Early works on the Hagen-Poiseuille flow

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This paper introduces to the early pioneers in the field of laminar flow described by the Hagen Poiseuille law. After giving some biographical information, the experimental setups are briefly explained and the original data are given in the form of diagrams scaled to modern units. Then the way of argument of Hagen, Poiseuille, Hagenbach and others is reviewed. The early historical development of, corrections to, and the scope and limits of the Hagen Poiseuille law receive thus due attention.

Keywords: Hydrodynamics, Laminar Flow, Viscosity, Hagen Poiseuille Flow, Hagenbach Correction

I. Introduction

The fundamental law describing laminar flow in pipes has been discovered 162 years ago by a German hydraulician G.H.L. Hagen and a French physiologist J.L.M. Poiseuille. It is fundamental for all branches of technology and science where laminar flow through pipes occurs. It also applies to problems as the blood flow through veins and arteries. It has been experimentially confirmed for liquids of a wide range of viscosities.

21 years after its discovery the theoretical explanation was given by E. Hagenbach (and F. Neumann). Its limits at the transition to turbulent flow were successfully analyzed by the English physicist O. Reynolds 44 years after its discovery. Laminar flow in pipes is studied by most students in the world nowadays. It is a beautiful example of the agreement of theory and experiment in classical physics.

This historical review first introduces some of the main players (Hagen, Poiseuille and Hagenbach). This selection may be representative, but certainly many other hydraulicians and scientists would deserve mentioning in this context [36].

After giving a short derivation of the Hagen Poiseuille law (HP law) as it is found in modern undergraduate text books, old physical units are explained. Then the historical experiments by Hagen and by Poiseuille are explained, and the original data given in the easy to digest form of diagrams converted to modern international units.

The last major section reviews the reasoning of Hagen, Poiseuille, Hagenbach and others who improved and tested the limits of the HP law in theory and experiment.

The list of original literature at the end – though far from complete – may give the interested reader an occasion to study the sources himself. For readers of French, especially the original works of Poiseuille make a very good, clearly structured reading.

II. Biographical information

II.1 Gotthilf Heinrich Ludwig HAGEN

Gotthilf Heinrich Ludwig Hagen was born on 3rd March 1797 in Königsberg in Prussia as son of a local official. In high school his math teacher became interested in him and introduced him to higher mathematics. Beginning in 1816 he studied under Bessel, doing some astronomic works. He became land surveyor and architect by 1822. Hagen hiked on a study trip through the Netherlands, France, Switzerland, Italy and Tirol, especially interested in hydraulic structures. He started to work in hydraulics in Danzig, Pillau and Berlin (1830), where he worked also for the navy port Wilhelmshaven. In 1842 he became member of the Berlin Academy of Science, and in 1843 honorary doctor at the University of Bonn. Parallel to this career, he worked as a teacher for hydraulics, bridge and road construction. He published a "Manual of Hydraulics", and the "Principles of Statistics" and performed hydrodynamic research on laminar and turbulent[1] flow. In his very last days he measured the air resistance of plane disks. Hagen died on 2nd February 1884. [2]

II.2 Jean Louis M. POISEUILLE

Jean Louis M. Poiseuille was born 1799 in Paris. He studied medicine, devoting special attention to experimental physiology. The title of his doctoral thesis (1828) read: "Research about the force of an aortic heart" for which he received a gold medal by the French Academy of Sciences. Also another work "Research about the origin of motion of the blood in the veins" (1832) received a prize. In 1839 he published a work entitled: "Research about the origin of motion of the blood in the capillary vessels" and in 1840 he presented short reports about his "Research on the liquids in pipes of small diameters" to the French Academy of Sciences. In 1842 he became a member of the French Academy of Medicine. Poiseuille worked as the editor of the "Dictionary of common medicine" and finally died on 25th December 1869. [3]

II.3 Eduard HAGENBACH-BISCHOFF

Eduard Hagenbach-Bischoff was born on 20th February 1833 in Switzerland as son of a professor on church history. He studied in Basel, Berlin, Geneva, Paris especially with Merian, Dove, Magnus and Jamin. He became doctor in 1855 and started to teach physics and chemistry at the Polytechnic of Basel. In 1859 he became professor of mathematics at Basel University and in 1862 for physics. He died on 23rd December 1910 in Basel. At Basel University he displayed a certain skill as a science organizer.

