Chains of Smarandache Semifields

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Abstract
In this paper we have constructed two chains of semifields. All semifields in the
chains are Smarandache semifields. Every member of the chain is an extension
semifield of Ordered equilateral Integral triangles with Zero triangle such that it is a
semivector space over \( R_e \).

Key words: Ordered integral triangle, Zero triangle, Equilateral integral triangle
Smarandache semiring, Smarandache semifield, Smarandache semivector space.

1. Introduction
Recently there has been an increasing interest in the study of Smarandache semirings
and associated structures. We propose to construct two chains of infinite Smarandache
semifields by defining Equilateral triangles.

An ordered integral triangle as defined in [1] is a triplet \((a,b,c)\) where \((a,b,c)\) are
positive integers satisfying \( a \geq b \geq c, b + c > a \).

Let us consider a set \( R'_e = \{(a,b,c) | a,b,c \in \mathbb{I}^+, a \geq b \geq c, b + c > a \} \cup \{0\} \) where
0 = (0,0,0). We shall call 0 as a Zero triangle.

We define the sum + and the product \((\cdot)\) of triangles as
\[
(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)
\]
and
\[
(a_1, b_1, c_1)(a_2, b_2, c_2) = (a, b, c)
\]
where
\[
a = \Sigma a_i a_j - (b_i c_j + c_i b_j)
\]
\[
b = \Sigma a_i a_j - (a_i c_j + c_i a_j)
\]
\[
c = \Sigma a_i a_j - (a_i b_j + b_i a_j)
\]
\[ \sum a_ia_2 = a_1a_2 + b_1b_2 + c_1c_2 \]

It is not difficult to see that:

i) \((R, +, \cdot)\) is a commutative semigroup with identity \((0,0,0)\).

ii) \((R, \cdot)\) is a semigroup (in fact a monoid).

iii) Multiplication distributes over addition.

iv) \((1,1,1)\) is the multiplicative identity.

v) Commutativity holds for multiplication.

Thus, \((R, +, \cdot)\) is a commutative semiring.

Also,

vi) \((a_1, b_1, c_1) + (a_2, b_2, c_2) = (0,0,0) \Rightarrow a_1 = a_2 = b_1 = b_2 = c_1 = c_2 = 0\)

Thus, \((R, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\). See [2].

vii) Let \(x, y = 0\) where \(x, y \in R\). Then \(x = 0\) or \(y = 0\)

We conclude that

- **A1** \((R, +, \cdot)\) is a semifield.

A triplet \((a,b,c)\) where \((a,b,c)\) are positive rational numbers satisfying \(a \geq b \geq c, b + c > a\) is called an ordered rational triangle.

Consider the following set

\[ R^\circ = \{(a,b,c)/a,b,c \in Q^+, a \geq b \geq c, b + c > a\} \cup \{0\} \]

where \(0\) is a zero triangle. Then, it can be verified that \((R^\circ, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\).

Also, \(R^\circ\) is without zero divisors.

Thus,

- **A2** \((R^\circ, +, \cdot)\) is a semifield.

A triplet \((a,b,c)\) where \((a,b,c)\) are positive real numbers satisfying \(a \geq b \geq c, b + c > a\) is called an ordered real triangle.

Consider the set

\[ R^\circ = \{(a,b,c)/a,b,c \in R^+, a \geq b \geq c, b + c > a\} \cup \{0\} \]

where \(0\) is a zero triangle. Then, it can be verified that \((R^\circ, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\).

Also, \(R^\circ\) is without zero divisors.

Thus,

- **A3** \((R^\circ, +, \cdot)\) is a semifield.

Consider the set,

\[ R^\circ = \{(a,b,c)/a,b,c \in R^+, a \geq b \geq c\} \cup \{0\} \]

where \(0\) is a zero triangle. Then, it can be verified that \((R^\circ, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\).

Also, \(R^\circ\) is without zero divisors.

