A Modification of the Standard Cosmological Metric

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In this article we firstly present an explicit dynamical equation satisfying the general principle of relativity under the framework of classical mechanics. In light of this fact, the necessity of Einstein's equivalence principle for the gravity being geometrized should be reexamined. Especially, Einstein's (strong) equivalence principle claims that the inertial force is equivalent to the gravitational force. But in fact the new dynamical equation proves that the essence of the inertial force is the real force exerted on the reference object, which can actually be all kinds of forces such as the gravitational force, electromagnetic force and so on. Therefore, in this context we only retain the numerical equality between the inertial mass and gravitational mass and abandon Einstein's (strong) equivalence principle. Consequently, the candidate for the standard clock should be corrected into the mathematical clock which duplicates the real clock equipped by the observer himself. Then a new physical picture for how to convert the gravitational force into a geometric description on spacetime is presented. On the other hand, we point out that all cosmological observations are made by the observer at present on the earth, instead of any other observers including the comoving observers in the earlier unverse. On this basis, we introduce an extra factor $b(t)$ in FRW cosmological metric to depict the gravitational time dilation effect since the local proper clock may run in a faster and faster rate with the expanding of the universe. In this way, we may obtain a positive value of $\rho + 3p$ and avoid the introduction of dark energy in the current universe.
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1 INTRODUCTION

As an alternative theory for Newton's gravity law, Einstein's geometric theory of gravity was used in studying the universe shortly after the theory was established in 1915, thus initiated the beginning of modern cosmology. In Einstein's geometric theory of gravity, the curvature of the spacetime is determined by energy and momentum distributions[1–4]. The predicted gravitational redshift effect is proved by a series of gravity tests in the solar system[5–7]. This effect has adequately illustrated that geometric description is more accurate to describe gravity than Newton's gravity law. In recent decades, cosmology has gradually developed into a science [8, 9] including observations of supernova, cosmic microwave background, etc. However, as soon as current standard cosmology was compared with cosmological observation data, it was confronted with some rigorous challenges such as dark energy[10, 11], dark matter[11]. All of these challenges have shaken the existing knowledge of physics. Especially dark energy, which may drastically change the direction of scientific research. Further considering that the dynamics of the universe can not be repeated in laboratory, therefore it is necessary for us to make repeated inspection on the foundation of the standard cosmology.

2 MOTIVATIONS AND REASONS

2.1 an explicit realization for general principle of relativity

General principle of relativity can actually be realized in Classical Mechanics. In the classical framework, a particle dynamical law satisfying the general principal of relativity can be constructed as[12],

$$\mathbf{f}|_{p-O} = m_p \mathbf{a}|_{p-O} = \mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O. \quad (1)$$

Here $\mathbf{F}|_p$ denotes the total forces from the whole universe exerted on the particle p, and $\mathbf{F}|_O$ denotes the total forces from the whole universe exerted on the reference object O. Both of them can be calculated respectively according to existing knowledge of all types of interactions. $\mathbf{a}|_{p-O}$ denotes the relative acceleration between particle p and reference object O. m_p and m_O respectively denote the mass of the particle p and the reference object O. The reference object O in above equation can correspond to any real object in the universe, meanwhile any physical reference frame must be fixed by at least one real object. Therefore, the equation (1) can be directly applied in all reference frames(namely the general principle of relativity), and naturally interpret the essence of inertial force. Any real object or some part of an object can be identified as a reference object. Moreover, according to the equation (1), the inertial force is essentially the real force exerted on the reference object that must be deducted in the relative counting of forces.

The general principle of relativity is essentially a practical requirement. On one hand, we are never able to know about the actual state of motion of our terrestrial reference frame where our observers exist. On the other hand, we are always able to determine the rotation of any reference frame relative to the spatial background of the whole universe by resorting to the galaxies far enough away, since the spatial background of the whole universe is objective and motionless. Therefore, what the practical observation really requires is that dynamical laws must keep form invariant to all irrotational reference frames. Furthermore, in particle dynamics, the rotation phenomena can always be attributed to the relative motion between different particles. But for single particle, there is no concept of rotation. Just as the problem of variable mass system under the framework of Newtonian mechanics, there the variable mass phenomena should be attributed to the relative motion between different particles in the system of particles. Thus the fundamental law for particle dynamics is still $\mathbf{F}|_p = m_p\mathbf{a}|_{p-O}$, while $\mathbf{F}|_p = \frac{d(\mathbf{p}|_{p-O})}{dt}$ can be generalized from the former equation. Therefore, the problem of reference frames' rotation is actually a mathematical problem. Ultimately, the problem of the rotation of reference frames can be separated from the problem of dynamical relativity.

Above analysis illustrates that the general principle of relativity for particle dynamics can be realized in a very simple and natural approach which is different with Einstein's view[13]. Because the concept of inertial force still exists in Einstein's theory of relativity, and Einstein's (strong) equivalence principle further claims that the inertial force is equivalent to the gravitational force. However, we can see from the natural and symmetric dynamical equation (1), the essence of the inertial force is the real force exerted on the reference object, which can actually be all kinds of interactions such as the gravitational interaction, electromagnetic interaction and so on. Therefore, the necessity of Einstein's equivalence principle for the physical picture of gravity being geometrized should be scrutinized. Our discussion is just based on giving up Einstein's (strong) equivalence principle and only accepting the equality between the gravitational mass and inertial mass. Consequently, the running rate for a clock free falling under gravity is no longer assumed to be constant. In principle, the gravitational field is responsible to change the rate of local clocks since gravitational time dilation effect has been verified in solar gravity tests[5–7]. In light of this new physics, here the standard clock is no longer the usual free falling clocks in Einstein's gravity theory. What is the qualified candidate for the standard clock defined to make the comparison on the running rate of local clocks? Our new approach to realize the general principle of relativity demonstrates an one to one correspondence between the counting of forces and the selection of reference frame. Once the counting of forces exerted on the particle under study being given, we have the only one option for the reference frame. In other words, it is not able for us to make a further transformation on observers. Besides, according to the gravitational time dilation effect, the running rate of local clocks depends on the strength of gravitational field at their positions. On the one hand, the realization of general principle of relativity has no longer need of Einstein's (strong)equivalence principle, so theoretically it is no longer necessary to assume that the running rate of any free falling clock under changing gravity is the same constant. On the other hand, the clock which is relatively rest in the gravitational field and the clock which is free falling under changing gravity, they differ only in a non-gravitational force and the resulting acceleration. If there is really no gravitational time dilation effect exists for the free falling clock under changing gravity, it must imply that a non-gravitational force and the resulting acceleration can also bring time dilation effects for clocks. However, by now there is no such a sign has been observed and verified in the all past experiments. Therefore, in this article we suppose that any clock in the gravitational field, regardless of its state whether it is free falling or not, its running rate will change with the difference in the strength of gravitational fields. For above reasons, we propose that the clock equipped by the observer himself is the only qualified candidate for the standard clock which is used as the reference to make the comparison on the running rate of local clocks.