Hagenbach researched fluorescence, electric discharging, glaciers and hydrodynamics. He theoretically established the Hagen-Poiseuille law and gave first considerations to boundary layers with respect to turbulent motion. A smaller work of him in hydrodynamics is about a bullet on a water jet.

III. Modern treatment of the Hagen-Poiseuille law

The liquid in a pipe sticks to the wall, i.e. $v(R)=0$ and flows most rapidly in the center. The velocity gradient has radial direction. We consider the forces on a coaxial liquid volume cylinder of radius r and length l. The net cylinder mantle friction force is

$F_f = 2\pi r l \eta dv/dr$, the cross section balancing pressure force is $F_p = \pi r^2 \Delta p$, with pressure difference $\Delta p = p_{begin.} - p_{end}$, viscosity η and velocity v. The cylinder has the mantle surface $2\pi rl$, and the cross section area πr^2. The situation is stationary for $F_f+F_p=0$ which results in

$$\frac{dv}{dr} = -\frac{r\Delta p}{2\eta l} .$$

Integration yields

$$v = -\frac{r^2 \Delta p}{4\eta l} + C .$$

C is an integration constant. Considering that $v(R)=0$ we end up with

$$v = \frac{(R^2 - r^2)\Delta p}{4\eta l} .$$

This shows that the velocity has the profile of a paraboloid. A liquid volume of $dV = 2\pi l dr v(r) t$ passes in the time t through the circular cross section ring between r and $r+dr$. Integration over the whole pipe cross section yields a total flow of

$$\frac{V}{t} = \frac{1}{t}\int_0^R dV = \frac{\pi}{8}\frac{\Delta p R^4}{\eta l} .$$

This is the modern form of the Hagen-Poiseuille law. [4]

IV. The historic measurements of Hagen and Poiseuille

Especially Hagen used outdated physical units when he reported his results [5]. Therefore a list of these units together with the modern equivalents is given in Table 1.

old unit		new unit	
value	name	value	name
Temperature			
0	degree Reaumur	0	degree Celsius
80	degree Reaumur	100	degree Celsius
Length			
144	Pariser Linien	0,3248	m
1	Pariser Linie	2,26	mm
1	Pariser Fuss	324,8	mm
1 Pariser Fuss = 12 Pariser Zoll			
1	Pariser Zoll	27.066	mm
Weight			
1	Preussisches Loth	15	g
Pressure			
1	mmHg = 1 Torr	101325/760	Pa (Pascal)
1	cmHg	101325/76	Pa
1	cmH_2O	98.0665	Pa

Table 1: Old and new physical units. [2,6,7,8,9]

IV.1. The historic measurements of Hagen (1839)

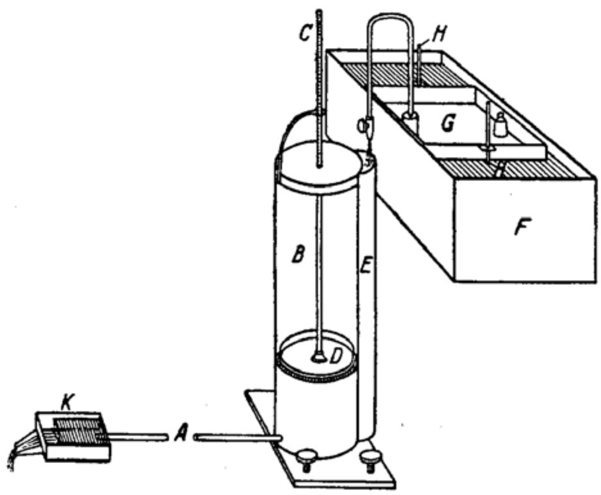

Fig. 1. Experimental setup of Hagen. [2]

The apparatus used by Hagen [2] is shown in Fig. 1. The letters in Fig. 1 stand for:

A test pipe
B water pressure cylinder
C measuring rod
D swimming brass bowl
E half cylinder, providing the water for B through a bottom connection
F water reservoir
G swimming sheet metal case guided by
H two vertical wires guiding G

K   water outflow sheet metal case receiving the out-flowing water

The measured values of Hagen[2] are displayed in the Figures 2a – 2f. The units have been converted to modern metric SI units.

