Transformation of the fine structure constant α equation into an α-quantized mass-generating Einstein equation

Malcolm H. Mac Gregor*
130 Handley Street
Santa Cruz, CA 95060
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e-mail: mhmacgreg@aol.com

Abstract

The equation for the fine structure constant $\alpha \sim 1/137$ is expanded to include (1) radii (as α_r), and (2) Compton constituent masses (as α_m). It then has the $E=mc^2$ form of a dynamical Einstein mass generator, wherein a reservoir of electromagnetic energy $E = e^2/r$ undergoes an α_m-defined phase transition in which it is adiabatically expanded radially by a factor of 137 and transformed into a Compton-sized mechanical (non-electromagnetic) unit mass quantum mc^2. The classical electron (Thomson) radius $r_e = e^2/m_e c^2$ anchors the mass spectrum. Four phase-transition channels (electron, boson, fermion, gauge boson) accurately create lepton, constituent-quark, ground-state hadron, and average-gauge-boson masses.

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*Formerly at Lawrence Livermore National Laboratory

A central problem in elementary particle physics today is to understand the structure of the elementary particle mass spectrum. This involves determining nature of the mechanism that generates particle masses. In this paper we address these problems by extending the scope of the fine structure constant α. When we do this, we discover that it transforms into an expanded Einstein-like equation, $E(r) = mc^2$. This transformation is a two-step process. The first step is the introduction of a radius r, and it defines the energy term $E(r)$ for any vale of r. The second step is the identification of r as a Compton radius, $r_C = \hbar / mc$, and it defines the mass/energy term mc^2 for that value of r_C. The quantities $E(r_C)$ and mc^2 are linked together by a factor of 137 adiabatic radial expansion—an α-boost phase transition—that accompanies the transformation of energy into mass. This extended Einstein equation applies initially to the creation of an electron-positron pair, and then, using the $e^- e^+$ pair as a ground state, to the creation of higher-mass states, including a second α-boost to the gauge bosons and top quark t.

The fine structure constant α is defined by the equation $\alpha = e^2/\hbar c = 1/137.036$. It plays a dominant role in atomic spectra, and also in quantum electrodynamics. The mysterious dimensionless number 137 has fascinated physicists ever since its identification by Sommerfeld in 1916. It ties together a well-known trio of α-spaced particle radii:

$$r_{\text{Thomson}} \times 137 = r_{\text{Compton}}, \quad r_{\text{Compton}} \times 137 = r_{\text{Bohr}} . \tag{1}$$

The Thomson scattering length $r_{\text{e}} = e^2/m_e c^2$ (also known as the *classical electron radius* or *Lorentz radius*) appears in the cross section for *low-energy* x-ray elastic scattering off electrons, and also in the relativistic Klein-Nishina equation. The Compton radius $r_C = \hbar/m_e c$ emerges from the equations for *high-energy* photon inelastic collisions with electrons. The Bohr radius $a_o = \hbar^2/m_e e^2$ is the ground state radius of the Bohr hydrogen atom. Theoretically, Eq. (1) represents the following sequence:

$$(e^2/ m_e c^2) \times (\hbar c / e^2) = (\hbar / m_e c) ; \quad (\hbar / m_e c) \times (\hbar c / e^2) = (\hbar^2 / m_e e^2) , \tag{2}$$

which is exact in powers of $\alpha = e^2/\hbar c$, and which involves both radii and masses as well as α. Empirically, we will see that this α-chain anchors the entire elementary particle mass spectrum.

Historically, the classical electron radius r_e was originally studied with the aim of attributing the mass/energy of the electron, $m_e c^2$, to the self-energy E_e of an extended electric charge e. The calculated self-energy is

$$E_e = Ae^2 / r_e , \tag{3}$$

where A = 1/2 for a surface charge and 3/5 for a uniform volume charge. If Poincaré forces are added in, these values for A become 2/3 and 4/5, respectively. For convenience, a "classical electron radius" was defined by setting A = 1, which gives

$$r_e = e^2 \big/ m_e c^2 = 2.82 \times 10^{-13}\,\text{cm}. \tag{4}$$

This procedure (setting A = 1) was regarded at the time as just a handy approximation, with no experimental significance. However, when the Thomson scattering cross section $\sigma = (8\pi/3)\, r_e^{\,2}$ was later calculated and found to be in accurate agreement with experiment, it became apparent that the radius r_e is in fact an important experimental quantity, which is reinforced here by the fact that it appears as r_{Thomson} in Eqs. (1) and (2). When we expand the scope of the fine structure constant, we discover that r_e determines the mass of the electron, and thereby provides the mass normalization for the particle mass spectrum.