For the advantage in understanding, it is necessary to establish a rigid and homogeneous coordinate system for the reference frame on the spatial region under study, the spacetime scale inside it is determined by observer's own clock and ruler. However the gravitational effect on the spacetime geometry has been verified in many gravity tests, hence such an observer's coordinate system for the reference frame can only be mathematical. Therefore, the spacetime interval measured by this set of coordinate system is mathematically equivalent to be measured by the observer's own clock and ruler.

2.2 the existence of an absolute cosmic spatial background

The new approach to realize the general principle of relativity implies the existence of an absolute cosmic spatial background. Any particle dynamics law must obey the causality principle. For classical mechanics, we may regard the force as the cause, and regard the acceleration as the effect. To the total force exerted on the particle p from the whole universe, how should we express the corresponding effect? The total force of a single particle should be objective, and not change with the transformation of observers. Therefore, the corresponding effect should also be objective, and not relevant with any reference frame. In fact the objective effect can only correspond to the acceleration in the spatial background of the whole universe(called as cosmic spatial background),

$$\mathbf{F}|_p = m_p \frac{d^2}{dt^2}(X,Y,Z)|_p. \tag{2}$$

Particularly, the objective position of the particle p in the cosmic spatial background here is denoted by $(X,Y,Z)|_p$ and in fact it can not be numerically measured. As the name implies, the spatial background is what left in a space region after all objects inside it were moved away. And the cosmic spatial background is just what left in the whole universe after all concrete objects in the universe were moved away. If we believe that the motion state for any object is objective, not only the state of relative motion as we have known is objective and definitely determinable, but the state of any object's motion itself should also be objective. That is to say there exists absolute state of motion for every object. Therefore, we believe that there exists an absolute and eternally unchangeable background for cosmic space. Any motion of any object in the universe must be confined in this common absolute three-dimensional space background. Here *absolute* means that the cosmic spatial background has no dynamical evolution and is irrelative to the existence or the movement of any object. The reason is that both Newton's view of absolute spacetime and Einstein's spacetime theory of relativity are mainly concerned with the change of spacetime scales. But the spacetime scales are radically defined by the intervals of proper events occurred in objects, and so belong to the property of concrete objects. However, the cosmic spatial background itself do not contain any concrete object. Therefore, the existence of absolute cosmic spatial background will not be conflicted with modern physics, and we must distinguish the spatial background of the universe from the spatial intervals in Einstein's theory of relativity. Definitely, the cosmic spatial background can be included into Newton's view of absolute spacetime. But in contrast, the former do not require that the spacetime intervals are absolute, which is the key point only for the latter.

In summary, the cosmic spatial background really exists for our universe although it is not concrete like ordinary matter. For example, we have known that the dimensions of cosmic spatial background should be of three. But there is still a problem that the objective position in cosmic spatial background can not be directly measured. What we can really measure is the difference between any two objective positions, which substantially constructs a mathematical vector,

$$\mathbf{x}|_{p-O} = (X,Y,Z)|_p - (X,Y,Z)|_O. \tag{3}$$

All objects in the universe, including objects under study (such as p) and their reference objects (such as O), should be of equal status in the most fundamental law of dynamics. For example, the dynamics of any real reference object should also satisfy $\mathbf{F}|_O = m_O \frac{d^2}{dt^2}(X,Y,Z)|_O$. The introduction of reference frames is to make relative measurements on kinematical quantities. As a causal correspondence, the forces should be relatively counted in nature.

$$m_O\mathbf{F}|_p - m_p\mathbf{F}|_O = m_p m_O \frac{d^2}{dt^2}((X,Y,Z)|_p - (X,Y,Z)|_O) = m_p m_O \frac{d^2\mathbf{x}|_{p-O}}{dt^2}. \tag{4}$$

Obviously, it turns out to be the new dynamical equation (1). From above discussion we can see that the new dynamical equation (1) can be logically and exactly derived, only if we postulate the existence of an absolute cosmic spatial background. The key point which must be distinguished here is that the existence is proposed for absolute background, instead of absolute space-time.

2.3 Einstein's (strong) equivalence principle is discarded

In light of above explicit realization of general principle of relativity, we only retain the assumption that the gravitational mass equals the inertial mass. In fact, this most weak equivalence principle is already enough to account for why the gravity can be depicted by a geometric theory. For example, we can illustrate this idea by using the trajectory of satellites around the earth. To simplify the question, we assume the trajectory is a circle. According to Newtonian mechanics, we have

$$\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}. \tag{5}$$

So it is obvious to see that the unique parameter for the trajectory (r) does not depend on the mass of the satellite. That is to say, the dynamical law for satellites in gravity field has no direct correlation with the magnitude of gravitational forces exerted on these satellites. Therefore, the dynamical law of satellites can be equivalently expressed by a kinematical law. This is a concrete example to account for that the physical effect of gravity can be geometrized on the spacetime.