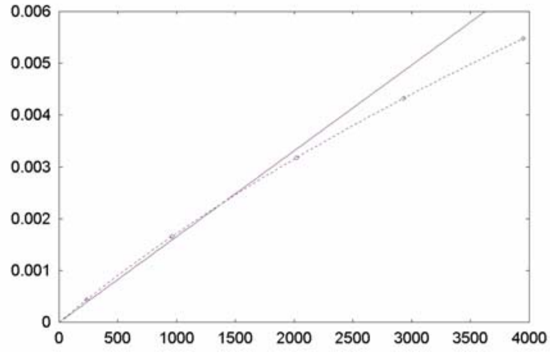

Fig. 2a, T=10.3°C, l=0.47320m, r=0.00128m

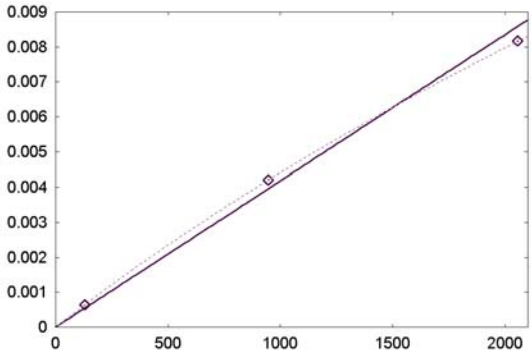

Fig. 2b, T=10.5°C, l=1.0897m, r=0.00201m

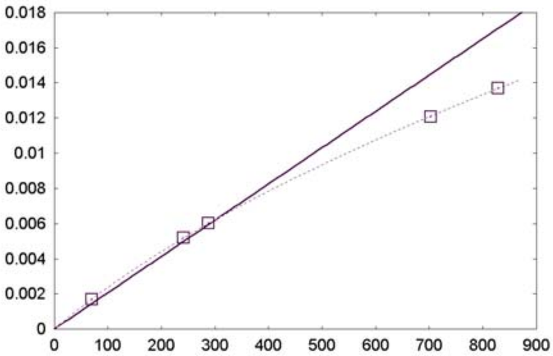

Fig. 2c, T=8.8°C, l=1.0466m, d=0.0029515m

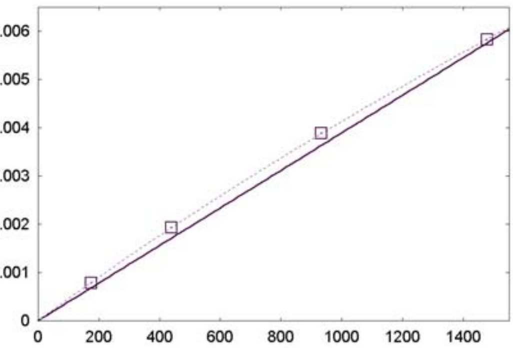

Fig. 2d, T=7.6°C, l=1.0897m, r=0.00201m

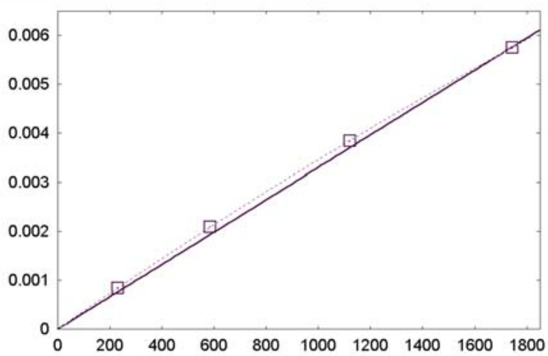

Fig. 2e, T=1.25°C, l=1.0897m, r=0.00201m

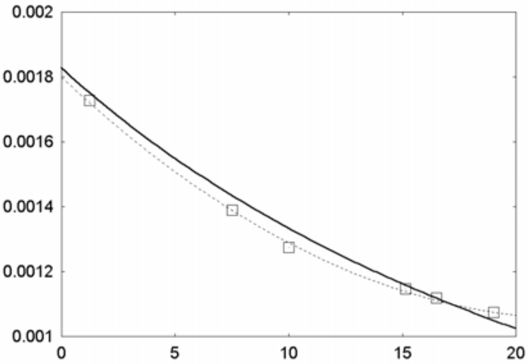

Fig. 2f, x: T in °C , y: viscosity η(T) in Pa s

Fig. 2: Hagen's measurements (squares) and his quadratic approximation formulas (dotted curves) according to [2]. (a-e) x: pressure in Pa, y: flow in kg/s. The solid lines show the Hagen-Poiseuille law [5]. (f) viscosity measurements and calculations by Hagen [2], and viscosity according to Pery [10].

