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Sequences of pyramidal numbers[1]

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Abstract Shyam Sunder Gupta [4] has defined Smarandache consecutive and reversed Smarandache sequences of Triangular numbers. Delfim F.M.Torres and Viorica Teca [1] have further investigated these sequences and defined mirror and symmetric Smarandache sequences of Triangular numbers making use of Maple system. One of the authors A.S.Muktibodh [2] working on the same lines has defined and investigated consecutive, reversed, mirror and symmetric Smarandache sequences of pentagonal numbers of dimension 2 using the Maple system. In this paper we have defined and investigated the s-consecutive, s-reversed, s-mirror and s-symmetric sequences of Pyramidal numbers (Triangular numbers of dimension 3.) using Maple 6.

§1. Introduction

Figurate number is a number which can be represented by a regular geometrical arrangement of equally spaced points. If the arrangement forms a regular polygon the number is called a polygonal number. Different figurate sequences are formed depending upon the dimension we consider. Each dimension gives rise to a system of figurate sequences which are infinite in number.

In this paper we consider a figurate sequence of Triangular numbers of dimension 3, also called as Pyramidal numbers.

The nth Pyramidal number $t_n, n \in N$ is defined by:

$$t_n = \frac{n(n+1)(n+2)}{6}$$

We can obtain the first k terms of Pyramidal numbers in Maple as;

```
> t:= n->(1/6)*n*(n+1)*(n+2):
> first := k -> seq (t(n), n=1...20):
> first(20);

  1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364, 455, 560, 680,
        816, 969, 1140, 1330, 1540
```

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For constructing Smarandache sequence of Pyramidal numbers we use the operation of concatenation on the terms of the above sequence.This operation is defined as ;

```
> conc :=(n,m)-> n*10^length(m)+m:
```

We define Smarandache consecutive sequence $\{scs_n\}$ for Pyramidal numbers recursively as;
$scs_1 = u_1$,
$scs_n = conc(scs_{n-1}, u_n)$
Using Maple We have obtained first 20 terms of Smarandache consecutive sequence of Pyramidal numbers;

```
>conc :=(n,m)-> n*10^length(m)+m:
> scs_n := (u,n)-> if n = 1 then u(1)else conc(scs_n(u,n-1),u(n))fi:
> scs := (u,n)-> seq (scs_n(u,i),i=1...n):
> scs(t,20);
```

```
1, 14, 1410, 141020, 14102035, 1410203556, 141020355684,
      141020355684120, 141020355684120165, 141020355684120165220,
      141020355684120165220286, 141020355684120165220286364,
      141020355684120165220286364455,
      141020355684120165220286364455560,
      141020355684120165220286364455560680,
      141020355684120165220286364455560680816,
      141020355684120165220286364455560680816969,
      1410203556841201652202863644555606808169691140,
      14102035568412016522028636445556068081696911401330,
      141020355684120165220286364455560680816969114013301540
```

Display of the same sequence in the triangular form is;

```
> show := L -> map(i ->print(i),L):
> show([scs(t,20)]);
```

```
1
14
1410
141020
14102035
1410203556
141020355684
141020355684120
141020355684120165
141020355684120165220
141020355684120165220286
```

```
14102035568412016522028636
14102035568412016522028636
```

$$\begin{gathered}
\texttt{141020355684120165220286364} \\
\texttt{141020355684120165220286364455} \\
\texttt{141020355684120165220286364455560} \\
\texttt{141020355684120165220286364455560680} \\
\texttt{141020355684120165220286364455560680816} \\
\texttt{141020355684120165220286364455560680816969} \\
\texttt{1410203556841201652202863644555606808169691140} \\
\texttt{14102035568412016522028636445556068081696911401330} \\
\texttt{141020355684120165220286364455560680816969114013301540}
\end{gathered}$$

The reversed Smarandache sequence (rss)associated with a given sequence $\{u_n\}, n \in N$ is defined recursively as:

$$rss_1 = u_1,$$
$$rss_n = conc(u_n, rss_{n-1}).$$

In Maple we use the following program;

```
> rss_n :=(u,n) -> if n=1 then u(1) else conc(u(n),rss_n(u,n-1)) fi:
> rss := (u,n) -> seq(rss_n(u,i),i=1..n):
```

We get the first 20 terms of reversed smarandache sequence of Pyramidal numbers as;

```
> rss(t,20);

  1, 41, 1041, 201041, 35201041, 5635201041, 845635201041,
        120845635201041, 165120845635201041, 220165120845635201041,
        286220165120845635201041, 364286220165120845635201041,
        455364286220165120845635201041,
        560455364286220165120845635201041,
        680560455364286220165120845635201041,
        816680560455364286220165120845635201041,
        969816680560455364286220165120845635201041,
        1140969816680560455364286220165120845635201041,
        13301140969816680560455364286220165120845635201041,
        154013301140969816680560455364286220165120845635201041
```