On the other hand, according to gravitational time dilation effect which is verified by the experiments on the solar system, the running rate of clocks is related to the strength of local gravitational field. But on the logic, the validity of Einstein's field equation does not depend on Einstein's equivalence principle[14]. Therefore, in this article we still adopt Einstein's gravitational field equation to be a mathematical correct equation in converting gravity into geometric description of spacetime.

According to the experiments on the solar gravity, the interval of local proper events occurred in gravitational field will show the difference compared with that occurred on the reference object. Therefore, a consistent approach to convert the gravity into geometric description of spacetime should be carried out as follows. First, a suitable reference frame such as sun-centered reference frame must be selected, which will bring convenience in relatively counting of forces. Second, a rigid and homogeneous coordinate system should be established. The reference object is naturally defined as the origin of coordinate system. At the same time, the space and time scale in this coordinate system is defined by the clock and the ruler equipped by the observer fixed on the reference origin point and duplicated into the whole coordinate system. Finally, according to the comparison between the local proper clock and ruler located at every position in gravitational field and that located on the reference origin point, the curved geometry of spacetime is determined based on the background of this coordinate system. If from mathematics we retain Einstein's field equation to be the correct equation converting gravity into geometric description of spacetime, the geometry of the curved spacetime can be solved quantitatively. In fact, if we recall the solar gravity test and will find that the above approach for converting gravity into geometric description of spacetime can self-consistently explain the gravitational redshift effect. Besides, there is a different point which distinguish our present physics picture from that of Einstein's traditional theory of gravity. In present physical picture for gravity being geometrized, the strength of gravitational field should be counted relatively, rather than the total contribution from all the matter in the universe. More specifically, in the derivation of Schwarzschild metric, the counting of the gravitational force is actually restricted to that exerted by objects inside the solar system. Meanwhile, the reference origin is fixed at the center of the solar system. Therefore, what the solar gravity test has essentially satisfied is our approach to convert the gravity into geometric description of spacetime.

2.4 who is the real observer for cosmology

To the whole history of the evolution of our universe, the cosmological observer is the observer at the present time on the earth, rather than the comoving observer evolving with the universe. The new approach on the realization of general principle of relativity shows that the counting of forces must be in an one-to-one correspondence with the selection of the reference frame. Therefore, an arbitrary choice of the reference frame would not affect the validity of dynamical laws, but every term in dynamical equations is closely related to concrete observers. To make clear the dynamical state for observers is the key point for correctly application of dynamical laws. Firstly, all observations from cosmology and all determinations on the redshift value are processed by the observer at the present time, instead of a comoving observer which evolves with the expanding of the universe. Therefore, in cosmology we must emphasize an important feature for observations that is "the present time". The observer is at the present time, and the determination of the value of redshift is implemented at the present time. In other words, cosmological observer is not the observer at any other time of the universe, including not the comoving observer on the earth.

In considering that the strength of gravitational field is changing continuously, the clock equipped by the observer on the earth should run faster and faster with the expanding of the universe. Therefore, the construction of a rigid and homogeneous reference coordinate system, based on which the gravity is geometrized, should be in strict accordance with the clock and ruler equipped by the observer at the present time on the earth.

3 NEW COSMOLOGICAL METRIC AND DYNAMICAL EQUATIONS[15]

According to previous discussion, there are two points of physical considerations should be fully incorporated in the construction of cosmological metric. Firstly, it has been demonstrated by an explicit form that the general principle of relativity can be realized in a very concise picture, so Einstein's (strong) equivalence principle is no longer indispensable to be introduced for the geometric theory of gravity. We only retain the numerical equality between the inertial mass and gravitational mass since it has a solid foundation from experiments. But the assumption that all free falling clocks under gravity run in a uniform rate is discarded. Therefore, the candidate for the standard clock which is used to compare the rate of clocks should also be changed. According to the gravitational time dilation effect and the new realization approach for the general principle of relativity, the running rate for the clock equipped by the observer himself must be set as the reference value in cosmological metric. In mathematics, the clock equipped by the observer himself can be duplicated by imagination on to all spacetime points. In theory, the geometrical effect resulted by the gravitational field is depicted by the difference between above mathematically defined standard clock and the local clocks. Secondly, we know the matter density in the universe has changed a lot from the beginning of the unverse, so the strength of gravitational field has also changed appreciably. There exists an evolution of the running rate for every local clock fixed on the comoving galaxies of the universe. Therefore, being relative to the long evolution history of the universe which is studied in cosmology, the construction of cosmological metric must exactly distinguish the present observer's clock on the earth from the comoving observer's clock on the earth.

3.1 the coordinate system of reference frame for cosmological metric

First, as for spatial coordinates of cosmological metric, there is a Hubble's principle which predicts a predominate spatial coordinate system, which can be named as spatial comoving coordinate system. In principle the observer can duplicate his ruler at every spatial point and build up a rigid and homogeneous spatial coordinator system. We give the ruler of the observer at present on the earth the name of "the cosmological observation ruler". The spatial interval in the comoving coordinate system is just measured by this cosmological observation ruler,

$$dl^2 = a^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2]. \tag{6}$$

Although the comoving coordinates are selected as spatial coordinates in cosmological metric, but the cosmological observer is still fixed to be the observer at the present time on the earth. Therefore, the distance between cosmological comoving coordinate points should be measured by the present observer's ruler on the earth. In principle we can only obtain the local spacetime property by solving cosmological dynamics equations around the comoving point where the earth exists. But by resorting to the cosmological principle, the local spacetime property can be easily extended into the global spacetime property for the whole universe.