Some discussion of Hagen's experimental method, his calculations for Fig. 2, his theoretical interpretation, and for his version of the Hagen-Poiseuille law will follow in section. V.1.

IV.2. The historic measurements of Poiseuille (1840)

The apparatus used by Poiseuille[2] is shown in Fig. 3. The letters in Fig. 3 stand for:

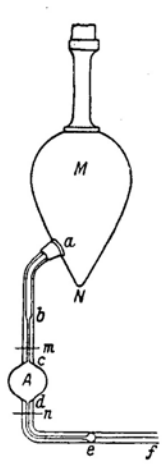

Fig. 3

M bulb shaped glas container with a vertical copper pipe on top and three glas pipes emerging from M:

1. The first pipe is connected with a compressor pump
2. The second with an open manometer
3. The third with a 60 liter compressed air reservoir made of thick copper sheet

N conic sink in order to collect impurities

A spherical ampoulla

m,n two levels. The volume between *m* and *n* has been thoroughly determined in advance. The levels are observed through a sliding telescope. Everything below *m* is immerged in a thermal bath for temperature stabilization.

e-f the narrow experimental flow pipe connected to M through *a-b-c-d-e*

e spherical widening after which the experimental flow pipe begins

Poiseuille went through an elaborate quality control program in order to select capillary pipes with constant circular cross sections. For measuring the pressure he employed a water manometer, and several mercury manometers depending on the pressure range.

The measured values of Poiseuille [2,11,12,13] are compiled in Fig. 4. Instead of millimeters and mmHg, we strictly use meters and Pa, respectively. This should help to compare the measurements of Hagen and Poiseuille with each other and with modern measurements.

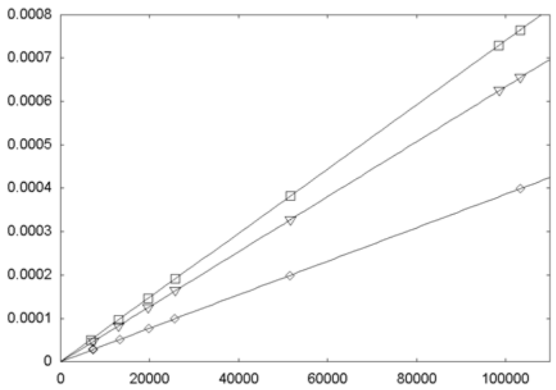

Fig. 4a, 1/time ~ pressure, three different pipes.