The fine structure constant α has the basic form

$$\alpha \equiv e^2 \big/ \hbar c \cong 1/137. \tag{5}$$

It is the dimensionless ratio of three fundamental constants, and is valid in any coordinate system. But the occurrence of α as a scaling factor in the experimental sequence displayed in Eqs. (1) and (2) shows that α also involves lengths (radii) and a particle mass (the electron). Thus it should be possible to reformulate α so as to include lengths and masses, as we now demonstrate.

In order to expand the scope of its physical content, we restructure α in two steps. First, we insert a radius r, which gives the extended fine structure equation α_r:

$$\alpha_r \equiv \frac{(e^2/r)}{(\hbar c/r)} \cong 1/137. \qquad . \tag{6}$$

Second, we equate r to the electron Compton radius $r_C = \hbar \big/ m_e c$, which gives the extended fine structure equation α_m:

$$\alpha_m \equiv \frac{(e^2/r_C)}{(\hbar c/r_C)} \cong 1/137. \tag{7}$$

This enables us to introduce the mass $m_e = \hbar / c\, r_C$ into the equation.

Eq. (6) for α_r can be recast and evaluated as follows:

$$\begin{aligned} \alpha_r \to E_r(r_{\text{fm}}) &\equiv (e^2 / r_{\text{fm}}) = (\hbar c / r_{\text{fm}}) / (137) = \\ &(197.33\,\text{MeV-fm}) / 137\, r_{\text{fm}} = 1.4400\ \text{MeV} / r_{\text{fm}}\,, \end{aligned} \tag{8}$$

where one fermi (fm) = 10^{-13} cm, and where $\hbar c = 197.33\ \text{MeV fm}.$ This equation *defines* the energy content $E_r(r_{\text{fm}})$ of an "energy reservoir" of *electromagnetic* potential energy $e^2 \big/ r_{\text{fm}}$ that is confined within a sphere of radius r_{fm}. It applies for any value of r_{fm}. In particular, if

r_{fm} is the Thomson radius $r_{\text{Thomson}} = 2.82$ fm, the calculated energy is $E_r(2.82\text{ fm}) = 0.511$ MeV, which is the electron energy. Thus the α_r-extended fine structure equation

$$E_r(r_{\text{fm}}) = 1.44 \text{ MeV} / r_{\text{fm}} \tag{9}$$

verifies the correctness of setting $A = 1$ in Eqs. (3) and (4).

In order to enter a mass term into α_m, we rewrite Eq. (7) in the form

$$\alpha_m \to (e^2 / r_{\text{Compton}}) = (\hbar c / r_{\text{Compton}}) / 137. \tag{10}$$

Then we insert the Compton radius $\hbar / mc$ into the right-hand side of Eq. (10), which gives

$$(e^2 / r_{\text{Compton}}) = m_e c^2 / 137. \tag{11}$$

We thus obtain the equation

$$E_r(r) = \frac{e^2}{\left(r_{\text{Compton}} / 137\right)} = m_e c^2, \tag{12}$$

where $r = r_{\text{Compton}} / 137$. Since the electron Compton radius r_{Compton} is a factor of 137 larger than the Thomson radius r_{Thomson} (Eqs. 1-2), we can rewrite Eq. (12) as the *adiabatic electron mass-generation equation*

$$E_r(r) = \frac{e^2}{r_{\text{Thomson}}} = m_e c^2 = 0.511 \text{ MeV}. \tag{13}$$

This equation quantitatively defines the direct conversion of stored electromagnetic energy E_r into "mechanical" (non-electromagnetic) electron mass/energy $m_e c^2$. In quantum electrodynamics (QED), the fine structure constant α serves as the coupling constant for the interactions between photons and electrons, including Feynman diagrams that convert photons into electron-positron pairs. In Thomson scattering, which is the low-energy scattering of x-rays off bound electrons, the Thomson radius gives the magnitude of the Thomson scattering cross section, which is a measure of the (coupling) strength of the interactions between x-rays and electrons. Thus it is not surprising to see the Thomson radius appearing in Eq. (13).

