces.

As the Super Brauer Group bears direct relation to supersymmetry, so too does the Qi Men Dun Jia model, given the isomorphic relations between the two concepts.

Moreover, the emergent field of newforms, Hecke Algebras, L Functions, etc. demonstrate clear parallels to the Qi Men Dun Jia Model, especially in the concept of the octagonal cuspidal newform and the direct mathematical relationships to the alternative group A5 and the icosahedrons. We further note the importance of the PGL 2 (C) relationship noted above by Bhargava and Ghate.

The 60 Stellated Icosahedrons form an extremely important component of the Qi Men Dun Jia Model as they lend temporal, frequency and tonal aspects, as well as qualities of the Five Elements, to all forms of visible matter.

Many thanks to John Baez for his help with this clock via emails.