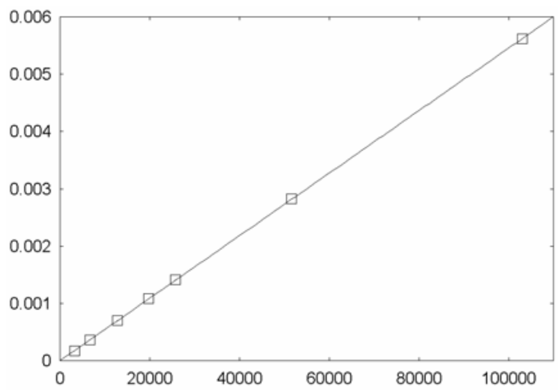

Fig. 4b 1/time ~ pressure

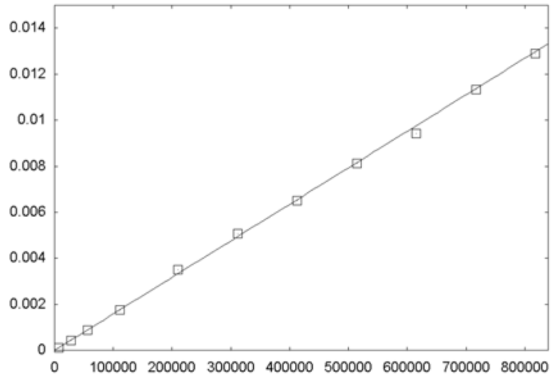

Fig. 4c, 1/time ~ pressure

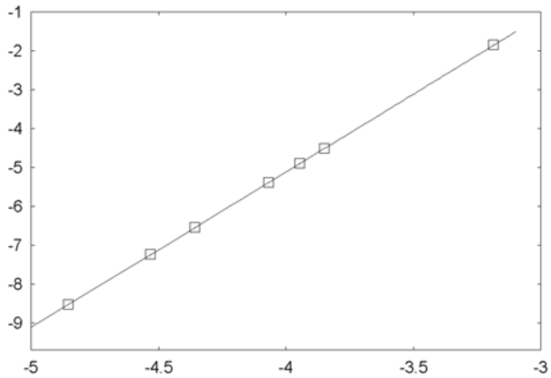

Fig. 4d, log-log plot: (diameter)[4] to flow

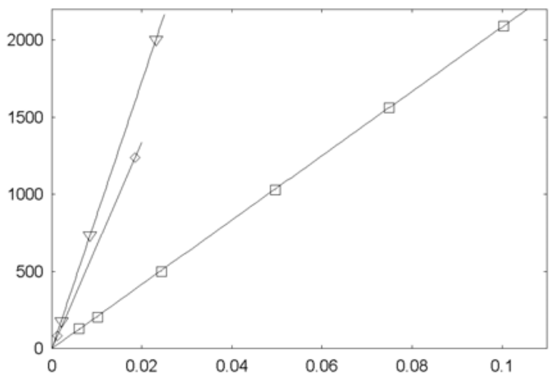

Fig. 4e, time ~ length for three different diametres

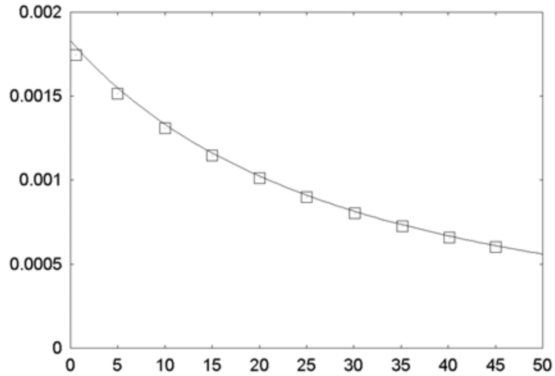

Fig. 4f, water viscosity η(T)

Fig. 4 Measurements of Poiseuille. (a-c) 1/time proportional pressure, x: pressure in Pa, y: 1/time in 1/s.[2,11,13] (a) squares: length l=0.0758m, diameter d=0.000142 m, triangles: l=0.07505m, d=0.000113m, diamonds: l=0.050225m, d=0.000043m. (b) l=0.00210m, d=0.000029m. (c) l=0.364m, d=0.0001316m. (d) x: $\log_{10}$(d in m), y: $\log_{10}$(flow in kg/s) showing the proportionality flow~d^4.[11,13,2] (e) time of flow proportional length, squares: d=0.000085m, triangles: d=0.000029m, diamonds, d=0.00001394m, x: length in m, y: time of flow of same ampulla volume in s.[11] (f) water viscosity η(T) [12,13,2], x: T in °C, y: η in Pa s, the solid curve η(T) according to Pery [10].

V. The reasonings of Hagen, Poiseuille, Hagenbach and others

V.1. The arguments of Hagen

Hagen starts by referring to even earlier research by Gestner in 1796 [14] by Prony in 1804 [15] and Eytelwein in 1814/15 [16]. He remarks that he is limiting his research to laminar flow and continues then to give a detailed technical description of his experimental apparatus, part of which is shown in Fig. 1. He measured temperature in °Reaumur, length in Parisian inches and Parisian lines, and mass in terms of the Prussian Lot. He seemed to be not to critical about the quality of the pipes he used, he rather determined their inner volume with a scale (filled minus empty).

After conducting five experiments with three pipes of varying diameter, length and temperature (comp. Fig. 2a-e), he fitted quadratic approximations for the pressure in terms of the flow (dotted curves in Fig. 2a-e). In the second half of his paper he exclusively concentrates on arguing with the approximation coefficients. Hagen remarks that especially the linear coefficient exhibits a strong temperature dependence.