The α-generation process in Eq. (13) can be described as a sequence in which a self-interacting distributed electric charge e is initially contained in a sphere of radius r_{Thomson}, with an electromagnetic energy of 0.511 MeV (Eq.8), and then expands adiabatically by a radial factor of 137 (volume factor of $137^3 = 2.57 \times 10^6$) and transforms into a Compton-sized electron mass. This large adiabatic expansion can be regarded as a "phase transition" in which the *electromagnetic* field energy e^2/r is converted into the *mechanical* mass/energy

$m_e c^2$ of an electron that has the Compton radius r_C. We can denote this α-expanded adiabatic phase transition symbolically as the electron α_m equation

$$E_r(r) = \frac{e^2}{r_{\text{Thomson}}} \overset{137}{\Rightarrow} m_e c^2 = 0.511 \text{ MeV}. \tag{14}$$

This α_m equation is for just the *particle* channel of the electron generation process. In order to conserve quantum numbers, electrons and positrons must be produced from electromagnetic energy in matching (e^-, e^+) pairs. This simultaneous phase transition is described by the *electron-positron* α_m equation

$$E_r(r) = 2e^2 \big/ \left(r_{\text{Thomson}}\right) \overset{137}{\Rightarrow} (m_e c^2 + \overline{m}_e c^2) = 1.022 \text{ MeV}. \tag{15}$$

The α_m equations (14) and (15) each have the form of an α-*quantized Einstein-type equation* that bridges two domains which are separated by a phase change and a radial scaling factor of 137:

$$E_r\left(r\right) \overset{137}{\Leftrightarrow} mc^2(137r), \tag{16}$$

where $E_r(r) \equiv e^2 \big/ \left(r\right)$ is the available (conserved) energy, and r and $137r$ are the length scales of the two domains. The mass m is the *inertial mass* of the particle (the total mass), which is denoted in particle physics as the *constituent mass*. Thus we are dealing here with a *constituent-mass formalism*, which applies to particles and their quark substates, and also to leptons, so that leptons and hadrons can combine together in the same *α-generation* process. The available potential energy $E_r(r) = e^2/r$ displayed here is electromagnetic. However, in accelerator particle production, the available energy is primarily the beam kinetic energy. But when beam particles collide and violently decelerate, their kinetic energy is logically converted in a bremsstrahlung-like manner back into electromagnetic energy.

The fact that the electron occurs as a Compton-sized spherical mass m_e in the extended α_m equations (11-14) has another physical consequence. It implies that the electron mechanical mass is uniquely in the form of a relativistically spinning sphere (RSS), whose properties have been well-documented [1,2]. An RSS is a spinning solid sphere (of rest-mass m_o) whose equator is moving at (or infinitesimally below) the limiting velocity $\text{v} = c$. Its calculated spinning mass is $m_s = {}^3\!/\!_2\, m_o$, and its moment-of-inertia is $I = \frac{1}{2} m_s r^2$. These results hold for any radius r_i and mass m_i of the RSS. If we now require the spinning mass m_s to be equal to the observed *electron* mass m_e, and the circumference $2\pi r$ of the observed spherical envelope to be equal to one de Broglie wavelength $\lambda = h/m_e c$, then the radius of the RSS is the Compton radius $r = r_{C_i}$, and the calculated spin is $J = \frac{1}{2}\hbar$. If a massless point charge e is placed on the equator, it acts as a current loop and gives rise to a calculated magnetic moment

$\mu = e\hbar/2m_e c$. These results seem to be given uniquely by the RSS. They apply not only to the electron, but also to spin 1/2 constituent quarks, as demonstrated in the calculation of hyperon magnetic moments [3].

We can in principle apply these results to fermions of any constituent mass by means of a generalized *electromagnetic-energy* to *constituent-mass* α-generation equation:

$$E_r\left(r_{C_i}/137\right) = e^2\Big/\left(r_{C_i}/137\right) \overset{137}{\Rightarrow} m_i c^2,$$
$$\text{where } r_{C_i} = \hbar/m_i c = 197 \text{ MeV-fm}/m_i c^2. \tag{17}$$

However, experimental evidence for this energy-to-mass phase transition exists at only a few particular energies and radii. The most basic energy is that of the electron-positron pair (Eq. 15), which is anchored on the Thomson radius r_{Thomson}. This channel produces just electrons and positrons, which have no nearby excited states. But the $e^- e^+$ electron pair serves in turn as the entrance channel for three higher-mass α-generated phase transition excitation channels, which each have expanded production capabilities. These channels have several *signature features* in common:

(1) the "α-boost" factor of 137 in energy is from an experimentally-well-defined particle-antiparticle-symmetric ground state energy;

(2) the particle-antiparticle mass pairs that are generated in this α-boost serve as the basic "building block" templates for higher-mass particle states in this channel, which occur as accurate multiples of the building-block masses;

(3) the energy region between the ground-state energy and the building-block energy is a void in which no particles of this channel type appear;

(4) the required energy to produce these particle-antiparticle masses comes from an electromagnetic potential energy reservoir, and the types of particles that can occur in a channel depend mainly on their spin states (fermion or boson), and are essentially independent of their family lineage, so that we can have mixed particle types (lepton, quark, hadron) all appearing interleaved in the same production-channel energy stream;

(5) particles that contain both quark and antiquark substates have hadronic binding energies (HBE) of 2-3% at particle energies of 1 GeV and below, with the HBE values decreasing at higher energies and vanishing above 6 GeV (as expected from *asymptotic freedom*).