Second, as for the coordinate time, we must also define it from the viewpoint of observation. We all know that the study of the universe is mainly based on the observation of light signals emitted from the earlier universe. The redshift of light signals is a particularly important quantity to investigate the evolution of the universe. The redshift is determined by comparing the received light signal with the same type of light signals on the earth at the present time. Therefore, the coordinate time should be defined resorting to the clock of the observer at present on the earth. We take the present observer's clock rate as the standard one, duplicate this clock rate on every time-point and build up a rigid and homogeneous coordinate time system. We give this time system the name of "the cosmological observation clock". We use t to denote the reading number of the cosmological observation clock. On the other hand, we know the matter density in the universe has changed a lot from the beginning of the unverse, so the strength of gravitational field has also changed appreciably. Hence a local (proper) clock in the earlier universe, regardless of its state whether it is free falling or not, will run in a different rate relative to the clock under null gravity. To be distinguished from the cosmological observation clock, the reading number of the local (proper) clock which is fixed on the comoving galaxy in the earlier universe is denoted by τ. The time dilation effect is expressed by

$$d\tau = b(t)dt. \tag{7}$$

It must be noticed that if we set $b(t_0) = 1$, which also means that the coordinate time t is equivalently measured by the proper clock of the present observer on the earth. In other words, the interval of coordinate time is actually a kind of proper time interval but all of them are measured by the clock at the present time. For the observer at the present time, dt can always be measured as his proper time interval.

According to the above definition of the time and spatial coordinates, a general cosmological metric can be written as[15],

$$ds^2 = -b^2(t)dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2]. \tag{8}$$

In fact, it is also the most general form of cosmological metric satisfying the cosmological principle. We propose the metric (8) to replace the well known Friedman-Robertson-Walk (FRW) metric in processing the observation data. The reason is what we reiterated in this article that the cosmological observations are made by the observer at the present time on the earth, instead of any other observers including the local observer in the earlier unverse.

3.2 the dynamical equation of cosmology

According to the new cosmological metric (8), The non-zero components of Ricci tensor can be easily derived:

$$R_{00} = -3\frac{\ddot{a}}{a} + 3\frac{\dot{a}\dot{b}}{ab} \tag{9}$$

$$R_{ij} = \frac{1}{b^2}(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab})g_{ij} + \frac{2k}{a^2}g_{ij} \tag{10}$$

A consistent energy-momentum tensor may take the following form due to the cosmological principle[13],

$$T_{\mu\nu} = \rho U_\mu U_\nu + p(\eta_{\mu\nu} + U_\mu U_\nu). \tag{11}$$

Substituting the equations (9-11) into Einstein's gravitational field equation, the fundamental equations of cosmology are derived[15]:

$$\frac{\dot{a}^2}{a^2b^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{12}$$

$$\frac{1}{b^2}(\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab}) = -\frac{4\pi G}{3}(\rho + 3p). \tag{13}$$

where k is the curvature of space. In essence, k is not a dynamical variable but a certain value decided by the primary conditions. The cosmic energy density $\rho(t)$ and pressure $p(t)$ can be treated in principle as the functions of two geometrical variables $a(t)$ and $b(t)$. As a complementarity, we need to put forward a specific method to solve the above system of equations in further research. However, since the expansion of the universe soon becomes slow enough, in a tentative investigation we might suppose $b(t) = \sqrt{1 - \frac{k'}{a(t)}}$ in analogous to the counterpart $b(r) = \sqrt{1 - \frac{2GM}{rc^2}}$ in Schwarzschild metric. Here k' is also an undetermined constant.

3.3 physical predicts under the new cosmological metric

Without explicitly solving equations (12-13), we are still able to understand the kinematic effects of the expansion (see details in [9]) from the metric (8). We have[15]

$$1 + z' = \frac{a(t_0)}{a(t_1)}. \tag{14}$$

This is just the same result in the standard cosmology. However, above redshift effect should be only attributed to the kinematical effect from the expansion which belongs to the Doppler effect, rather than gravitational redshift effect. An observational redshift effect in cosmology does actually further incorporate the gravitational time dilation effect which should be counted into when the light signal being emitted and received. According to the gravitational time dilation effect in the solar gravity test, we have

$$1 + z'' = \frac{b(t_0)}{b(t_1)}. \tag{15}$$

Then we obtain the final value for redshift[15]:

$$1 + z = (1 + z')(1 + z'') = \frac{b(t_0)}{b(t_1)}\frac{a(t_0)}{a(t_1)}. \tag{16}$$

Since $b(t_0)/b(t_1)$ is greater than 1 and will increase with the decrease of the time t_1 when the universe is expanding, it may change the value of apparent coordinate acceleration d^2a/dt^2 according to the current observation data[9, 10].

Furthermore, we discuss the mathematical formula for the acceleration of the universe. As we know, acceleration is not invariant under coordinate transformation, and it may change explicitly in different coordinate reference frames. When the cosmological metric (8) is adopted, the definition of a phenomenological acceleration is revised to be $\frac{d^2a}{dt^2}$, instead of $\frac{d^2a}{d\tau^2}$, because t is the only proper time coordinate for the observer at the present time on the earth. $\frac{d^2a}{dt^2}$ can be called as the coordinate acceleration and $\frac{d^2a}{d\tau^2}$ can be called as the proper acceleration. The relationship between these two definitions of acceleration is given by[15]

$$\frac{d^2a}{d\tau^2} = \frac{1}{b^2(t)}\frac{d^2a(t)}{dt^2} - \frac{1}{b^3(t)}\frac{da(t)}{dt}\frac{db(t)}{dt}. \tag{17}$$

In considering that all comparisons of the frequency of light signals are implemented at the present time, then all the redshift are intrinsically measured by the clock of the present observer on the earth. That is to say, all redshifts are practically evaluated by a coordinate time t, which has been defined to run at the same rate with the clock of the present observer on the earth. Hence the value of the phenomenological acceleration of expanding, which is resulted from current observational data, is directly related to $\frac{d^2a}{dt^2}$. It can be shown that the sign of this formula may be in different with that of $\frac{d^2a}{d\tau^2}$. To illustrate this point, we may investigate the evolution property of $b(t)$ in analogy to the gravitational time dilation effect in Schwarzschild metric. There the time dilation factor $\sqrt{1-\frac{2GM}{rc^2}}$ will increase with the distance r. So we may expect that the factor $b(t)$ will increase with the decrease of the gravitational field strength. With the expanding of the universe, the gravitational field strength decreases, then $b(t)$ will increase with time. It indicates $\frac{db(t)}{dt} > 0$. On the other hand, we have $\frac{da(t)}{dt} > 0$ for an expanding universe. Hence it is possible to have a negative $\frac{d^2a}{d\tau^2}$ according to the equation (17) even $\frac{d^2a}{dt^2} > 0$ holds. It further means a possibility that $\rho + 3p > 0$ according to the equation (13). Therefore, we must keep in mind that there are two different concepts of the acceleration when we talk about the accelerated expanding of the universe. The acceleration indicated directly from the practical observation data is $\frac{d^2a}{dt^2}$, rather than $\frac{d^2a}{d\tau^2}$.

