A possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes

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Abstract. I was studying the Poulet numbers of the form n*p - n + 1, where p is prime, numbers which appear often related to Fermat pseudoprimes (see the sequence A217835 that I submitted to OEIS) when I discovered a possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes (I pointed out 4 such Poulet numbers).

It is known the following relation between the Fermat pseudoprimes to base 2 (Poulet numbers) and the Wieferich primes: the squares of the two known Wieferich primes, respectively 1194649 = 1093^2 and 12327121 = 3511^2, are Poulet numbers. I discovered yet another relation between these two classes of numbers:

Conjecture 1: For every Wieferich prime p there is an infinity of Poulet numbers which are equal to n*p – n + 1, where n is integer, n > 1.

Note: Because there are just two Wieferich primes known (it's not even known if there are other Wieferich primes beside these two), we verify the conjecture for these two and few values of n (until n < 31).

: 1093*3 – 2 = 3277, a Poulet number;
: 1093*4 - 3 = 4369, a Poulet number;
: 1093*5 - 4 = 5461, a Poulet number;

: 3511*14 – 13 = 49141, a Poulet number.

Observation 1: The formula n*p – n + 1, where p is Wieferich prime and n is integer, n > 1, leads often to semiprimes of the form q*(m*q – m + 1) or of the form q*(m*q + m – 1):

: 1093*11 – 10 = 5*2621 and 2621 = 5*655 - 654;
: 3511*4 – 3 = 19*739 and 739 = 19*41 – 40;
: 3511*9 – 8 = 7*4593 and 4593 = 7*752 – 751;
: 3511*10 – 9 = 11*3191 and 3191 = 11*319 – 318;
: 3511*12 – 11 = 73*577 and 577 = 73*8 – 7;
: 3511*14 – 13 = 157*313 and 313 = 157*2 – 1;

: 3511*21 – 20 = 11*6701 and 6701 = 11*670 - 669;
: 3511*24 – 23 = 61*1381 and 1381 = 61*23 - 22;
: 3511*28 – 27 = 29*3389 and 3389 = 29*121 - 120;

: 1093*11 – 10 = 41*293 and 293 = 41*7 + 6;
: 1093*18 – 17 = 11*1787 and 1787 = 11*149 + 148;
: 1093*29 – 28 = 11*2879 and 2879 = 11*240 + 239;
: 3511*4 – 3 = 19*739 and 739 = 19*37 + 36;
: 3511*19 – 18 = 17*3923 and 3923 = 17*218 + 217;
: 3511*31 – 30 = 233*467 and 467 = 233*2 + 1;
: 3511*28 – 27 = 29*3389 and 3389 = 29*113 + 112.

Note: Every Poulet number obtained so far through the formula above (until n < 31) is semiprime, in other words a 2-Poulet number.

Note: The class of primes p that can be written in both ways, like p = n*q – n + 1 and like m*q + m – 1, where q is prime and m and n are integers larger than 1, seems to be interesting to study. Such primes p are, for instance, 739 = 19*41 – 40 = 19*37 + 36 and 3389 = 29*121 – 120 = 29*113 + 112. Maybe is not a coincidence that both pairs of primes (p,q) are of the form (10k + 9,10h + 9).

Observation 2: Most of the 2-Poulet numbers (for a list with Fermat pseudoprimes to base 2 with two prime factors see the sequence A214305 in OEIS) can be written as d*(d*n – n + 1) or as d*(d*n + n – 1), where d is obviously one of the two prime factors and n is integer, n > 1: for instance 341 = 11*31 = 11*(11*3 – 2) and 1387 = 19*73 = 19*(19*4 – 3). But not all 2-Poulet numbers can be written in one of these two ways: for instance 23377 = 97*241, the 18th 2-Poulet number, can’t be written this way.

Observation 3: I also noticed that two semiprimes obtained from the Wieferich primes through the formula above can be written as q*(q*38 + 17):

: 14041 = 19*739 = 19*(19*38 + 17);
: 52651 = 37*1423 = 37*(37*38 + 17).

Note: That would be also interesting to study the pairs of primes (p,38*p+17); such pairs of primes are, for instance, (7,283), (19,739), (37,1423), (73,2791), (79,3019), (103,3931).