The building blocks in the three α-quantized particle production channels are:

$$\textit{boson}\ (J=0):\ m_e/\alpha = m_b = 70.025 \text{ MeV}. \tag{18}$$

$$\textit{fermion}\ (J=1/2):\ 3m_e/2\alpha = m_f = 105.038 \text{ MeV}. \tag{19}$$

$$\textit{gauge boson}\ (J=1/2):\ m_{u,d}/\alpha = m_{gb} = 43.17 \text{ GeV}. \tag{20}$$

We now summarize the range and accuracy of the particle states that are generated in these three production channels.

The *boson production channel* of Eq. (18) contains the (π, η, η', K) spin $J = 0$ pseudoscalar mesons. The $(\pi^{\pm}, \pi^0)$ pi meson doublet is the lowest-mass hadron state, with an average energy of 137.27 MeV, which is a factor of ~137 larger than the 1.022 MeV energy of the (e^-, e^+) electron-positron ground state. The electron mass m_e itself is generated from the electromagnetic potential $E_r(r_{\text{Thomson}})$ (Eq. 13). To obtain a factor-of-137 increase in potential energy $E_r(r)$, we decrease r_{Thomson} by a factor of 137, which defines the "boson radius" r_{boson}. This radius extends the α-spaced trio of radii in Eq. (1), as follows:

$$r_{\text{boson}} \times 137 = r_{\text{Thomson}}, \quad r_{\text{Thomson}} \times 137 = r_{\text{Compton}}, \quad r_{\text{Compton}} \times 137 = r_{\text{Bohr}}. \tag{21}$$

The α_{m} generation equation

$$E_r(r) = e^2 / \left(r_{\text{boson}}\right) = m_b c^2 = 70.0 \text{ MeV} \tag{22}$$

defines the 70 MeV boson building block m_b. It cannot be produced separately so as to conserve the required particle quantum numbers. Instead, it is formed as an $m_b + \overline{m}_b$ particle-antiparticle pair, the 140 MeV pion, which is created by the α_m generation equation

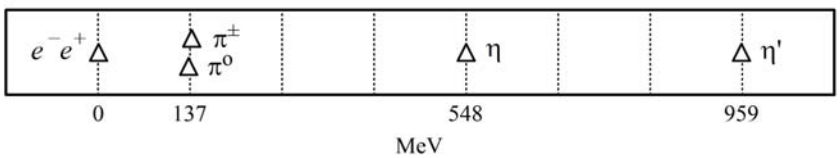
$$2E_r(r) = 2e^2 / r_{boson} = m_b + \overline{m}_b = 140.0 \text{ MeV}. \tag{23}$$

We denote the pion symbolically as $\pi = m_b \overline{m}_b$, where the mass quanta $m_b + \overline{m}_b$ are bound together hadronically. The 137.27 MeV average energy of the $\pi^{\pm} = 139.57 \text{ MeV}$ and $\pi^0 = 134.98 \text{ MeV}$ mesons is 2% smaller than value of 140 MeV shown in Eq. (23), where this difference is attributed to hadronic binding energy HBE (see *signature feature* (5) above). By expanding the trio of a-spaced radii in Eq. (1) to also include r_{boson} (Eq. 21), we have extended the α_m α-generation process, with its E-to-mc^2 phase transition, to accurately encompass the average pion mass, so that the experimental radius r_{Thomson} provides absolute mass values for both the electron and pion masses.

The mass and lifetime regularities of the (π, η, η', K) pseudoscalar mesons are clear-cut. The boson mass m_b = 70 MeV is the *pion constituent-quark*: $m_b = m_{q_\pi}$, $q_\pi \equiv (u_\pi, d_\pi)$. The u_π and d_π pion quarks carry the same u and d quark fractional charge states as in the Standard Model, but with constituent-quark (inertial) masses. The *non-strange* (π, η, η') mesons occur in an accurate (1::4::7) mass ratio, as do the non-strange meson constituent quarks