4 CONCLUSION

In this article we present a feasible and natural approach to realize the general principle of relativity. In light of this new approach, we propose a consistent physical picture in converting gravity into geometric description of spacetime. Firstly, according to the observer, a spatial region from which gravitational forces are counted should be firstly determined. And the clock and the ruler equipped by the observer should be duplicated all over the selected spatial region. Thus a rigid and homogeneous coordinate system is established. Secondly, only based on the background of this mathematical coordinate system, the relatively counted gravity can be correctly calculated. Besides, since we discard the assumption that all free falling clocks under different gravity still run at the same rate, the standard clock should be changed to the mathematical clock which duplicates the observer's clock all over the spatial region.

Finally, we investigate the foundation of cosmology and introduce a new cosmological metric in which the coordinate time is defined referring to the proper clock equipped by the observer at present on the earth. The present observer on the earth is the only qualified reference observer to determine the redshift value of light signals that were emitted from the earlier universe. Furthermore, we know that the matter density in the universe has changed a lot from the beginning of the universe, so the strength of gravitational field has also changed appreciably. Therefore the proper clock at the present time on the earth must run at a different rate comparing with that in the earlier universe because of the existence of gravitational time dilation effect. Consequently, there are two different concepts of acceleration are introduced to describe the evolution of the universe. One is coordinate acceleration d^2a/dt^2, the other is proper acceleration $d^2a/d\tau^2$. It is significant for us to distinguish these two accelerations. Because even a positive coordinate acceleration is directly evaluated from current observation data, it is still possible to have a negative proper acceleration according to gravitational time dilation effect. Then $\rho + 3p$ may turn out to be a positive value according to the equation (13). In this way, the dark energy is possible to be ultimately removed from the present universe.

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APPENDIX A

THE ESSENCE OF INERTIAL FORCE AND THE REALIZATION OF GENERAL PRINCIPLE OF RELATIVITY

A1 Introduction

In the framework of Newtonian classical mechanics, the fundamental dynamical equation is Newton's second law. But Newton's second law is only valid in so-called inertial reference frames. If we want to invoke the form of Newton's second law in non-inertial reference frames, there is a fictitious force—inertial force must be introduced additionally. And the magnitude of the inertial force is essentially defined by the relative acceleration between the non-inertial reference frame and a certain inertial reference frame. Therefore, the classical particle dynamics is totally established on the basis of inertial reference frame. However, as we well know, we are never able to find a true inertial reference frame in the practical application. This situation is surely not satisfying.

In fact, every reference frame must be established based on a concrete and real reference object. Otherwise, there would be no reference value in measuring any object's motion in the natural world. As for the relationship between the reference object and the reference frame, in practical cases if we assign the reference object first, the reference frame can be naturally established by identifying the reference object as its origin point. Otherwise, the reference frame is assigned first, then in principle any real object which is fixed in the reference frame can be identified as its reference object.

A2 Physical Motivations and New Particle Dynamical Equation

A2.1 what problems exist in Newton's second law

In theory, Newton's second law is valid only for inertial reference frames. Even we introduce the inertial force to make a compulsory application of Newton's second law in non-inertial reference frames, the calculation of inertial forces still depends on finding out an inertial reference frame firstly. In practice, we are not able to find out even one real object which can exactly correspond to the inertial reference frame. Besides, the essence of inertial forces is not clear. There are widespread controversies about the essence of inertial forces. However, the natural philosophy basis for classical mechanics is definite and clear. So these problems illustrate that Newton's second law still has somewhere for improvement.

A2.2 why these problems exist in Newton's second law

The reason is that Newton's second law is not causal symmetric in its form. The Newton's second law can be generally expressed as

$$\mathbf{F}|_p = m_p \mathbf{a}|_{p-O}. \tag{18}$$

The left hand side of this equation is given by the total force exerted on the particle p, and which only depends on p. But the right hand side of this equation not only depends on the particle p but also depends on the reference object O which corresponds to the origin point of the reference frame. If we regard the force as the cause, and regard the acceleration as the effect (namely result), the causality of Newton's second law is not symmetric and consistent in form. This is the just point to result that Newton's second law is theoretically valid only in inertial reference frames, but none of them can be found in practice.

On the other hand, what is the exact information that all experiments of classical mechanics tell us? In fact, the experiments of classical mechanics didn't directly guarantee the validness of Newton's second law since every term of force appeared in Newton's second law must be defined to count force totally. But any empirical dynamics law from ground based experiments can only depict the quantitative relation between the new additionally exerted force and the resulting relative acceleration. Therefore, the classical mechanics experiments did not actually tell us that Newton's second law is correct at all situations. Inversely, we find that Newton's second law can not be exactly applied in any situation. Now we know the reason is that there exists a causal inconsistency problem in Newton's second law.