He explains the quadratic coefficient to be due to the acceleration of the water. He rightly says that the water in the center is faster than the water at the pipe wall, but as we saw in section III, his assumption of a conic as opposed to parabolic velocity profile was premature.

The last third of his paper concentrates on the linear coefficient, which corresponds to the linear term of the Hagen-Poiseuille law (HP law). Here he first establishes the temperature dependence of the linear coefficient, which is equivalent to determining the temperature dependence of the viscosity η(T). Next he tries to explain the dependence on the radius of the pipe with an integer exponent, which he concludes to be –4. In the following theoretical consideration, he obtains the right dependencies ([2], p. 88), but with wrong numeric factors due to his conic velocity profile. As in section III he also tries to analyze the friction forces on thin coaxial water cylinders.

Hagen finishes his considerations by remarking what influence turbulence will have in bigger pipes. He later devoted more research to this [17].

V.2. The arguments of Poiseuille

The early works of Poiseuille were published in the reports (Compte Rendu) of the French Academy of Sciences in four parts. The first three parts are his own research divided into four sections: on the influence of the pressure[11], the length[11], the diameter[11] in 1840, and the fourth section on the temperature[12] in 1841, respectively. The fourth part is a report by a commission of four French researchers Arago, Babinet, Piobert and Regnault, who had to rigorously verify Poiseuilles results, published in 1844. L. Schiller reprinted it in 1933 together with Hagen's and Hagenbach's papers [2].

Poiseuille starts off by referring to earlier works of Prony, Bossut, Couplet, Dubuat[18], Gerstner[19], Girard[20] and Navier. He states that he has a strong physiologically motivated interest in the flow through pipes with diameters less than 0.01mm (20 times smaller than Hagen's narrowest pipe).

In his first section on the pressure he extends the experiments of Hagen to pressures 200 times higher values of the pressure. As section IV.2 shows he used technically more advanced equipment. Whilst Hagen's smallest and longest pipe differ just by a factor of two, this factor is 50 for Poiseuille's pipes. The thermal bath (legend to Fig. 3) gives Poiseuille the advantage, that he was able to operate at well defined temperatures. He concludes that the flow must be directly proportional to the pressure difference (see Fig. 4a-c). At the end of the first section he makes a remarks about the lower length limit for laminar flow.

In the section on the length, he keeps pressure, total flow volume and temperature constant. He used the technique of taking a long pipe and chopping of portions until the lower limit for laminar flow is reached. He concludes that the flow must be inversely proportional to the length of the pipe (see Fig. 4e).

He then keeps pressure, temperature, total flow volume and length constant and investigates pipes of different diameters (see Fig. 4d). He notes that his result of the flow through capillary pipes shows the proportionality of the flow to the 4th power of the diameter and not the square, as derived earlier by Navier.

In his final section on the temperature dependence of the proportionality relationship he steps the temperature in steps of five °C from 0 to 45°C. He carefully takes the volume variations of the ampulla between the levels *m* and *n* in Fig. 3 into account. The reasonable agreement of his quadratic approximation in the temperature with modern values is shown in Fig. 4f.

In the report on their examination of Poiseuilles research, Arago, Babinet, Piobert and Regnault give a more detailed description of previous works and of Poiseuilles experimental apparatus. They point out that previous works didn't find the d^4 dependence, since they operated with lengths below the laminar flow limit. The explain in detail why the influence of density variations of water with temperature is negligible. New experimental results of Poiseuille about the influence of admixture of pure alcohol, using pure ethanol itself or mercury were also included in the report. The four examiners seem also to have undertaken a number of experiments themselves in order to verify Poiseuilles claims.

V.3. The arguments of Hagenbach

Hagenbach's focus [21,2] is on viscosity. Up till 1860 it were experimentalists rather than theorists who introduced it. Hagenbach discards the treatment for velocity in the cases of plates vibrating in liquids or liquids oscillating in U-shaped pipes as mathematically too complicated. Thus he is left with the simple flow through pipes and he further restricts himself to liquids that wet the pipe wall. As Wiedemann before (see the next subsection) he analyzes the motion in terms of coaxial liquid cylinders with mantle thicknesses of molecular diameters. Yet as Schiller [2] remarks, this latter choice is not really essential. Hagenbach shows that a finite adhesive force between liquid and boundary wall necessitates v(R)=0 (boundary condition of section III.)