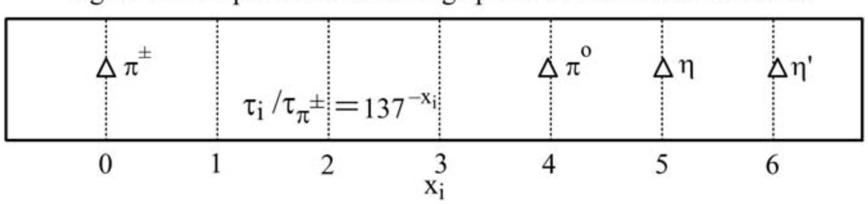
$(m_{q_\pi} = m_\pi/2,\ m_{q_\eta} = m_\eta/2,\ m_{q_{\eta'}} = m_{\eta'}/2)$. The $\pi = m_b \bar{m}_b \cong 137$ MeV mass combination (which incorporates the 2% HBE correction) serves as the boson channel bound-state building block. Thus the η and η' masses also have HBE = 2%. The mass interval between particle states on each side of the $m_b \bar{m}_b$ excitation channel is the excitation quantum $X_b \equiv 3m_b = 210$ MeV (before the HBE is applied). The accuracy of this procedure is illustrated graphically in Fig. (1), where the π, η and η' masses are plotted on a 137 MeV mass grid. The average accuracy of the experimental mass fits to the 137 MeV mass grid is 0.12%. This attests not only to the mass linearity of these three states, but also to their absolute values, which follow computationally from the boson radius r_{boson} in Eqs. (21-23), combined with a uniform 2% HBE applied to these states.

Fig. 1. The α-quantized nonstrange pseudoscalar meson masses



The spin $J = 0$ *non-strange* π, η, η' mesons have 2, 8, 14 $(m_b$ and $\bar{m}_b)$ total building block masses, respectively. The spin J = 0 *strange* K mesons each contain 7 $(m_b$ and $\bar{m}_b)$ total mass units, which is an *odd* number. From this we conclude that the spin of the m_b building block itself is $J = 0$, so that m_b is a *boson* constituent quark, as its name suggests.

Fig. 2. The α-quantized nonstrange pseudoscalar meson lifetimes



The α quantization of the π, η and η' masses is also manifested in their lifetimes, which are displayed in Fig. 2. This is a plot of mean lifetimes τ_i relative to the reference $\tau_{\pi^\pm}$ lifetime, using a logarithmic lifetime grid spaced by factors of 137. The factor of 137^{-4} spacing between the π^0 and $\pi^\pm$ lifetimes is due to the long-lived $\pi^\pm$ electroweak decay ($\pi^\pm$) versus the short-lived π^0 radiative decay (see Fig. 4). The linear (in powers of α) $\pi^o, \eta,\ \eta'$ *lifetime* ratios in Fig. 2 echo the linear π^o, η, η' *mass* ratios in Fig. 1. The significance of these α-spacings is that the decaying masses "remember" their α-quantized α_m excitation history.

The *fermion production channel* of Eq. (19) contains the spin J = 1/2 mass units of the constituent quarks, leptons, proton-neutron pair, and vector meson ground states. The fermion building block $m_f = 105$ MeV is the spin 1/2 counterpart of the spin 0 boson building block $m_b = 70$ MeV. It is defined by the α_m generation equation

$$E_r(r) = e^2 / (r_{\text{fermion}}) = m_f c^2 = 105 \text{ MeV}, \tag{24}$$

where $r_{fermion} = (2/3)\, r_{boson}$, and is created in particle-antiparticle pairs by the equation

$$E_r(r) = 2e^2 / r_{\mathrm{E}_{fermion}} = \mu^+ + \mu^- = 210 \text{ MeV}, \tag{25}$$

where m_f is observed directly as the muon. Higher-mass fermion states appear as multiples of m_f. We can describe these by considering just the particle production channel. Since the mass quantum m_f carries spin J = 1/2 as a conserved quantity, the higher-mass states are *odd* multiples of m_f, and the excitation units that add to the m_f ground state are *even* multiples of m_f. Interestingly, the $X_b \equiv 3m_b = 210$ MeV excitation unit of the π, η, η' boson channel also appears here, but as the configuration $X_f \equiv 2m_f = 210$ MeV. A mixed group of important fermion states are generated sequentially in the following 210 MeV excitation-doubling sequence:

$$m = m_f + nX_f \ (n = 0,1,2,4,8) = \mu(105); (u\text{-}d)(315); (s)(525); (\text{p-n})(945); \tau(1785). \tag{26}$$

This sequence is composed of the two leptons (μ and τ), two constituent-quark states (*u-d* and *s*), and a nucleon pair (p-n). Their masses are all odd multiples of 105 MeV, and the calculated muon, tauon and nucleon mass values displayed in Eq. (26) are accurate to 0.6%. The *u-d* and *s* constituent-quark masses, from their relationships to gauge bosons and vector mesons, respectively (as shown in Fig. 3), are also at this same level of accuracy, and they agree well with the *u-d* and *s* masses deduced from quark magnetic moments [3].