A2.3 how to solve the problem of causal inconsistency

We suppose that the particle dynamics is certainly to be a theory with causal principle. We regard the force as the cause, and regard the acceleration as the effect(result). Then, to the total force exerted on the particle p from the whole universe, how should we express the corresponding effect? The total force exerted on a single particle should be objective, and not change with the transformation of observers. Therefore, the corresponding effect should be objective, and be not relevant with any reference frame. Therefore, a completely objective acceleration can only be expressed as the acceleration in the spatial background of the whole universe(called cosmic spatial background),

$$\mathbf{F}|_p = m_p \frac{d^2}{dt^2}(X, Y, Z)|_p. \tag{19}$$

Particularly, here the objective position of the particle p in the cosmic spatial background is denoted by $(X, Y, Z)|_p$. In fact, the cosmic spatial background can be understand intuitively. As the name implies, the spatial background is just what still exist in a space region after all objects inside it were moved away. And the cosmic spatial background is just what still exist in the whole universe after all concrete objects in the universe were moved away. If we believe that the motion state for any object is objective, not only the state of relative motion as we have known is objective and definitely determinable, but the state of any object's motion itself should also be objective. That is to say there exists absolute state of motion for every object. Therefore, we postulate that there exists an absolute and eternally unchangeable spatial background for the whole universe. The motion of all objects in the universe must be located in this common absolute three-dimensional space background, because only on this basis then the existence of objective dynamical laws is possible. Strictly speaking, the existence of an absolute comic spatial background is an assumption at the natural philosophy level. The key point which must be distinguished here is that the existence is proposed for absolute background, instead of absolute space-time. Here *absolute* means that the cosmic spatial background has no dynamical evolution and is irrelative to the existence or motion of any object. The reason is that both Newton's view of absolute spacetime and Einstein's spacetime theory of relativity are mainly concerned with the rules on the change of spacetime scales. But the spacetime scales are radically defined by the intervals of proper events occurred in objects, and so belong to the property of concrete objects. However, the cosmic spatial background do not contain any concrete object. Therefore, the existence of absolute cosmic spatial background will not be conflicted with the core ideas of modern physics, and we must distinguish the spatial background of the universe from the spatial intervals in Einstein's theory of relativity. Definitely, the cosmic spatial background can be included into Newton's view of absolute spacetime. But in contrast, the former do not require that the spacetime intervals are absolute, which is the key point only for the latter.

In summary, the cosmic spatial background really exists for our universe although it is not concrete like general objects. For example, we have known that the dimensions of cosmic spatial background should be of three. But there is still a problem that the objective position in cosmic spatial background can not be directly measured. What we can really measure is the difference between any two objective positions, which substantially constructs a mathematical vector,

$$\mathbf{x}|_{p-O} = (X, Y, Z)|_p - (X, Y, Z)|_O. \tag{20}$$

A2.4 how to construct a particle dynamical equation which is really applicable for any observer

All objects in the universe, including objects under study (p) and reference objects (O), should be of equal status in the most fundamental law of dynamics. For example, the dynamics of any real reference object should also satisfy $\mathbf{F}|_O = m_O \frac{d^2}{dt^2}(X, Y, Z)|_O$. The introduction of reference frames is to make relative measurements on kinematical quantities. As a causal correspondence, the forces should also be relatively counted in nature.

$$m_O \mathbf{F}|_p - m_p \mathbf{F}|_O = m_p m_O \frac{d^2}{dt^2}((X, Y, Z)|_p - (X, Y, Z)|_O) = m_p m_O \frac{d^2 \mathbf{x}|_{p-O}}{dt^2}. \tag{21}$$

Here the reference object O naturally corresponds to the origin point of a reference frame. Finally, we obtain

$$\frac{\mathbf{F}|_p}{m_p} - \frac{\mathbf{F}|_O}{m_O} = \mathbf{a}|_{p-O}. \tag{22}$$

This is just the new dynamical equation under the framework of classical mechanics. Obviously, above equation (22) can be naturally obtained without resorting to Newton's second law and the concept of inertial reference frame. Here the definition of the force and the acceleration is just the same as that in Newtonian mechanics. $\mathbf{F}|_p$ and $\mathbf{F}|_O$ denote the total forces from the whole universe exerted on the particle p and the reference object O respectively. m_p and m_O denote the mass of the particle p and the reference object O respectively. Since the reference object is definitely fixed in the reference frame, therefore, the acceleration ($\mathbf{a}|_{p-O}$) of the particle p relative to the reference object, equals to that relative to the origin point of the reference frame. This equation is also discussed in detail in our special paper[12]. The equation (22) is proposed to replace Newton's second law under the framework of classical mechanics since the new dynamical law (22) keeps form invariant in any irrotational reference frame, namely the general principle of relativity.

The general principle of relativity is essentially a practical requirement. On one hand, we are never able to know about the actual state of motion of our terrestrial reference frame where our observers exist. On the other hand, we are always able to determine the rotation of any reference frame relative to cosmic spatial background by resorting to the galaxies far enough away since the cosmic spatial background is objective and motionless. Therefore, what the practical observation really requires is that dynamical laws must keep form invariant to all irrotational reference frames. Furthermore, in particle dynamics, the rotation phenomena can always be attributed to the relative motion between different particles. But for single particle, there is no concept of rotation. In other words, once any reference object is regarded as a particle, no problem of rotation exists for reference particle. Just as the problem of variable mass system under the framework of Newtonian mechanics, the variable mass phenomena should be attributed to the relative motion between different particles in the system of particles, so the fundamental particle dynamics is still $\mathbf{F}|_p = m_p\mathbf{a}|_{p-O}$. But for $\mathbf{F}|_p = \frac{d(\mathbf{p}|_{p-O})}{dt}$, it actually can be generalized from the former equation. Therefore, the problem of reference frames' rotation is actually a mathematical problem. Ultimately, the problem of the rotation of reference frames can be separated from the problem of dynamical relativity.