Hagenbach goes on to explicitly acknowledge the convincing agreement of Poiseuilles experiments [11,12,13,2] with the Hagen-Poiseuille law as stated in section III. His own experiments (conducted in Wiedemann's laboratory) serve him rather to get an insight in the practical circumstances of such measurements and to get data for relatively short pipes, where the HP law as originally given by Poiseuille fails. After this reassurance of the theoretical and practical aspects of the HP law, he uses it to define the viscosity η as:

$$\eta = \frac{\pi}{8}\frac{\Delta p R^4}{lV/t}.$$

He then derives an expression for the case that the kinetic energy of the moving liquid itself is not negligible:

$$\frac{V}{t} = \frac{2^{8/3}}{g\rho}\pi\left(-gl\eta + \sqrt{2^{-14/3}R^4 g\Delta p\rho g + g^2 l^2\eta^2}\right).$$

Solving this equation for the viscosity η we obtain:

$$\eta = \frac{\pi R^4 \Delta p}{8lV/t} - \frac{\rho V/t}{2^{11/3}\pi l}.$$

The expressions given by Hagenbach himself missed a factor of $2^{1/3}$ in the Ansatz for the kinetic energy, as remarked by Schiller[2]. This is the reason for the slight differences in exponents of 2 compared with the formulas in Hagenbach's original publication [21,2]. Schiller's annotations to the rest of Hagenbach's paper [2] show that this $2^{1/3}$ factor further improves the agreement between experiment and theory. The second term in the equation for η is called kinetic correction or Hagenbach correction[22]. It has even become part of a German Industrial Norm [23] for correctly measuring viscosity.

Next Hagenbach uses the improved formula for η to show that it is more appropriate for shorter pipes by recalculating the previous experiments. He then applies the new formula for η also to past measurements of Poiseuille and Hagen, clearly improving their agreement with the theory, i.e. $\eta(T)$=const. for constant temperature T. Poiseuille used glass pipes and Hagen copper pipes. That η doesn't depend on that demonstrates that the flow is governed by internal cohesion forces and not by adhesive wall forces, once the boundary layer with $v(R)=0$ has developed.

Hagenbach goes on to compare his formula with the quadratic approximation formula of Hagen. Schiller remarks that taking the correct $2^{1/3}$ factor into account brings both formulas into reasonable agreement.

In the final quarter of his paper, Hagenbach analyzes measurements for wide pipes previously conducted by Dubuat [24] and Darcy [25]. He mentions that engineering formulas by Gerstner[26], Prony[15], Eytelwein[27] and Weisbach[28] apply, but fail to give a microscopic explanation. Hagenbach now assumes a material dependent turbulent flow resistance proportional to the square of the velocity: av^2. The resulting differential equation for v is non-trivial. He therefore just treats the case for strong turbulence (large a). He argues that the parabolic velocity profile should deform into an elliptic one.

With this in hand, he takes wide pipe measurements by Darcy[25], inserts the viscosity obtained by interpolation from other measurements of Poiseuille and calculates the turbulence resistance coefficient a. a varies only 3% around a constant value. In a final step Hagenbach compares his elliptic velocity distribution with a velocity distribution observed by Darcy[29] with a Pitot pipe. This shows that close to the wall, the elliptic velocity distribution needs to be heightened.

It may be interesting to remark that it seems that an error free derivation of the kinematic correction has first to been given by another physicist, Franz Neumann (Königsberg, 1798-1895) [2,35].

V.4. Wiedemann, Glaser and Schiller

Wiedemann[30], in whose laboratory Hagenbach is working, is motivated to find the force necessary to drive an electrolyte through porous clay. He views it as a system of capillaries. Assuming viscosity to be the force necessary to pull one liquid molecule past another, he deduces the HP law in fairly the same way as in section III.

He conducts a series of measurements with the same experimental geometry, but varying electrolytes: sulfuric acidic copper oxide, nitrous acidic copper oxide, nitrous acidic silver oxide, sulfuric acid, potassium, nitrous acidic ammonia. The temperatures in his experiments range between 14 and 20 °C, except for the sulfuric acidic copper oxide (T between 15°C and 74°C).

He observes that the viscosity increases stronger than the salt concentration. Something, which is especially valid for sulfuric acid. The only exception is nitrous acidic ammonia, with declining viscosity for increasing salt concentrations. Wiedemann remarks (giving explicit values for sulfuric copper oxide), that in general increase of temperature strongly reduces the viscosity.

