Special relativity as a possible consequence of quantized spacetime

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Abstract. – The behavior of the electrons in crystals exhibits some properties well known from solid state physics, like an upper limit for the particle velocity and an effective mass, which goes to the infinity as the particle velocity goes to that limit. This reminds us the relativistic mass. Indeed, both effective and relativistic masses have the same dependence on the velocity. The result is applicable to a general lattice – quantized space could be one –, which suggests that special relativity can be a consequence of a quantized spacetime. Even that is not the case, a similar approach could be used in the search for the Quantum Gravity.

1. Motivation

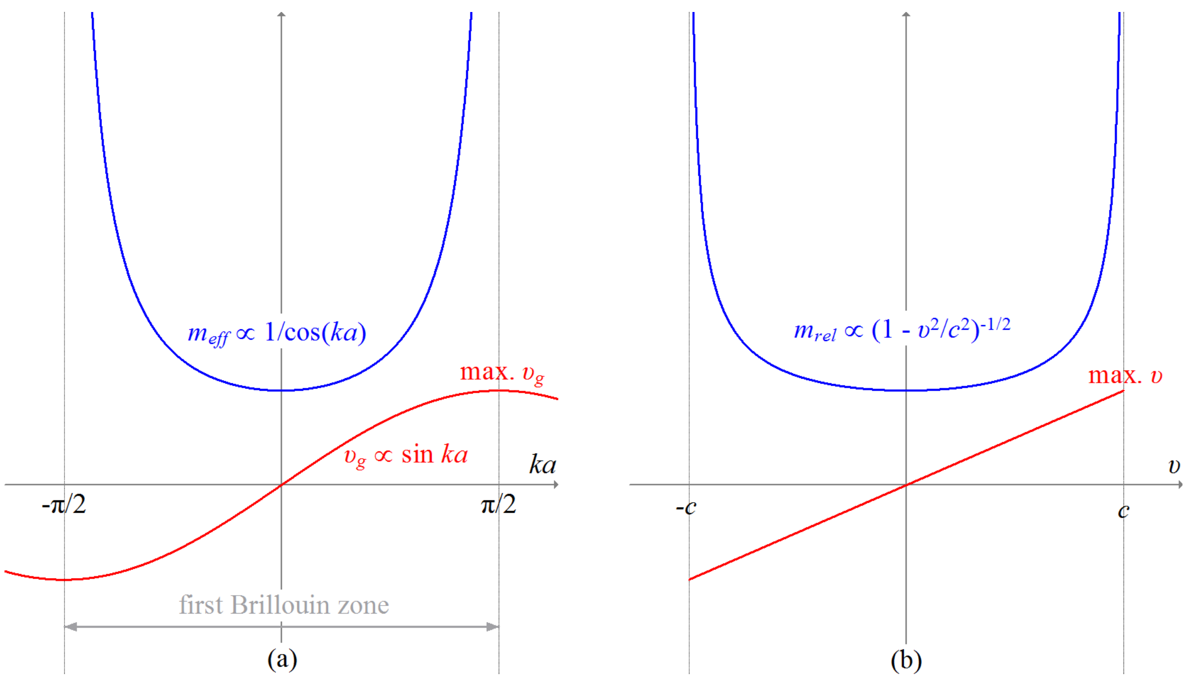

Fig. 1. (a) One-dimensional approximation for the effective mass m_{eff} and the group velocity υ_g of an electron in a semiconductor in terms of ka (electron wavenumber and lattice constant); (b) Plot of the relativistic mass m_{rel} of a body in terms of its velocity υ relative to the observer.

Fig. 1 shows two similar plots from rather unrelated areas. (a) comes from solid state physics and (b) comes from special relativity. Although they seem to be different curves, they share some peculiar behaviors:

- there is a maximum velocity;
- as the particle velocity approaches the maximum, its mass grows to infinity.

In fact, it can be shown that they are the same curve.

2. The one-dimensional tight binding approximation

It's well known from solid state physics that when an electron is subject to periodic potential there will be regions of forbidden energies and bands of allowed energies, after Kronig and Penney [1]. Unlike for free electrons, its energy ε is not proportional to the square of its wavenumber k [2]. In the one-dimensional case, we can express that relation as:

$$\varepsilon(k) = \varepsilon_0 - A \cos ka \; . \tag{1}$$

where a is the spatial periodicity of the potentials, ε_0 and A are constants and $A > 0$.

The relevant values of k is inside the first Brillouin zone, that is, $-\pi/a \leq k \leq +\pi/a$, since other values of k can be reduced to the that interval by a displacement of $n\frac{2\pi}{a}$ with no changes to the solution (n is integer).

The velocity $\upsilon_g(k)$ of the particle is equal to the group velocity of the waves which represent the particle, that is, $\upsilon_g(k) = d\omega/dk$. By using the relations $\varepsilon = \hbar\omega$ and (1), we have:

$$\upsilon_g = \frac{1}{\hbar}\frac{d\varepsilon}{dk} = \frac{Aa}{\hbar}\sin ka = c_g \sin ka \tag{2}$$

where the constant $c_g \equiv Aa/\hbar$ represents the maximum allowed velocity in that band.

To accelerate a particle subject to a periodic potential, it behaves as having an *effective mass*, given by (see, for instance, [3]):

$$m_{eff} = \frac{\hbar^2}{d^2\varepsilon/dk^2} \tag{3}$$

By applying (1) to (3) we obtain:

$$m_{eff} = \frac{\hbar^2}{Aa^2}\frac{1}{\cos ka} = \frac{m_0}{\cos ka}, \tag{4}$$

with $m_0 \equiv \hbar^2/Aa^2$.

The equations (2) and (4) are plotted on Fig. 1a.

We can rewrite the effective mass (4) in terms of the group velocity of the particle. From (2) we have:

$$\sin ka = \frac{\upsilon_g}{c_g} \qquad (5)$$

and therefore

$$m_{eff} = \frac{m_0}{\cos ka} = \frac{m_0}{\sqrt{1-\sin^2 ka}} = \frac{m_0}{\sqrt{1-\upsilon_g^2/c_g^2}}\ , \qquad (6)$$

which is the same dependence on velocity as the relativistic mass.

3. Consequences

That relationship is valid for a particle in any medium containing periodic potential barriers, apparted from each other by a distance *a*. We can exchange the electron in an atom by a particle in a well, and the mathematics will remain the same. The particle doesn't even need be electrically charged. Although the derivation for effective mass is generally presented through the application of an external electrical force, it doesn’t need to be an electrical one.

In particular, spacetime itself can be such a lattice, if it is quantized in a manner that the potential barriers represent the discontinuity between two adjacent quanta of spacetime, and hence each spacetime quantum can be regarded as a potential well.

This link between solid state physics and special relativity can hint us of a new direction for Quantum Gravity research: to derive general relativity from quantized spacetime or other appropriate concept.

4. References

[1] Kronig R. de L. and Penney W. G., *Proc. Roy. Soc* (*London*), **A130** (1930) 499.
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[3] Bube R. H., *Electrons in Solids: An Introductory Survey*, 2nd. ed. (Academic Press, New York) 1988, pp. 121-122.