A3 The Physical Logic of the New Dynamical Equation

In fact, the new dynamical equation (22) is also a logical result from Newton's second law. In all concepts of reference frame, besides the property of space-time scales, there is only the reference object is physical. Therefore under the framework of classical mechanics, since the space-time scales have been assumed to be constant for all reference frames, any object's dynamical problem in any reference frame can be substantially regarded as a two-body problem. Therefore, even in the framework of Newtonian mechanics, the new dynamical equation (22) can also be logically derived. Firstly, we assume there really exists an inertial reference frame which is denoted by Ω. According to Newton's second law, an arbitrary object p satisfies the particle dynamics as

$$\mathbf{F}|_p = m_p\mathbf{a}|_{p-\Omega}. \tag{23}$$

Similarly, to an arbitrary reference object O, which must also obey the same natural law,

$$\mathbf{F}|_O = m_O\mathbf{a}|_{O-\Omega}. \tag{24}$$

Performing a simple algebraic manipulation, we obtain

$$\mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-\Omega} - \frac{m_p}{m_O}(m_O\mathbf{a}|_{O-\Omega}) = m_p[\mathbf{a}|_{p-\Omega} - \mathbf{a}|_{O-\Omega}] = m_p\mathbf{a}|_{p-O}. \tag{25}$$

Therefore, the new dynamical equation (22) is logically derived. In this sense, the new dynamical equation must be right under the framework of classical mechanics if we assume the existence of an inertial reference frame. But according to above discussions, the new dynamical equation has definite advantages comparing with Newton's second law. Especially, we remove the necessity of the existence of inertial reference frames and inertial forces from the formulism of the theory. Therefore, if furthermore we want to logically remove the special status of inertial reference frames, in principle the equation (22) should not be derived on the basis of Newton's second law.

The new dynamical equation (22) embodies a formal symmetry on the causal relationship. The right hand side of this equation is a relative acceleration, meanwhile the left hand side of this equation is a relative counting of forces. We suppose that the every natural principle which has been described as a pure relative law, must exist an absolute basis. In this spirit, the most simple natural philosophy basis which is logically required to construct the new dynamical equation (22) is nothing but the existence of an absolute cosmic spatial background.

A4 The Advantages and Physical Effects of the New Dynamical Equation

A4.1 the realization of general principle of relativity and the essence of inertial force

The equation (22) can be directly applied in all reference frames, namely the general principle of relativity, and naturally interprets the essence of inertial force. $\mathbf{F}|_p$ and $\mathbf{F}|_O$ in the equation (22) can be calculated respectively according to existing knowledge of all types of interactions. In principle any real object or some part of object can be identified as a reference particle. Therefore, the equation (22) can be applied in all reference frames. Moreover, according to the equation (22), the inertial force is essentially the real force exerted on the reference object, which must be deducted in a relative counting of forces. To illustrate this point, we assume that there are two relatively accelerated reference frames. Their reference objects are denoted by O and O'. For above two reference frames, it is the difference in their forces being exerted that results in a relative acceleration between these two reference frames. In Newtonian mechanics, such a relative acceleration is depicted as a so-called inertial force. But now the relative acceleration between reference frames is naturally interpreted by the difference in their forces being exerted,

$$\frac{m_p}{m_{O'}}\mathbf{F}|_{O'} - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-O} - m_p\mathbf{a}|_{p-O'} = m_p\mathbf{a}|_{O'-O} = \mathbf{f}|_{inertial}. \tag{26}$$

The equation (26) self-consistently goes back to the relative dynamical equation given by new dynamical law (22) when it is applied on reference objects O and O'. Above analysis illustrates that the new dynamical law (22), compared with Newton's second law, possesses more complete and symmetric theoretical structure. Therefore, it is in possession of real universality.

A4.2 The correctness of the new dynamical equation

It can be proved that all successful applications of classical mechanics can also be naturally accounted for by the new dynamical equation (22). In general, we assume that there are two arbitrary particles denoted by 1 and 2(such as the Moon and Venus), exist in the solar system. The total forces from the whole universe exerted on the particle i can be written down as (Here we assume that non-gravitational forces are ignorable)

$$\mathbf{F}|_i = (\mathbf{f}_i)_{NonGrav} + (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav} \approx (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav}. \tag{27}$$

Here $(\mathbf{f}_i)_{SolarGrav}$ means the gravitational forces exerted on the particle i from the matter in solar system, and $(\mathbf{f}_i)_{OutSolarGrav}$ means the gravitational forces exerted on the particle i from the matter outside the solar system. According to the equations (22) and (27), the relative dynamical equation between 1 and 2 can be expanded as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \simeq [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] + [(\mathbf{f_1})_{OutSolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{OutSolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{28}$$

Owing to both 1 and 2 being bound in the same solar system which is just one of innumerable gravitational bound systems in our universe, the gravitational forces exerted from the matter outside the solar system satisfy,

$$\frac{(\mathbf{f_1})_{OutSolarGrav}}{(\mathbf{f_2})_{OutSolarGrav}} \approx \frac{m_1}{m_2}. \tag{29}$$

So the relative dynamics (28) between particles 1 and 2 can be approximately rewritten as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{30}$$

Specifically, if we select the center of sun as the particle 2 in above equation, at that time the reference object can be the whole sun, or can be any real object at the center of sun. For example, if we choose the whole sun as the reference object, the second term in above equation would be drove down since $m_1 \ll m_2$. Otherwise, we may choose a real object at the center of sun as the reference object 2, which has a comparative mass as the object 1. Because for the reference object 2, the distribution of the matter in the solar system has a high approximated spherical symmetry, the forces exerted on the reference object 2 from the solar system may usually be approximated to be zero, namely $(\mathbf{f_2})_{SolarGrav} \approx 0$. Therefore, if we don't desire a high precision, the dynamics of particle 1 can always be expressed as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx (\mathbf{f_1})_{SolarGrav} = m_1\mathbf{a}|_{1-2}. \tag{31}$$

That is the true formula which we always conform with, when the classical mechanics is practically applied on objects in the solar system. Obviously, Newton's second law is substantially an approximation of new dynamical law (22) when it is applied in gravitational bound systems(It undergoes about two steps of approximations). The approximation such as (31) is valid for usual sun-centered reference frame and earth-centered reference frame. Even for the most common ground-based reference frame, it substantially comes down to the earth-centered reference frame. Therefore, all successful applications of classical mechanics can be naturally accounted for again by the new dynamical law (22). Besides, why the gravitational forces exerted from the outside of the solar system do not have to be taken into account? The direct reason is not the magnitude of these forces being so small as to be ignored, but in fact the forces are deducted in the relative counting of forces.