Thus,

- **A4** \((R^\circ, +, \cdot)\) is a semifield.

Result: From A1, A2, A3 and A4 we obtain a chain of semifields as

- **(A)** \(R^\circ \supset R^\circ \supset R^\circ \supset R^\circ\)

Where \(R^\circ\) is a real equilateral triangle defined in (A17)

Ordered equilateral triangles lead us to a new chain of semifields. A triplet \((a,a,a)\) where \(a \in R^+\) is called an ordered equilateral real triangle.
Consider the following set
\[ \mathbb{R}^R_+ = \{(a,a,a)/a \in \mathbb{R}^+ \} \cup \{0\} \] where \(0 = (0,0,0)\).
Then, \((\mathbb{R}^R_+, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\) and is without zero divisors.
Thus,

- **A5** \((\mathbb{R}^R_+, +, \cdot)\) is a semifield.

Similarly, triplet \((a,a,a)\) where \(a \in Q^+\) is called an ordered equilateral rational triangle.
Consider the following set
\[ \mathbb{R}^Q_+ = \{(a,a,a)/a \in \mathbb{Q}^+ \} \cup \{0\} \] where \(0 = (0,0,0)\).
Then, \((\mathbb{R}^Q_+, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\) and is without zero divisors.
Thus,

- **A6** \((\mathbb{R}^Q_+, +, \cdot)\) is a semifield.

Similarly, a triplet \((a,a,a)\) where \(a \in I^+\) is called an ordered equilateral Integral triangle.
Consider the following set
\[ \mathbb{R}^I_+ = \{(a,a,a)/a \in \mathbb{I}^+ \} \cup \{0\} \] where \(0 = (0,0,0)\).
Then, \((\mathbb{R}^I_+, +, \cdot)\) is a strict commutative semiring with unity \((1,1,1)\) and is without zero divisors.
Thus,

- **A7** \((\mathbb{R}^I_+, +, \cdot)\) is a semifield.

Result: From A1, A2, A5, A6 and A7 we obtain a chain of semifields as

**(B)** \(\mathbb{R}_+ \supset \mathbb{R}^R_+ \supset \mathbb{R}^Q_+ \supset \mathbb{R}^I_+\)

2. Some Observations

1. Members of ordered equilateral triangles act as scalar multiples for every semifield in the chain.
   E.g. let \((a,a,a) \in \mathbb{R}^R_+\) and \((x,y,z) \in \mathbb{R}_+\). Then
   \[ (a,a,a)(x,y,z) = (ax,ay,az) = a(x,y,z). \]
   Thus, multiplication by \((a,a,a) \in \mathbb{R}^R_+\) amounts to component wise multiplication. Hence, we call \((a,a,a)\) a magnifier.

2. There is a chain of magnifiers
   \(\mathbb{R}_+ \supset \mathbb{R}^R_+ \supset \mathbb{R}^Q_+ \supset \mathbb{R}^I_+\)

3. Every semifield in the chains (A) and (B) is of characteristic \(0\).

4. Every semiring except \(\mathbb{R}^I_+\) in chains (A) and (B) is a Smarandache semiring.

5. Every semifield in the chains (A) and (B) is an extension semifield of \(\mathbb{R}^I_+\).

6. \(\mathbb{R}^I_+\) is a prime semifield as it has no proper subsemifield.

7. All the members in the chains are semivector spaces over the semifield \(\mathbb{R}^I_+\).

8. All the semifields in the chains (A) and (B) are Smarandache semi fields because they contain \(\mathbb{A}\) as a proper subset where \(\mathbb{A}\) is
   a. \(\mathbb{A} = \{(0,0,0),(p,p,p),(2p,2p,2p),...(rp,rp,rp)\}\) which is isomorphic
with \( A' = \{0, p, 2p, \ldots, rp, \ldots\} \) which is a k-semi algebra [2].

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