The *strange* quark *s*(525) displayed in Eq. (26) serves as a *secondary building block*, which generates the *c*(1575) and *b*(4725) *charm* and *bottom* constituent-quark masses by successive $s \to c \to b$ mass triplings. These quarks then pair together to form the vector meson ground states:

$$s\overline{s} = \phi(1050)(+3.0\%),\ c\overline{c} = \mathrm{J}/\psi_{1\mathrm{s}}(3150)(+1.7\%);\ b\overline{b} = \Upsilon_{1\mathrm{s}}(9450)(-0.1\%), \tag{27}$$

and the mixed-quark excitation

$$b\overline{c} = \mathrm{B_c}(6300)(0.4\%). \tag{28}$$

The errors displayed for these calculated mass values reflect their HBE hadronic binding energies, which are 3% for the ϕ at 1 GeV, and then decrease monotonically for increasing energies and vanish at 9 GeV and above (asymptotic freedom).

The *gauge boson production channel* of Eq. (20) has a well-defined ground state, which acts as the platform for a secondary α-boost that accurately extends the low-energy α-quantized mass excitations up into the region of the $W^{\pm}$ and Z^0 gauge bosons and top quark *t*. The existence of this secondary α-boost is suggested by the well-explored 69 GeV "particle void" that extends from the $\Upsilon = b\overline{b}$ excited states at 11 GeV up to 80 GeV, where the $W^{\pm}$ gauge boson appears. The idea of a linkage between the low-energy mass spectrum and the very-high-energy gauge bosons and top quark is also suggested by the unexpected experimental discovery of a mass relationship between the gauge bosons and top quark:

$$m_{W^{\pm}} + m_{Z^{o}} = m_t \text{ (1.1\% accuracy)}. \tag{29}$$

This equation is similar in form to the pseudoscalar meson equation

$$m_{\pi^{\pm}} + m_{\pi^{o}} = m_{q_{\eta}} \equiv m_{\eta}/2 \text{ (0.3\% accuracy)}, \tag{30}$$

where q_η is the constituent quark in the η meson. The similarities and accuracy of these two equations suggests that their generation mechanisms may be related.

Experimentally, the high energy $(W^{\pm}, Z^0, t)$ particle states are produced by $p - \overline{p}$ collisions at the Tevatron and LHC. At these TeV energies, a proton is flattened relativistically, and the *uud* proton quarks are essentially independent, so the collisions are between individual q_p and $\overline{q}_{\overline{p}}$ quarks, where $q_p = (u,d)$ collectively represents the *u* and *d* proton constituent quarks. (The small *u-d* mass difference averages out in the collisions, so $m_{q_p} = m_p/3$ is an exact relationship.) Once in every 10^{10} scattering events, a head-on $q_p\overline{q}_{\overline{p}}$ collision occurs where the quark pair absorbs enough collision energy to create gauge bosons and top quarks. If we reproduce this event as an α-generated α_m phase transition, the factor-of-137 increase in $q_p\overline{q}_{\overline{p}}$ mass should coincide with the appearance of a particle state at the α-boosted energy, which is

$$(m_q + \overline{m}_{\overline{q}})/\alpha = (m_p + \overline{m}_{\overline{p}})/3\alpha = 85.72 \text{ GeV}. \tag{31}$$

No direct particle state appears at this energy, but it closely matches the average energy $\overline{WZ}$ of the $W^{\pm} = 80.4$ GeV and $Z^{o} = 91.2$ GeV gauge bosons, which is

$$m_{\overline{WZ}} \equiv (m_{W^{\pm}} + m_{Z^{o}})/2 = 85.79 \text{ GeV}. \tag{32}$$

This is agreement to an accuracy of 0.08%. Hence we have experimentally established the mass relationship

$$(m_p + \overline{m}_{\overline{p}})/3\alpha = m_{\overline{WZ}} \text{ (0.08\% accuracy)}. \tag{33}$$