Now we consider the case that the equation (22) is applied in ground-based reference frame. We assume that A is the particle under study which is moving on the ground and B denotes the reference object which is at rest relative to the earth's surface. According to the equation (22), the relative dynamics between A and B can be directly expressed as

$$\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B = m_A\mathbf{a}|_{A-B}. \tag{32}$$

In theory, the total forces exerted on the A and B can be expanded as

$$\begin{aligned}\mathbf{F}|_A &= (\mathbf{f}_A)_{OutEarthGrav}(\propto m_A) + (\mathbf{f}_A)_{EarthGrav}(\propto m_A) + (\mathbf{f}_A)_{NonGrav},\\ \mathbf{F}|_B &= (\mathbf{f}_B)_{OutEarthGrav}(\propto m_B) + (\mathbf{f}_B)_{EarthGrav}(\propto m_B) + (\mathbf{f}_B)_{NonGrav}.\end{aligned} \tag{33}$$

The non-gravitational forces exerted on B can be equivalently calculated from the case that B is at rest relative to a smooth and horizontal surface. Therefore, the non-gravitational force must numerically equal to the support force from the ground,

$$(\mathbf{f}_B)_{NonGrav} = -m_B\mathbf{g}. \tag{34}$$

Substituting it into the equation (32), at the same time we adopt an approximation that the gravitational forces from the earth and outside of the earth can be counterbalanced, we finally obtain,

$$\begin{aligned}\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B &= (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}((\mathbf{f}_B)_{NonGrav}) = (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(-m_B\mathbf{g})\\ &= (\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g} = m_A\mathbf{a}|_{A-B}.\end{aligned} \tag{35}$$

This is the actual application expression of Newton's second law in ground-based reference frames. There is one point should also be emphasized. The change from $(\mathbf{f}_A)_{EarthGrav}$ in (33) to $m_A\mathbf{g}$ in (35), is usually interpreted by a fictitious inertial force owing to the rotation of the earth. But now we can see that it actually should be attributed to the real force from the ground exerted on the reference object B. Broadly speaking, the surface of the earth is rigid in spatial distance relative to the center of the earth, so the measurement of kinematical effect in ground-based reference frames is equivalent to that relative to the center of earth. Accordingly, for the part of counted force, the gravitational forces exerted from the outside of the earth system should not be considered.

Furthermore, if we assume the reference object B is an arbitrary object moving over the ground(including the case of acceleration), at that time $(\mathbf{f}_B)_{NonGrav} \neq -m_B\mathbf{g}$, so the expression for an available particle dynamics (35) should be changed as,

$$(\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(\mathbf{f}_B)_{NonGrav} = m_A\mathbf{a}|_{A-B} = [(\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g}] - \frac{m_A}{m_B}[(\mathbf{f}_B)_{NonGrav} + m_B\mathbf{g}]. \tag{36}$$

Therefore, in all senses, the practical Newton's second law can be regarded as an approximation of the new dynamical equation (22) under some special conditions.

A4.3 the universality of the new dynamical equation

The equation (22) is directly applicable in the relative dynamics between galaxy clusters. For instance, we assume that there are only two particles (such as 1 and 2) exist in the whole universe, and there is only gravitational interaction exists between them. In the framework of Newtonian mechanics, none of real particles can be approximated as a inertial reference frame, so Newton's second law can not be directly applied in this situation. Even someone resorts to the center of mass for this system, but essentially the center of mass method is finally attributed to an assumption that the center of mass is rest relative to a certain inertial reference frame. But according to (22), the relative dynamics between above two particles can be directly written down as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \; = \; (\frac{Gm_1m_2}{r^3}\mathbf{r}_{1\to 2}) - \frac{m_1}{m_2}(\frac{Gm_1m_2}{r^3}\mathbf{r}_{2\to 1}) = m_1\mathbf{a}|_{1-2} = m_1\frac{d^2\mathbf{r}_{2\to 1}}{dt^2}. \tag{37}$$

A5 The physical significance of the new dynamical equation

First, the new dynamical equation may improve the precision on the physical application. In principle, we should always make an approximation on the inertial reference frame before Newton's second law can be applied. Actually, such an approximation is made in theory, rather than in practical measurement. Therefore, if the forces exerted on the reference origin can not be ignored, the error would be significant. By contrast, if we adopt the equation (22) as the new particle dynamical law, it at least has entirely solved the problem of the approximation on inertial reference frames.

Second, the new dynamical equation (22) illustrates that the general principle of relativity for particle dynamics can be realized in a very simple and natural approach which is obviously different with Einstein's view[13]. Because the concept of inertial force still exists in Einstein's theory of relativity, and Einstein's (strong) equivalence principle further claims that the inertial force is equivalent to the gravitational force. However, we can see from the equation (22), which obviously is more natural and simple in physical picture, the essence of the inertial force is the real force exerted on the reference object. Therefore, the so-called inertial force can actually be all kind of interactions such as the gravitational interaction, electromagnetic interaction and so on.

Third, the new approach to realize the general principle of relativity must bring about modifications[15] on physical pictures under non-classical frameworks. The existence of an absolute cosmic spatial background is actually not conflicted with the principle of invariant light speed and the idea of gravity being geometrized. The reason is that both Einstein's special relativity and geometric theory of gravity are mainly concerned with the laws on the change of space-time scales which all belong to the property of concrete objects, instead of the change on the cosmic spatial background[1, 4, 13]. Besides, the idea that forces should be relatively counted in any reference frame embodies the principle of causal consistency, so it is a very fundamental requirement and should also be retained.

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