Smarandache Mirror Sequence (sms)is defined as follows:

$$sms_1 = u_1,$$
$$sms_n = conc(conc(u_n, sms_{n-1}), u_n).$$

The following program gives first 20 terms of Smarandache Mirror sequence of Pyramidal numbers.

```
> sms_n := (u,n) -> if n=1 then
> u(1)
```

```
> else
> conc(conc(u(n),sms_n(u,n-1)),u(n))
> fi:
> sms :=(u,n) ->seq(sms_n(u,i), i=1..n):
> sms(t,20);

  1, 414, 1041410, 20104141020, 352010414102035, 5635201041410203556,
        845635201041410203556 84, 120845635201041410203556 84120,
        16512084563520104141020355684120165,
        220165120845635201041410203556841201 65220,
        286220165120845635201041410203556841201 65220286,
        364286220165120845635201041410203556841201 65220286364,
        455364286220165120845635201041410203556841201 65220286364455,
        56045536428622016512084563520104141020355684120165 22028636\
        4455560, 6805604553642862201651208456352010414102035568412 0\
        165220286364455560680, 8166805604553642862201651208456352 01\
        0414102035568412016522028636445556068 0816, 969816680560455 3\
        6428622016512084563520104141020355684120165220286364455560\
        680816969, 11409698166805604553642862201651208456352010414 1\
        0203556841201652202863644555606808169691140, 1330114096981 6\
        68056045536428622016512084563520104141020355684120165220 28\
        636445556068081696911401330, 154013301140969816680560455364\
        2862201651208456352010414102035568412016522028636445556068\
        0816969114013301540
```

Finally Smarandache Symmetric sequence (sss) is defined as:

$$sss_{2n-1} = conc(bld(scs_{2n-1}), rss_{2n-1}),$$
$$sss_{2n} = conc(scs_{2n}, rss_{2n}), n \in N,$$

where the function "bld" (But Last Digit) is defined in Maple as

```
> bld := n->iquo(n,10):
```

First 20 terms of Smarandache Symmetric sequence are obtained as

```
> bld :=n-> iquo(n,10):
> conc := (n,m)-> n*10^length(m)+m:
> sss_n := (u,n) -> if type(n,odd) then
> conc(bld(scs_n(u,(n+1)/2)),rss_n(u,(n+1)/2))
> else
> conc(scs_n(u,n/2),rss_n(u,n/2))
> fi:
> sss := (u,n) -> seq(sss_n(u,i), i=1..n):
> sss(t,20);
```

```
1, 11, 141, 1441, 1411041, 14101041, 14102201041, 141020201041,
      141020335201041, 1410203535201041, 1410203555635201041,
      14102035565635201041, 14102035568845635201041,
      141020355684845635201041, 1410203556841212084563520104l,
      14102035568412012084563520104l,
      1410203556841201616512084563520104l,
      14102035568412016516512084563520104l,
      14102035568412016522220165120845635201041,
      141020355684120165220222016512084563520104l
```

We find out primes from a large (first 500) terms of various Smarandache sequences defined so far. We have used Maple 6 on Pentium 3 with 128Mb RAM. We first collect the lists of first 500 terms of the consecutive, reversed, mirror and symmetric sequences of Pyramidal numbers;

```
> st :=time(): Lscs500:=[scs(t,500)]: printf("%a seconds",round(time()-st));
15 seconds
> st :=time(): Lrss500:=[rss(t,500)]: printf("%a seconds",round(time()-st));
20 seconds
> st :=time(): Lsms500:=[sms(t,500)]: printf("%a seconds",round(time()-st));
58 seconds
> st :=time(): Lsss500:=[sss(t,500)]: printf("%a seconds",round(time()-st));
12 seconds
```

Further we find the number of digits in the 500th term of each sequence.

```
> length(Lscs500[500]),length(Lrss500[500]);

                              3283, 3283

> length(Lsms500[500]),length(Lsss500[500]);

                              6565, 2846
```

There exist no prime in the first 500 terms of Smarandache consecutive sequence of Pyramidal numbers;

```
> st:= time():select(isprime,Lscs500);

                                  []

> printf("%a minutes",round((time()-st)/60));
9 minutes
```

There is only one prime in the first 500 terms of reversed Smarandache sequence of Pyramidal numbers;

```
> st:= time():
> select(isprime,Lrss500);

                              [41]

> printf("%a minutes",round((time()-st)/60));
119 minutes
```

There is no prime in the first 500 terms of Smarandache mirror sequence;

```
> st:= time():
> select(isprime,Lsms500);
> printf("%a minutes",round((time()-st)/60));

                              []

177 minutes
```

There is only one prime in the first 500 terms of Smarandache symmetric sequence;

```
> st:= time():
> select(isprime,Lsss500);

                              [11]

> printf("%a minutes",round((time()-st)/60));
90 minutes
```

§2. Open problems

1) How many Pyramidal numbers are there in the first 500 terms of Smarandache consecutive, mirror, symmetric and reverse symmetric sequences of Pyramidal numbers ?

2) What are those numbers ?

References

[1] Delfim F.M., Viorica Teca, Consective, Reversed, Mirror and Symmetric Smarandache Sequences of Triangular Numbers, Scientia Magna, **1**(2005), No.2, 39-45.

[2] Muktibodh A.S., Smarandache Sequences of Pentagonal Numbers, Scientia Magna, **2**(2006), No.3.

[3] Muktibodh A.S., Figurate Systems, Bull. Marathwada Mathematical Soc., **6**(2005), No.1, 12-20.

[4] Shyam Sunder Gupta, Smarandache Sequence of Triangular Numbers, Smarandache Notions Journal, **14**, 366-368.