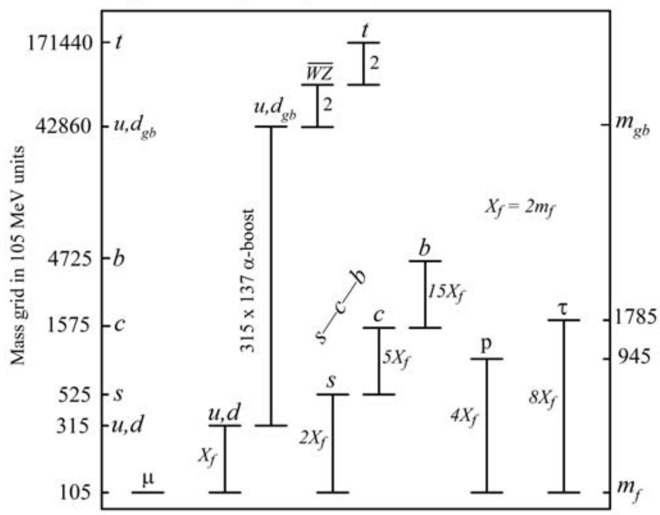
Eq. (29) extends this result upwards in energy so as to include the top quark mass [4]:

$$m_t = m_{W^\pm} + m_{Z^0} = 2\, m_{\overline{WZ}} = 171.6 \text{ GeV}. \tag{34}$$

The measured top quark mass is m_t=173.5 GeV, which matches Eq.(34) to 1.1%. Thus we have experimentally linked the gauge boson average mass and the top quark mass to the proton constituent-quark average mass via the α-boost shown in Eq. (31). The *gauge boson unit mass* that accurately reproduces the $m_{\overline{WZ}}$ and m_t masses is the quark state [5]

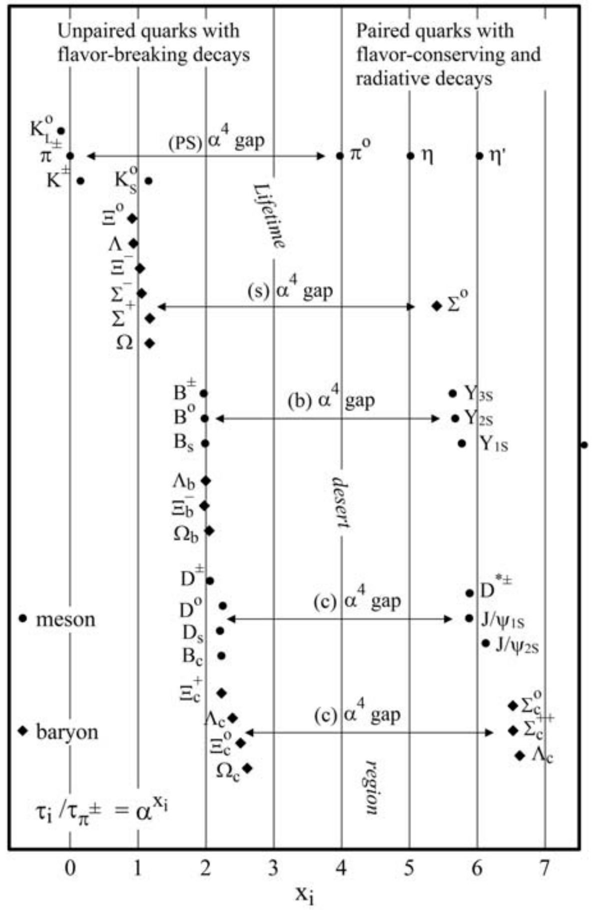
$$m_{gb} = m_p / 3\alpha = 42.86 \text{ GeV}. \tag{35}$$

Fig. 3. Fermion plot of quark and ground-state hadron masses.



The fermion masses described in Eqs. (24-36) are shown graphically in Fig. 3, where they are plotted on a logarithmic mass scale that is in units of the mass $m_f = 105$ MeV for masses below 12 GeV, and $m_{gb} = 42.86$ GeV for masses above 12 GeV. The fermion masses and constituent-quark states shown in Fig. 3, together with the pseudoscalar masses displayed in Fig. 1, represent the fermion quark structure and basic *ground states* of the hadronic mass spectrum. The unit masses m_b = 70 MeV and $m_f = 105$ MeV are α-generated from the electron, and the electron itself is α-generated from and anchored on the experimental Thomson cross section. The high-energy gauge bosons and top quark masses are α-generated from the proton quarks in Tevatron and LHC $p - \overline{p}$ collisions. Thus the particle and constituent-quark masses displayed in Figs. 1 and 3 are all related to the electron, and they have calculated absolute mass values that are at an accuracy level of about 1%.