Heinrich Glaser [31] looks at the validity of the HP law including the kinetic correction given by Hagenbach (section V.3.) for a wide range of viscosity values. In order to do that he mixes turpentine oil (low viscosity) and colophony (high viscosity) in varying concentrations. He refers to a work by R. Reiger [32] showing the validity of the HP law for high viscosities $\eta(T)=4\times10^6$Pa s up to 10^{12}Pa s, at temperatures of 8.2°C to 9.0°C. Then Glaser quotes the viscosity formula of Hagenbach, including the kinematic correction and briefly explains about Reynolds [33] distinction between laminar and turbulent flow.

For turpentine oil he measures the viscosity for temperatures from 0 to 85°C in steps of 5°C. He then looked for limits to the validity of the HP law. In the range of 5 to 23.5 hPa (hecto Pascal) he finds no deviation but remarks on changes of the viscosity under very high pressures. At a pipe radius of 0.29mm he finds a lower limit of l=60mm for the validity of the kinematically corrected HP law (p=139 hPa, T=7.1°C). For pipes of fixed length=151mm with T=7.1°C and p=139 hPa, he varies the radius and finds that the HP law is valid up to R=0.4mm. He concludes that the validity of the HP law depends foremost on the pipe radius.

Then Glaser turns to mixtures of turpentine oil and colophony. He measures the viscosity for colophony concentrations from 0 to 100%. He finds that η increases rapidly, especially for higher concentrations of colophony (100%: $\eta(7.1°C)=10^{21}$Pas). He then tries to test the HP law for 80% colophony with $\eta=10^8$Pa s. For a pipe of R=4.9mm and l=105mm, he finds the law valid for all pressures (1.3×10^4 to 2.13×10^5 Pa s). For pipes of R=4.9mm, p=1925hPa and T=11.5°C he finds the HP law valid for all measured lengths (l=24mm to 206mm). It seems to him for such high values of η no lower length limit exists.

Operating at T=11.7°C and p=1925hPa he found (for 80% colophony) the HP law valid even for a radius as big as 15.2mm. At such high viscosities no turbulence developed. Glaser also tried to find a lower radial limit. For lengths around 150 to 250mm he found lower radial limits of R=1mm ($\eta=10^8$Pas), R=5mm ($\eta=10^{10}$Pas) and R=10mm ($\eta=10^{12}$Pas). Down to these radii the HP law proved valid, below them, η increased rapidly and the flow eventually stopped. Glaser further conducted experiments just at the lower radius limit and just below it at a low pressure of only 251hPa. After waiting up to two days, he found the HP law again valid. As an explanation for this strong radius dependence of η, he quoted Maxwell's relaxation hypothesis [34]:

$$\eta' = \frac{\eta}{1-\frac{a}{R}},$$

where η' is the viscosity to be used in the HP law for pipes of radius R, and *a* is a constant depending on the elastic properties of wall and liquid, and on dimensions of the wall. η is the true viscosity.

Through his experiments Glaser confirmed the HP law even for very high viscosities. It may be of interest to know that Glaser was a student of Wiedemann as well.

L. Schiller edited and commented three works of Hagen, Poiseuille and Hagenbach in 1933 [2]. His comments contain valuable corrections and information. He emphasizes the importance of Hagen's work. Maybe because of his time and because before him people like Hagenbach had called the HP law simply Poiseuille's law, omitting the name of Hagen. Schiller's own works are referred to in [2].

VI. Conclusion

This review concentrated on selected historical papers concerned with the HP flow. After giving some biographical background information about Hagen, Poiseuille and Hagenbach, the original experiments were shown. Then I presented the original data of Hagen and of Poiseuille in the form of diagrams. Following this I presented the main ideas of Hagen, Poiseuille, Hagenbach and a few other early researchers. The major achievements in this field seem all to have been arrived at during the course of the 44 years between the discoveries of Hagen and Poiseuille and the successful distinction of laminar and turbulent flow by O. Reynolds in 1883. The HP law keeps to be fundamental for many aspects of our daily life as well as for future technological developments. The laws of laminar flow are nowadays well understood, whereas the theory of turbulence is yet an open and vibrant field of research.

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