Fig. 4. Lifetime α quantization and α^4 gaps



In the case of the nonstrange pseudoscalar π, η and η' mesons, the clear-cut factor-of-137 *mass* quantization displayed in Fig. 1 is echoed by an equally clear-cut factor-of-137 *mean lifetime* quantization in Fig. 2. This indicates that the particle masses retain some record of the α-quantized way in which they are produced. This result also carries over to the fermion quark and particle states of Fig. 3. There are 37 metastable hadrons with lifetimes $\tau > 10^{-21}$ sec. These are the ground states of the various quark configurations. Their lifetimes are plotted in Fig. 4, using the same α-spaced logarithmic lifetime grid and same $\pi^\pm$ reference lifetime as in Fig. 2. There are several important regularities in these lifetimes: (1) The lifetimes fall into discrete groups, each dominated by a single quark. (2) The quark dominance rule for decay lifetimes is $c > b > s$. (3) The unpaired quarks have slow electroweak decays that are separated from the fast paired-quark decays by a factor of approximately α^4, with no lifetimes in the intervening *lifetime desert region*. (4) The non-strange, strange (s) and bottom (b) quark states occupy the $x_i = 0, 1, 2$ grid lines, respectively, and the charm (c) quark state lifetimes are a factor of 3 shorter than the b-quark lifetimes. (5) The meson and baryon lifetimes both exhibit the same quark regularities.

The grouping of the metastable particle lifetimes into quark-dominated families is so distinctive that the existence of quark-like substates could be deduced from the lifetimes alone if the quark model had not yet been discovered. The main reason for including the lifetimes here is to demonstrate the extent to which quantizations in factors of 137 have permeated the whole fabric of particle lifetimes and masses. This provides at least indirect confirmation of the validity of the α_m mass generation process, which is defined by the expanded α_m fine structure equation $E_r(r) \equiv e^2/r \overset{137}{\Leftrightarrow} mc^2(137r)$ (Eq. 16).

We conclude with two examples that illustrate the accuracy of the extended α_m equation in its electron-based generation of particle masses. Fig. 3 contains an excitation chain of particle masses that starts with the 105 MeV muon and leads up to the *b* quark and its associated particle state, the $\Upsilon_{1S} = b\overline{b}$ vector meson ground state. Fig. 3 also contains a similar excitation chain that starts with the muon, involves a factor-of-137 Tevatron-LHC α-boost, and leads to the top quark *t*. The muon itself is reached by a $(3/2\alpha)$ α-boost from the electron (Eq. 19). Thus we can start with the electron mass and arrive at both the Υ_{1S} and top quark masses without using any freely-adjustable parameters. The equation for the Υ_{1S} upsilon mass is

$$m_{\Upsilon_{1S}} = (3/2\alpha)(5)(3)(3)(2)\,m_e = \frac{135}{\alpha} m_e = 9453.4 \text{ MeV}. \tag{36}$$

The experimental upsilon mass is [4]

$$\left(m_{\Upsilon_{1S}}\right)_{\text{exper.}} = 9460.3 \text{ MeV}. \tag{37}$$

This is agreement to an accuracy level of 0.07%. The equation for the top quark mass is

$$m_{\text{top}} = (3/2\alpha)(9)(1/3)(1/\alpha)(4)m_e = \frac{18}{\alpha^2} m_e = 172.7 \text{ GeV}. \tag{38}$$

The experimental top quark mass is [4]

$$\left(m_{\text{top}}\right)_{\text{exper.}} = 173.5 \text{ GeV}. \tag{39}$$

This is agreement to an accuracy level of 0.46%.

There are two significant conclusions that can be drawn from Eqs. (36-39): (1) The constant α used here is the *renormalized* value $\alpha \cong 1/137$, and not the running QCD coupling constant $\alpha_s(q^2)$ [6] that increases in value to $\alpha_s(q^2) \approx 1/128$ at $q^2 \simeq m_{\mathrm{W}}^2$. (2) These very accurate no-free-parameter equations would not be possible without the inclusion of the factors α in Eq. (36) and α^2 in Eq. (38).

References

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[2] Mac Gregor, M. H., *The Power of Alpha* (World Scientific, Singapore, 2007), Ch. 4.

[3] Seiden, A., *Particle Physics* (Addison-Wesley, San Francisco, 2005), p. 196.

[4] The mass values used in this paper, including the top quark mass, are from Beringer, J. *et al*. (Particle Data Group), Phys.Rev. D86, 010001 (2012).

[5] The $m_{\overline{\mathrm{WZ}}}$ average mass contains two m_{gb} = 42.86 GeV mass units, and the top quark t contains four m_{gb} mass units. A state with three m_{gb} mass units would have an energy of 128.6 GeV, which is approximately the location (~126 GeV) where the sightings of a Higgs-like particle have emerged.

[6] Ref. (4), p. 101; Ref. (3), pp. 234-240.